Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Robust filtering for Itô stochastic Systems subject to sensor nonlinearities

Daniel W. C. Ho * **Yugang Niu** ** **C. W. Li** ***

* *Department of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong. E-mail: madaniel@cityu.edu.hk*
** *School of Information Science & Engineering, East China University of Science & Technology, Shanghai, 200237, P.R.C. E-mail:acniuyg@ecust.edu.cn.*
*** *Department of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong. E-mail: macwli@cityu.edu.hk*

Abstract: This paper will deal with the filtering problem for uncertain stochastic systems subject to sensor nonlinearities. There exist time-varying parameter uncertainties, and state and external-disturbance-dependent noise. The robust filters are constructed for Itô stochastic systems, and sufficient conditions are obtained such that the filtering error systems are robustly stochastically stable with a prescribed disturbance attenuation level despite sensor nonlinearities and all admissible uncertainties. A simulation example illustrating the proposed method is given.

Keywords: Filtering, stochastic systems, sensor nonlinearities, external disturbance, disturbance attenuation.

1. INTRODUCTION

In many industrial and commercial applications, measurements are often made under harsh environments that include both uncontrollable elements (e.g., variations in flow rates, temperatures, etc.) and aggressive conditions (e.g., corrosion, erosion, and fouling) [1]. These factors may yield significant limitations on various aspects of sensor performance, e.g., the range limitation that will result in the nonlinear characteristic of sensors. Besides, for some sensors, e.g., some accelerometers, temperature sensors and strain gauge, nonlinearity is also an inevitable feature. Moreover, it is known that in the image restoration, a practical image sensor usually has a nonlinear characteristic and has attracted a lot of attentions [2]. Hence, the study on the systems with sensor nonlinearities carries a great deal of interest and practical importance [3, 4, 5]. In this work, it will be shown (see Figure 5) that the filtering problem of systems with sensor nonlinearities cannot be effectively solved by the filter without considering the nonlinear characteristic of sensor.

On the other hand, the filtering problem for the Itô stochastic systems have been recently attracting an increasing attention, e.g., in [6, 7, 8, 9, 10], since the stochastic system design governed by Itô differential equations has extensive applications in practice [11]. However, it is worth noting that most of the aforementioned works are discussed only for the case of *linear* sensors. To the authors' best knowledge, up to now, the filtering problem for the Itô stochastic system subject to sensor nonlinearities is not well addressed. Especially, these results in the aforementioned works cannot be directly utilized to deal with the stochastic systems with sensor nonlinearities.

Motivated by the above discussions, this paper will be concerned with the filtering problem for uncertain Itô stochastic systems subject to sensor nonlinearities. In the systems under consideration, there exist time-varying parameter uncertainties, and state and external-disturbance-dependent noise. The robust filters are designed. By means of stochastic Lyapunov method, and sufficient conditions are obtained such that the resultant filtering error systems are robustly stochastically stable with a prescribed H_∞-disturbance attenuation performance despite sensor nonlinear and all admissible uncertainties. Very often, the external disturbance in real engineering systems may come from a deterministic source or a stochastic one (e.g., Brownian motion). Hence, both deterministic and stochastic disturbance signals are considered, respectively, in this simulation.

Notations: Throughout the paper, $|\cdot|$ is the standard Euclidean vector norm. For symmetric matrix, the notation $M > 0$ (< 0) is used to denote a positive definite (negative definite, respectively). $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the corresponding matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. THE DESIGN OF ROBUST FILTERING

In this section, we consider the uncertain stochastic systems modeled by the following Itô stochastic differential equation:

$$\begin{aligned}(\Sigma_c) : dx(t) = {} & [(A + \Delta A(t))x(t) + (B + \Delta B(t)) \\ & \times v(t)]\, dt + [(E + \Delta E(t))x(t) \\ & + (G + \Delta G(t))v(t)]\, dw(t), \qquad (1)\\ y(t) = {} & \phi(Cx(t)) + (D + \Delta D(t))v(t), \qquad (2)\end{aligned}$$

$$z(t) = Lx(t), \tag{3}$$

where $x(t) \in \mathbf{R^n}$ is the state; $y(t) \in \mathbf{R^q}$ is the output; $z(t) \in \mathbf{R^r}$ is the state combination to be estimated; $w(t)$ is a standard one-dimensional Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ relative to an increasing family $(\mathcal{F}_t)_{t>0}$ of σ-algebras $\mathcal{F}_t \subset \mathcal{F}$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$; $v(t) \in \mathbf{R^p}$ is an unknown exogenous stochastic disturbance signal. It is assumed that $v(t)$ is adapted and measurable processes with respect to $\mathcal{F}_t$, and belongs to $L_{E_2}([0,\infty);\mathbf{R^p})$, where $L_{E_2}([0,\infty);\mathbf{R^p})$ denotes the space of nonanticipatory square-integrable stochastic process $f(\cdot) = (f(t))_{t\in[0,\infty)}$ on $\mathbf{R^p}$ with respect to $(\mathcal{F}_s)_{t\in[0,\infty)}$ satisfying

$$||f||_{E2}^2 = \mathcal{E}\left\{\int_0^\infty |f(t)|^2 dt\right\} < \infty$$

with $\mathcal{E}\{\cdot\}$ the mathematical expectation.

In (1)–(3), A, B, E, G, C, D, L are known real constant matrices of appropriate dimensions; $\Delta A(t)$, $\Delta B(t)$, $\Delta E(t)$, $\Delta G(t)$ and $\Delta D(t)$ are unknown matrices representing time-varying parameter uncertainties with the following form,

$$[\,\Delta A(t), \Delta E(t)\,] = [\,M_A, M_E\,]\, F_1(t) N_1, \tag{4}$$

$$[\,\Delta B(t), \Delta G(t), \Delta D(t)\,] = [\,M_B, M_G, M_D\,]\, F_2(t) N_2 \tag{5}$$

where M_A, M_B, M_E, M_G, M_D, N_1 and N_2 are known real constant matrices, and $F_i(t)$ $(i = 1, 2)$ are unknown matrix functions with Lebesgue-measurable elements and satisfy

$$F_i^T(t) F_i(t) \le I, \forall t. \tag{6}$$

The parameter uncertainties $\Delta A(t)$, $\Delta B(t)$, $\Delta E(t)$, $\Delta G(t)$ and $\Delta D(t)$ are said to be admissible if expressions (4)–(6) hold.

As pointed out in Introduction, many actual applications will inevitably result in the nonlinear characteristic of sensors. Hence, the function $\phi(\cdot)$ in system (Σ_c) is assumed to belong to $[K_1,\, K_2]$, for some given diagonal matrices $K_1 \ge 0$ and $K_2 \ge 0$ with $K_2 > K_1$, and satisfies the following sector condition:

$$(\phi(u) - K_1 u)^T (\phi(u) - K_2 u) \le 0, \forall u \in \mathbf{R^q}. \tag{7}$$

Remark 2.1. It should be pointed out that the class of nonlinear functions satisfying (7) is general in practical application [12]. Moreover, the characteristics of the sensor directly depend on the function $\phi_s(u)$. As a special case, when $\phi_s(u)$ is a linear function, the sensor will reduce to a linear one.

For the stochastic system (Σ_c), we are concerned with obtaining the estimation $\hat{z}(t)$ of $z(t)$. To this end, we construct the following filter of order n

$$(\mathcal{F}_c) : d\hat{x}(t) = A_f x(t) dt + B_f y(t) dt, \tag{8}$$

$$\hat{z}(t) = L\hat{x}(t), \tag{9}$$

where $\hat{x}(t) \in \mathbf{R^n}$ and $\hat{z}(t) \in \mathbf{R^r}$, the matrices A_f and B_f are to be determined. And then, the above robust filtering problem can be stated as follows:

Robust Filtering (**RF**): Given a disturbance attenuation level $\gamma > 0$, the parameters A_f and B_f of filter (8)–(9) are designed such that the resultant filtering error system is asymptotically stable in probability for $v(t) = 0$ and any $\phi \in [K_1,\, K_2]$, and satisfies $||z - \hat{z}||_{E_2} < \gamma ||v||_{E_2}$ under zero initial conditions for all $v(t) \in L_{E_2}([0,\infty); R^p)$.

Remark 2.2. In the conventional design of filtering, the filtering error dynamics need firstly be constructed. However, the existence of nonlinear sensor (2) complicates the derivation of the error systems. In order to deal with the difficulty, a decomposition (10) of the nonlinear function $\phi(u)$ will be introduced. Moreover, it will be observed that this decomposition (10) plays an important role for the development of filter design later.

Now, we decompose the nonlinear function $\phi(u)$ as follows:

$$\phi(u) = \phi_s(u) + K_1 u, \tag{10}$$

where the nonlinearity $\phi_s(u)$ belongs to the set Φ_s given by

$$\Phi_s = \left\{\phi_s : \phi_s(u)^T (\phi_s(u) - Ku) \le 0\right\} \tag{11}$$

with $K = K_2 - K_1 > 0$.

Thus, it follows from the system (Σ_c) and filter $(\mathcal{F}_c)$ that:

$$\begin{aligned} d\tilde{x}(t) = \{&A_f \tilde{x}(t) + [A + \Delta A(t) - A_f - B_f K C] \\ &\times x(t) - B_f \phi_s(u) \\ &+ [B + \Delta B(t) - B_f (D + \Delta D(t)) v(t)]\}\, dt \\ &+ [(E + \Delta E(t)) x(t) + (G + \Delta G(t)) v(t)] \\ &\times dw(t), \end{aligned} \tag{12}$$

where $\tilde{x}(t) = x(t) - \hat{x}(t)$, and $u = Cx(t)$.

Define $e(t) = \begin{bmatrix} x(t)^T & \tilde{x}(t)^T \end{bmatrix}^T$, and $\tilde{z}(t) = z(t) - \hat{z}(t)$, then we obtain the filtering error dynamics as follows

$$\begin{aligned} (\Sigma_{ce}) : de(t) = &\left[(\bar{A} + \Delta\bar{A}(t)) e(t) + \bar{B}_f \phi_s(Cx(t)) \right. \\ &\left. + (\bar{B} + \Delta\bar{B}(t)) v(t)\right] dt \\ &+ \left[(\bar{E} + \Delta\bar{E}(t)) e(t) + (\bar{G} + \Delta\bar{G}(t)) \right. \\ &\left. \times v(t)\right] dw(t), \end{aligned} \tag{13}$$

$$\tilde{z}(t) = \bar{L} e(t), \tag{14}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ A - A_f - B_f K_1 C & A_f \end{bmatrix},$$

$$\bar{E} = \begin{bmatrix} E & 0 \\ E & 0 \end{bmatrix}, \bar{B}_f = \begin{bmatrix} 0 \\ -B_f \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B \\ B - B_f D \end{bmatrix}, \bar{G} = \begin{bmatrix} G \\ G \end{bmatrix},$$

$$\Delta\bar{A}(t) = \bar{M}_A F_1(t) \bar{N}_1, \quad \Delta\bar{B}(t) = \bar{M}_B F_2(t) N_2,$$

$$\Delta\bar{E}(t) = \bar{M}_E F_1(t) \bar{N}_1, \quad \Delta\bar{G}(t) = \bar{M}_G F_2(t) N_2,$$

$$\bar{M}_A = \begin{bmatrix} M_A \\ M_A \end{bmatrix}, \bar{M}_E = \begin{bmatrix} M_E \\ M_E \end{bmatrix},$$

$$\bar{M}_B = \begin{bmatrix} M_B \\ M_B - B_f M_D \end{bmatrix}, \bar{M}_G = \begin{bmatrix} M_G \\ M_G \end{bmatrix},$$

$$\bar{N}_1 = [\, N_1 \;\; 0 \,], \bar{L} = [\, 0 \;\; L \,].$$

Next, we shall provide a solution to **RF** problem.

Theorem 2.1. Consider the uncertain stochastic systems (Σ_c). For given disturbance attenuation level $\gamma > 0$, if there exist matrices $X > 0$, $Y > 0$, W, and S, and scalars $\varepsilon_1 > 0$, and $\varepsilon_2 > 0$ satisfying the following LMI:

$$\begin{bmatrix} \Theta_{c1} & * & * & * & * & * & * & * \\ \Theta_{c3}^T & \Theta_{c4} & * & * & * & * & * & * \\ B^T X & B^T Y - D^T S^T & \Theta_{c2} & * & * & * & * & * \\ KC & -S^T & 0 & -2I & * & * & * & * \\ XE & 0 & XG & 0 & -X & * & * & * \\ YE & 0 & YG & 0 & 0 & -Y & * & * \\ M_A^T X & M_A^T Y & 0 & 0 & M_E^T X & M_E^T Y & -\varepsilon_1 I & * \\ M_B^T X & M_B^T Y - M_D^T S^T & 0 & 0 & M_G^T X & M_G^T Y & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{15}$$

with

$$\Theta_{c1} = XA + A^T X + \varepsilon_1 N_1^T N_1, \tag{16}$$

$$\Theta_{c2} = -\gamma^2 I + \varepsilon_2 N_2^T N_2, \tag{17}$$

$$\Theta_{c3} = A^T Y - W^T - C^T K_1^T S^T, \tag{18}$$

$$\Theta_{c4} = W + W^T + L^T L, \tag{19}$$

then, the robust H_∞ filtering problem is solved by the filter $(\mathcal{F}_c)$. In this case, the parameters of the desired filter $(\mathcal{F}_c)$ are given as

$$A_f = Y^{-1} W, B_f = Y^{-1} S. \tag{20}$$

Proof: Firstly, we establish the robustly stochastic stability of the filtering error system (Σ_{ce}) under the condition of Theorem 2.1. It can be shown that the LMI (15) implies

$$\begin{bmatrix} \Theta_{c1} & * & * & * & * & * \\ \Theta_{c3}^T & \Theta_{c5} & * & * & * & * \\ KC & -S^T & -2I & * & * & * \\ XE & 0 & 0 & -X & * & * \\ YE & 0 & 0 & 0 & -Y & * \\ M_A^T X & M_A^T Y & 0 & M_E^T X & M_E^T Y & -\varepsilon_1 I \end{bmatrix} < 0. \tag{21}$$

with $\Theta_{c5} = W + W^T$.

Now, for the filtering error system (13) with $v(t) = 0$, select the stochastic Lyapunov functional candidate as

$$V(e(t), t) = e(t)^T \bar{P} e(t) \tag{22}$$

with

$$\bar{P} = \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}, \tag{23}$$

where $X > 0$ and $Y > 0$ satisfy (21).

By utilizing Itô's formula, we obtain the infinitesimal generator of $V(e(t), t)$ as

$$\begin{aligned} &\mathcal{L}V(e(t), t) \\ &= 2e(t)^T \bar{P} \left[(\bar{A} + \Delta\bar{A}(t))e(t) + \bar{B}_f \right. \\ &\quad \left. \times \phi_s(Cx(t)) \right] + e(t)^T (\bar{E} + \Delta\bar{E}(t))^T \\ &\quad \times \bar{P}(\bar{E} + \Delta\bar{E}(t))e(t) \\ &\le \left[\, e(t)^T \;\; \phi_s(Cx(t))^T \,\right] \Theta_c \\ &\quad \times \left[\, e(t)^T \;\; \phi_s(Cx(t))^T \,\right]^T, \end{aligned} \tag{24}$$

where

$$\Theta_c = \begin{bmatrix} \bar{P}(\bar{A} + \Delta\bar{A}(t)) + (\bar{A} + \Delta\bar{A}(t))^T \bar{P} \\ +(\bar{E} + \Delta\bar{E}(t))^T \bar{P}(\bar{E} + \Delta\bar{E}(t)) & * \\ \bar{B}_f^T \bar{P} + K\bar{C} & -2I \end{bmatrix} \tag{25}$$

and $\bar{C} = [\, C \;\; 0 \,]$.

It is seen that if $\Theta_c < 0$ holds, one has $\mathcal{L}V(e(t), t) < 0$. Thus, it follows by [13] that the filtering error system (13) with $v(t) = 0$ is asymptotically stable in probability. Further, by means of Schur's complement, it can be shown that if there exist matrices $X > 0$, $Y > 0$, W, and S, and scalar $\varepsilon_1 > 0$ satisfying LMI (21), one has $\Theta_c < 0$.

Next, we shall further show that the filtering error system (Σ_{ce}) satisfies

$$||\tilde{z}(t)||_{E_2} < \gamma ||v(t)||_{E_2}, \tag{26}$$

for all nonzero $v(t) \in L_{E_2}([0, \infty); R^p)$.

It is known by Itô's formula that under zero initial conditions,

$$\mathcal{E}\{V(e(t), t)\} = \mathcal{E}\left\{ \int_0^T \mathcal{L}V(e(t), t) dt \right\},$$

where $V(e(t), t)$ is given in (22)–(23) and the infinitesimal generator $\mathcal{L}V(e(t), t)$ is associated with (Σ_{ce}) for $v(t) \neq 0$.

Now, define

$$J(T) = \mathcal{E}\left\{ \int_0^T \left[\tilde{z}(t)^T \tilde{z}(t) - \gamma^2 v(t)^T v(t) \right] dt \right\}, \tag{27}$$

for any $T > 0$, and $v(t) \neq 0$.

It can be shown that

$$\begin{aligned} J(T) &= \mathcal{E}\left\{ \int_0^T \left[\tilde{z}(t)^T \tilde{z}(t) - \gamma^2 v(t)^T v(t) \right. \right. \\ &\quad \left. \left. + \mathcal{L}V(e(t), t) \right] dt \right\} - \mathcal{E}\{V(e(t), t)\} \\ &\le \mathcal{E}\left\{ \int_0^T \left[\, e(t)^T \;\; v(t)^T \;\; \phi_s(Cx(t))^T \,\right] \Xi_c \right. \\ &\quad \left. \times \left[\, e(t)^T \;\; v(t)^T \;\; \phi_s(Cx(t))^T \,\right]^T dt \right\} \end{aligned} \tag{28}$$

where

$$\Xi_c = \begin{bmatrix} \Xi_{c1} & \Xi_{c2} & \bar{P}\bar{B}_f + \bar{C}^T K^T \\ \Xi_{c2}^T & \Xi_{c3} & 0 \\ \bar{B}_f^T \bar{P} + K\bar{C} & 0 & -2I \end{bmatrix}$$

with

$$\Xi_{c1} = \bar{L}^T\bar{L} + \bar{P}(\bar{A} + \Delta\bar{A}(t)) + (\bar{A} + \Delta\bar{A}(t))^T\bar{P} + (\bar{E} + \Delta\bar{E}(t))^T\bar{P}(\bar{E} + \Delta\bar{E}(t)),$$
$$\Xi_{c2} = \bar{P}(\bar{B} + \Delta\bar{B}(t)) + (\bar{E} + \Delta\bar{E}(t))^T \times\bar{P}(\bar{G} + \Delta\bar{G}(t)),$$
$$\Xi_{c3} = -\gamma^2 I + (\bar{G} + \Delta\bar{G}(t))^T\bar{P}(\bar{G} + \Delta\bar{G}(t)).$$

By utilizing Schur's complement, and following a similar derivation, it can be shown that $\Xi_c < 0$ holds, if there exist matrices $X > 0$, $Y > 0$, W, and S, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and $\varepsilon_3 > 0$ satisfying LMI (15). Hence, from (27) and (28), one has that $J(T) < 0$, which implies that (26) holds for any nonzero $v(t) \in L_{E_2}([0,\infty); R^p)$. □

As a special case, when the sensor in (Σ_c) is linear, i.e., $\phi(Cx(t)) = Cx(t)$, the following corollary can be obtained from Theorem 2.1.

Corollary 1. For the uncertain stochastic systems (1)–(3) with $\phi(Cx(t)) = Cx(t)$ and a given disturbance attenuation level $\gamma > 0$, if there exist matrices $X > 0$, $Y > 0$, W, and S, and scalars $\varepsilon_1 > 0$, and $\varepsilon_2 > 0$ satisfying the following LMI:

$$\begin{bmatrix} \Theta_{c1} & * & * & * & * & * & * \\ \Theta_{c6}^T & \Theta_{c4} & * & * & * & * & * \\ B^TX & B^TY - D^TS^T & \Theta_{c2} & * & * & * & 0 \\ XE & 0 & XG & -X & * & * & * \\ YE & 0 & YG & 0 & -Y & * & * \\ M_A^TX & M_A^TY & 0 & M_E^TX & M_E^TY & -\varepsilon_1 I & * \\ M_B^TY & M_B^TY - M_D^TS^T & 0 & M_G^TX & M_G^TY & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \quad (29)$$

with

$$\Theta_{c6} = A^TY - W^T - C^TS^T,$$

and Θ_{c1}, Θ_{c2} and Θ_{c4} as in (16)–(19), then the robust filtering problem is solved by the filter $(\mathcal{F}_c)$ with the desired parameters given as

$$A_f = Y^{-1}W, B_f = Y^{-1}S.$$

3. SIMULATION EXAMPLE

In this simulation, two different kinds of external disturbances, i.e., deterministic and stochastic, will be considered.

Consider the Itô stochastic systems (1)–(3) with parameters as follows:

$$A = \begin{bmatrix} -1.8 & 0.6 & 0.2 \\ -2.5 & -3.8 & 0.1 \\ -2.3 & 1.0 & -3.6 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, C = \begin{bmatrix} 0.2 & 0.8 & -0.3 \\ 1.0 & 0.3 & 0.5 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}, E = \begin{bmatrix} 0.3 & 1.0 & 0.6 \\ 1.3 & 0.5 & -0.2 \\ 2.3 & 0.2 & -0.4 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.2 & 0.4 \\ 1.2 & 0.5 \\ 0.6 & 0.3 \end{bmatrix}, L = \begin{bmatrix} 0.5 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.2 \end{bmatrix},$$

$$M_A = [\,0.2\ 0.2\ 0.1\,]^T, M_D = [\,0.3\ 0.2\,]^T$$
$$M_B = [\,0.1\ 0.4\ 0.2\,]^T, F_1(t) = 0.5\sin(t),$$
$$M_E = [\,0.3\ 0.1\ 0.3\,]^T, \ F_2(t) = 0.2\sin(t),$$
$$M_G = [\,0.3\ 0.2\ 0.1\,]^T,$$
$$N_1 = [\,0.2\ 0.2\ 0.1\,], N_2 = [\,0.2\ 0.1\,],$$
$$K_1 = diag\,[\,0.6\ 0.5\,], K_2 = diag\,[\,1\ 0.8\,],$$
$$\phi(u) = \frac{K_1 + K_2}{2}u + \frac{K_2 - K_1}{2}\sin(u).$$

By Theorem 2.1, we solve LMI (15) for $\gamma = 0.8$ and obtain the filter in (8)–(9) with

$$A_f = \begin{bmatrix} -183.7871 & -146.4546 & -36.0467 \\ -236.9819 & -177.8053 & -59.1438 \\ -264.1377 & -177.2443 & -81.1756 \end{bmatrix},$$

$$B_f = \begin{bmatrix} 31.7819 & 188.5886 \\ 35.3266 & 246.4703 \\ 29.4426 & 279.5673 \end{bmatrix}.$$

In real engineering systems, the external disturbance may come from a deterministic source or a stochastic one. Hence, in the following simulations, two different kinds of disturbance signal $v(t)$ will be considered, respectively:

Case I: $v(t)$ is deterministic with $v(t) = \begin{bmatrix} \frac{1}{1+t^2} & \frac{1}{1+t^2} \end{bmatrix}^T$.

Case II: $v(t)$ is stochastic and is chosen as the truncated Brownian motion, i.e., $v(t) = [\,w(t)\ w(t)\,]^T$ for $t \leq 3$ or $v(t) = [\,0\ 0\,]^T$ for $t > 3$.

In this work, the simulation is undertaken by using the discretized approach as in [16, 17] with the simulation time $t \in [0, 5]$, the normally distributed variance $\delta t = 5/N$ with $N = 2^{12}$, step size $\Delta t = R \cdot \delta t$ with $R = 2$, the number of *discretized Brownian paths* $M = 10$, the initial state $x(0) = [\,1\ 0.5\ -1\,]^T$ and $\hat{x}(0) = x(0)$.

Figures 1–2 show the simulation results for case I, and Figures 3–4 show the simulation results for case II. It is seen that the proposed filter in this work can ensure a satisfying performance for the resultant filtering error systems.

4. CONCLUSIONS

In this paper, we have discussed the filtering problem in the presence of sensor nonlinearities for uncertain stochastic systems,and a desired robust filter has been provided.

ACKNOWLEDGEMENTS

The research was partially supported by a grant from CityU (Project 7001992), and the National Natural Science Foundation under Grant 60674015 and Shanghai Leading Academic Discipline Project (B504).

REFERENCES

[1] R. K. Pearson. Gray-box modeling of nonideal sensors. In *Proc. Amer. Contr. Conf.*, pages 4404–4409, Arlington, VA,2001.

[2] A. M. Tekalp and G. Pavlovic. Image restoration with multiplicative noise: incorporating the sensor nonlinearity. *IEEE Transactions on Signal Processing*, 39:2132–2136, 1991.

[3] G. Kreisselmeier. Stabilization of linear systems in the presence of output measurement saturation. *Systems & Control Letters*, 29:27–30, 1996.

[4] Z. Lin and T. Hu. Semi-global stabilization of linear systems subject to output saturation. *Systems & Control Letters*, 43:211–217, 2001.

[5] Y.-Y. Cao, Lin Z., and B. M. Chen. An output feedback H_∞ controller design for linear systems subject to sensor nonlinearities. *IEEE Transactions on Circuits and Systems – I: Fundamental Theory and Applications*, 50:914–921, 2003.

[6] D. Hinrichsen and A. J. Pritchard. Stochastic H_∞. *SIAM Journal on Control and Optimization*, 36:1504–1538, 1998.

[7] E. Gershon, D. J. N. Limebeer, U. Shaked, and I. Yaesh. Robust H_∞ filtering for stationary continuous-time linear systems with stochastic uncertianties. *IEEE Transactions on Automatic Control*, 46:1788–1793, 2001.

[8] S. Xu and T. Chen. Reduced-order H_∞ filtering for stochastic systems. *IEEE Transactions on Signal Processing*, 50:2998–3007, 2002.

[9] Z. Wang, D. W. C. Ho, and X. Liu. A note on the robust stability of uncertain stochastic fuzzy systems with time-delays. *IEEE Transactions on Systems, Man and Cybernetics, (Part A)*, 34:570–576, 2004.

[10] W. Zhang, B.-S. Chen, and C.-S. Tseng. Robust H_∞ filtering for nonlinear stochastic systems. *IEEE Transactions on Signal Processing*, 53:589–598, 2005.

[11] W. M. Wonham. Random differential equations in control theory. In *em Probabilistic Methods in Applied Mathematics*, volume 2, A. T. Bharucha Reid, Ed. New York: Academic, 1970.

[12] H. Khalil. *Nonlinear Systems*. Upper Saddle River, NJ:Prentice-Hall, 1996.

[13] V. Kolmanovskii and A. Myshkis. *Applied Theory of Functional Differential Equations*. Dordrecht, The Netherlands: Kluwer, 1992.

[14] A. EL Bouhtouri, D. Hinrichsen, and A. J. Pritchard. H_∞-type control for discrete-time stochastic systems. *Int. J. Robust & Nonlinear Control*, 9:923–948, 1999.

[15] S. Xu, J. Lam, and T. Chen. Robust H_∞ control for uncertain discrete stochastic time-delay systems. *Systems & Control Letters*, 51:203–215, 2004.

[16] Y. Niu, D. W. C. Ho, and J. Lam. Robust integral sliding mode control for uncertain stochastic systems with time-varying delay. *Automatica*, 41:873–880, 2005.

[17] D. Higham. An algorithmic introduction to numerical simulation of stochastic differential equations. *SIAM Review*, 43:525–546, 2001.

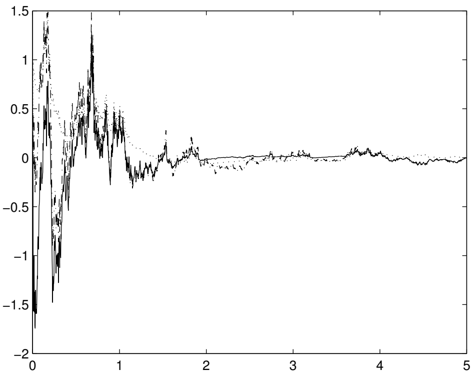

Figure 1. The trajectories of state $x(t)$ for case I.

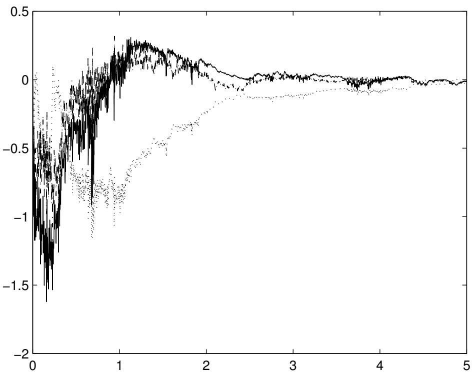

Figure 2. The trajectories of error $\tilde{x}(t)$ for case I.

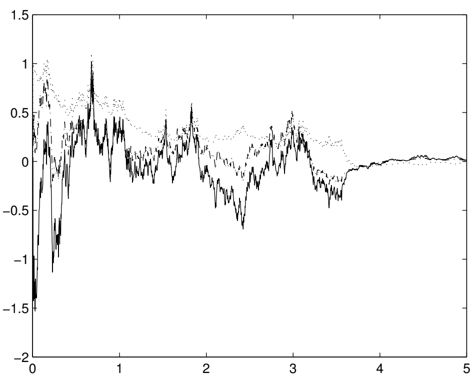

Figure 3. the trajectories of state $x(t)$ for case II.

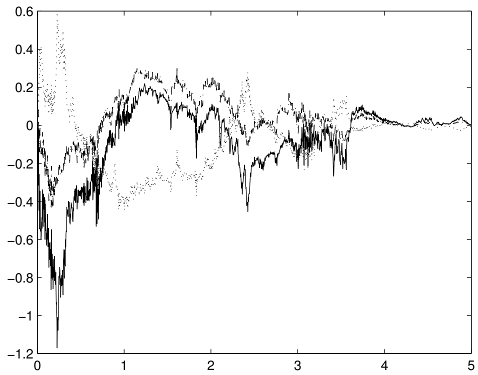

Figure 4. The trajectories of error $\tilde{x}(t)$ for case II.