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Robust Mixed H_2/H_∞ Control of Uncertain Neutral Systems with Time-Varying Delays

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Abstract: This paper considers the problem of robust mixed H_2/H_∞ delayed state feedback control for a class of uncertain neutral systems with time-varying discrete and distributed delays. Based on the Lyapunov-Krasovskii functional theory, new required sufficient conditions are established in terms of delay-range-dependent linear matrix inequalities (LMIs) for the stability and stabilization of the considered system using some free matrices. The desired robust mixed H_2/H_∞ delayed control is derived based on a convex optimization method such that the resulting closed-loop system is asymptotically stable and satisfies H_2 performance with a guaranteed cost and a prescribed level of H_∞ performance, simultaneously. Finally, a numerical example is given to illustrate the effectiveness of our approach.

1. INTRODUCTION

Delay systems represent a class of infinite-dimensional systems largely used to describe propagation and transport phenomena or population dynamics. Neutral delay systems constitute a more general class than those of the retarded type. Stability of these systems proves to be a more complex issue because the system involves the derivative of the delayed state. Especially, in the past few decades increased attention has been devoted to the problem of robust delay-independent stability or delay-dependent stability and stabilization via different approaches for linear neutral systems with delayed state and/or input and parameter uncertainties (see, Han, 2004; He et. al., 2007; Han and Yu, 2004; Lam et. al., 2005). Among the past results on neutral delay systems, the LMI approach is an efficient method to solve many control problems such as stability analysis and stabilization (Fridman, 2001; Chen and Zheng, 2007) and H_∞ control problems (Chen, 2005; Fridman and Shaked, 2003; Gao and Wang, 2003; Xu et. al., 2001; Chen, 2006; Xu et. al., 2002). It is also worth citing that some appreciable works have been performed to design a guaranteed-cost (observer-based) control for the neutral system performance representation (Karimi, 2008; Chen et. al., 2006; Lien, 2005; Park, 2003; Xu et. al., 2003). To the best of our knowledge, a robust mixed H_2/H_∞ delayed state feedback control for uncertain neutral systems with time-varying discrete and distributed delays has not been fully investigated in the past and remains to be important and challenging.

This paper develops an efficient approach for robust mixed H_2/H_∞ delayed state feedback control problem of uncertain neutral systems with discrete and distributed time-varying delays. The main merit of the proposed method is the fact that it provides a convex problem such the control gain can be found from the LMI formulations. New required sufficient conditions are established in terms of delay-range-dependent LMIs combined with the Lyapunov-Krasovskii method for the existence of the desired robust mixed H_2/H_∞ control such that the resulting closed-loop system is asymptotically stable and satisfies both, H_2 performance with a guaranteed cost and a prescribed level of H_∞ performance. A numerical example is given to illustrate the use of our results.

2. PROBLEM DESCRIPTION

Consider a class of neutral systems with discrete and distributed delays and norm-bounded time-varying uncertainties represented by

$$\begin{aligned}
\dot{x}(t) - A_2\,\dot{x}(t-d(t)) &= (A+\Delta A(t))\,x(t) + (A_1+\Delta A_1(t))\,x(t-h(t)) \\
&\quad + (A_3+\Delta A_2(t)) \int_{t-\tau(t)}^{t} x(s)\,ds + (B+\Delta B(t))u(t) + (B_1+\Delta B_1(t))\,w(t), \\
x(t) &= \phi(t), \qquad t\in[-\max\{h_2, d_2, \tau_2\},\ 0] \\
z(t) &= (C+\Delta C(t))\,x(t) + (D+\Delta D(t))\,u(t),
\end{aligned} \tag{1a-c}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$, $w(t)\in L_2^s[0,\infty)$ and $z(t)\in\Re^z$ are state, input, disturbance and controlled output, respectively. The time-varying function $\phi(t)$ is continuous vector valued initial function and the time-varying delays $h(t), d(t)$ and $\tau(t)$ are functions satisfying, respectively,

$$h_1 \le h(t) \le h_2, \qquad \dot{h}(t) \le h_3, \tag{2a}$$

$$0 \le d(t) \le d_1, \qquad \dot{d}(t) \le d_2 < 1, \tag{2b}$$

$$0 \le \tau(t) \le \tau_1, \qquad \dot{\tau}(t) \le \tau_2 < 1. \tag{2c}$$

Moreover, $\Delta A(t), \Delta A_1(t), \Delta A_2(t), \Delta A_3(t), \Delta B(t), \Delta B_1(t), \Delta C(t)$, and $\Delta D(t)$ are bounded uncertainties and defined as follows:

$$\begin{aligned}
[\Delta A(t) \quad \Delta A_1(t) \quad \Delta A_2(t) \quad \Delta B(t) \quad \Delta B_1(t)] &= H_1\Delta(t)[E \quad E_1 \quad E_2 \quad E_3 \quad E_4] \\
[\Delta C(t) \quad \Delta D(t)] &= H_2\Delta(t)[E_5 \quad E_6]
\end{aligned} \tag{3}$$

where the uncertain matrix $\Delta(t)$ satisfies $\Delta^T(t)\Delta(t) \le I$.

Definition 1: The H_2 and H_∞ performance measures of the system (1) are defined, respectively, as

$$J_2 = \int_0^\infty \chi^T(x(t))S_1\chi(x(t)) + u^T(t)S_2u(t) \quad dt, \tag{4a}$$

$$J_\infty = \int_0^\infty z^T(t)z(t) - \gamma^2 w^T(t)w(t) \quad dt, \tag{4b}$$

where the operator $\chi(x(t))$ in (4a) is defined by

$$\chi(x(t)) = x(t) - \tfrac{1}{1-d_2}A_2\, x(t-d(t)) \tag{5}$$

and $S_1 > 0$, $S_2 > 0$ and the positive scalar γ are given.

Assumption 1: The full state variable $x(t)$ is available for measurement.

In this paper, the authors' attention will be focused on the design of the following robust mixed H_2/H_∞ delayed state feedback control law,

$$u(t) = K\chi(x(t)) \tag{6}$$

where the matrix K of the appropriate dimension is to be determined such that for any delays satisfying (2) the resulting closed-loop system is asymptotically stable and $J_2 \le J_0$, where the constant scalar J_0 is an upper bound of the H_2 performance measure which satisfies an H_∞ norm bound γ. It can be easily seen that the resulting closed-loop system (1) and (6) is of the following form,

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A(t) + (B + \Delta B(t))K)\chi(t) + \tfrac{1}{1-d_2}(A + \Delta A(t)) \\ &\quad \times A_2\, x(t-d(t)) + (A_1 + \Delta A_1(t))x(t-h(t)) + (A_3 + \Delta A_2(t)) \\ &\quad \times \int_{t-\tau(t)}^{t} x(s)\,ds + A_2\,\dot{x}(t-d(t)) + (B_1 + \Delta B_1(t))w(t),\end{aligned} \tag{7}$$

Lemma 1: (Wang et. al., 1992) Given matrices $Y = Y^T$, D, E and F of appropriate dimensions with $F^TF \le I$, then the matrix inequality $Y + sym(DFE) < 0$, the operator $sym(A)$ represents $A + A^T$, holds for all F if and only if there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon DD^T + \varepsilon^{-1}E^TE < 0$$

3. ROBUST CONTROL SYNTHESIS

In this section, both the asymptotic stability and mixed H_2/H_∞ performance of the interconnection of plant and the control are investigated such sufficient stability conditions are derived for the existence of the control (6) combined with the Lyapunov method in terms of LMIs. In the literature, extensions of the quadratic Lyapunov functions to the quadratic Lyapunov-Krasovskii functionals have been proposed for time-delayed systems (Park, 1999). Now, we choose a Lyapunov functional candidate for the uncertain neutral system (1) as

$$V(t) = \sum_{i=1}^{8} V_i(t), \tag{8}$$

where

$$V_1(t) = \chi(x(t))^T P\chi(x(t)),\quad V_2(t) = \sum_{i=1}^{2}\int_{t-h_i}^{t} x(s)^T R_i\, x(s)\ ds$$

$$V_3(t) = \int_{t-h(t)}^{t} x(s)^T R_3\, x(s)\ ds,\quad V_4(t) = \int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{x}(s)^T R_4\,\dot{x}(s)\ ds\,d\theta,$$

$$V_5(t) = \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}(s)^T R_5\,\dot{x}(s)\ ds\,d\theta,\quad V_6(t) = \int_{t-d(t)}^{t} x(s)^T R_6\, x(s)\ ds,$$

$$V_7(t) = \int_{t-d(t)}^{t} \dot{x}(s)^T R_7\,\dot{x}(s)\ ds,\quad V_8(t) = \int_{t-\tau(t)}^{t}\int_{\beta}^{t} x(s)^T R_8\, x(s)\ ds\ d\beta.$$

Differentiating $V_1(t)$ in t we obtain

$$\begin{aligned}\dot{V}_1(t) &\le 2\chi(x(t))^T P\,\{(A + \Delta A(t) + (B + \Delta B(t))K)\chi(x(t)) \\ &\quad + (A_1 + \Delta A_1(t))x(t-h(t) + \tfrac{1}{1-d_2}(A + \Delta A(t))A_2x(t-d(t)) \\ &\quad + (A_3 + \Delta A_2(t))\int_{t-\tau(t)}^{t} x(s)\,ds + (B_1 + \Delta B_1(t))w(t)\end{aligned} \tag{9}$$

Differentiating other terms in (8) give ($h_{12} := h_2 - h_1$)

$$\dot{V}_2(t) = \sum_{i=1}^{2} x(t)^T R_i\, x(t) - x(t-h_i)^T R_i\, x(t-h_i) \tag{10}$$

$$\dot{V}_3(t) \le x^T(t)R_3x(t) - (1-h_3)x^T(t-h(t))R_3\,x(t-h(t)) \tag{11}$$

$$\dot{V}_4(t) = h_2\dot{x}(t)^T R_4\,\dot{x}(t) - \int_{t-h(t)}^{t} \dot{x}(s)^T R_4\,\dot{x}(s)\ ds - \int_{t-h_2}^{t-h(t)} \dot{x}(s)^T R_4\,\dot{x}(s)\ ds \tag{12}$$

$$\dot{V}_5(t) = h_{12}\dot{x}(t)^T R_5\,\dot{x}(t) - \int_{t-h_2}^{t-h(t)} \dot{x}(s)^T R_5\,\dot{x}(s)\ ds - \int_{t-h(t)}^{t-h_1} \dot{x}(s)^T R_5\,\dot{x}(s)\ ds \tag{13}$$

$$\dot{V}_6(t) \le x^T(t)R_6\,x(t) - (1-d_2)x^T(t-d(t))R_6\,x(t-d(t)) \tag{14}$$

$$\dot{V}_7(t) \le \dot{x}^T(t)R_7\,\dot{x}(t) - (1-d_2)\dot{x}^T(t-d(t))R_7\,\dot{x}(t-d(t)) \tag{15}$$

$$\dot{V}_8(t) \le \tau_1 x(t)^T R_8\, x(t) - (1-\tau_2)\int_{t-\tau(t)}^{t} x(s)^T R_8\, x(s)\ ds \tag{16}$$

Moreover, from the Leibniz-Newton formula, the following equations hold for $\{N_i\}_{i=1}^4$ with appropriate dimensions:

$$2(\chi^T(t)N_1 + x^T(t-h(t))N_2)(x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)\,ds) = 0 \tag{17}$$

$$2(\chi^T(t)N_3 + x^T(t-h(t))N_4)(x(t-h_1) - x(t-h(t)) - \int_{t-h(t)}^{t-h_1} \dot{x}(s)\,ds) = 0 \tag{18}$$

Now, to establish the H_∞ performance measure for the system (1), assume zero initial condition, then we have $V(t)\big|_{t=0} = 0$. Consider the index J_∞ in (4b), then along the solution of (1) for any nonzero $w(t)$ there holds

$$J_\infty \le \int_0^\infty z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t) \quad dt \tag{19}$$

Substituting (1c), (7) and from (8)-(16) and adding the left sides of equations (17) and (18) into (19), we obtain

$$J_\infty \le \int_0^\infty \vartheta^T(t)\,\Sigma\,\vartheta(t)\,dt \tag{20}$$

where $\vartheta(t) := col\{\chi(x(t)),\ x(t-h(t)), x(t-h_1), x(t-h_2), x(t-d(t)),$ $\dot{x}(t-d(t)),\ \int_{t-\tau(t)}^{t} x(s)\,ds,\ w(t)\}$ is an augmented state vector and the matrix Σ is given by

$$\Sigma = \Pi + \overline{A}^T(h_2R_4 + h_{12}R_5 + R_7)\overline{A} + h_2\,M_1R_4^{-1}M_1^T + h_{12}M_2R_5^{-1}M_2^T \tag{21}$$

where

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & N_3 & 0 & \Pi_{15} & 0 & P(A_3 + \Delta A_2(t)) & P(B_1 + \Delta B_1(t)) \\ * & \Pi_{22} & N_4 & 0 & \frac{1}{1-d_2}N_2A_2 & 0 & 0 & 0 \\ * & * & -R_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -(1-d_2)R_7 & 0 & 0 \\ * & * & * & * & * & * & -(1-\tau_2)R_8 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

with $M_1 = col\{N_1, N_2, 0\}$, $M_2 = col\{N_3, N_4, 0\}$ and

$$\overline{A} = [A + \Delta A(t) + (B + \Delta B(t))K, A_1 + \Delta A_1(t), 0, 0, \tfrac{1}{1-d_2}(A + \Delta A(t))A_2, A_2, A_3 + \Delta A_2(t), B_1 + \Delta B_1(t)]$$

$$\begin{aligned}\Pi_{11} &= sym\{P((A + \Delta A(t)) + (B + \Delta B(t))K) + N_1\} \\ &\quad + K^T (B + \Delta B(t))^T (R_3 + h_2 R_4)(B + \Delta B(t))K \\ &\quad + ((C + \Delta C(t)) + (D + \Delta D(t))K)^T ((C + \Delta C(t)) \\ &\quad + (D + \Delta D(t))K) + \sum_{i=1}^{3} R_i + R_6 + \tau_1 R_8, \\ \Pi_{12} &= P(A_1 + \Delta A_1(t)) - N_1 + N_2^T - N_3, \\ \Pi_{15} &= \tfrac{1}{1-d_2}(\sum_{i=1}^{3} R_i + R_6 + \tau_1 R_8)A_2 + \tfrac{1}{1-d_2}(P + N_1)A_2 \\ &\quad + \tfrac{1}{1-d_2}((C + \Delta C(t)) + (D + \Delta D(t))K)^T (C + \Delta C(t))A_2 \\ \Pi_{22} &= -(1 - h_3)R_3 - sym\{N_4 + N_2\},\end{aligned}$$

$$\Pi_{55} = -(1-d_2)R_6 + (1-d_2)^{-2} A_2^T ((C+\Delta C(t))^T (C+\Delta C(t)) + \sum_{i=1}^{3} R_i + R_6 + \tau_1 R_8)A_2$$

where the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. Thus, if the inequality $\Sigma < 0$ holds, the inequality $J_\infty < 0$ is satisfied. The inequality $\Sigma < 0$ yields (by Schur complement)

$$\begin{bmatrix} \Pi & h_2\overline{A}^T & h_{12}\overline{A}^T & \overline{A}^T & h_2 M_1 & h_{12}M_2 \\ * & -h_2 R_4^{-1} & 0 & 0 & 0 & 0 \\ * & * & -h_{12}R_5^{-1} & 0 & 0 & 0 \\ * & * & * & -R_7^{-1} & 0 & 0 \\ * & * & * & * & -h_2 R_4 & 0 \\ * & * & * & * & * & -h_{12}R_5 \end{bmatrix} < 0 \cdot \qquad (22)$$

Let $\xi = diag\{X, \overline{R}_3, \overline{R}_1, \overline{R}_2, \overline{R}_6, \overline{R}_7, \overline{R}_8, I, I, I, I, \overline{R}_4, \overline{R}_5\}$ where $X := P^{-1}$ and $\overline{R}_i := R_i^{-1}$ for $i = 1, \cdots, 8$. Pre-multiplying ξ and post-multiplying ξ^T to the matrix inequality (22) yield

$$\begin{bmatrix} \hat{\Pi}_1 & \hat{\Pi}_2 & h_2\widetilde{A}^T & h_{12}\widetilde{A}^T & \widetilde{A}^T & h_2 \overline{M}_1 & h_{12}\overline{M}_2 \\ * & \hat{\Pi}_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & -h_2\overline{R}_4 & 0 & 0 & 0 & 0 \\ * & * & * & -h_{12}\overline{R}_5 & 0 & 0 & 0 \\ * & * & * & * & -\overline{R}_7 & 0 & 0 \\ * & * & * & * & * & -h_2\overline{R}_4 & 0 \\ * & * & * & * & * & * & -h_{12}\overline{R}_5 \end{bmatrix} + sym\left(\Psi_1 (I_{21} \otimes \Delta(t))\Psi_2\right) < 0 \qquad (23)$$

and by Lemma 1 and applying Schur complement, the following inequality is obtained for any scalar $\varepsilon_1 > 0$,

$$\begin{bmatrix} \hat{\Pi} & \varepsilon_1 \Psi_1 & \Psi_2^T \\ * & -\varepsilon_1 I & 0 \\ * & * & -\varepsilon_1 I \end{bmatrix} < 0 \qquad (24)$$

where

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_1 & \hat{\Pi}_2 & h_2\widetilde{A}^T & h_{12}\widetilde{A}^T & \widetilde{A}^T & h_2 \overline{M}_1 & h_{12}\overline{M}_2 \\ * & \hat{\Pi}_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & -h_2\overline{R}_4 & 0 & 0 & 0 & 0 \\ * & * & * & -h_{12}\overline{R}_5 & 0 & 0 & 0 \\ * & * & * & * & -\overline{R}_7 & 0 & 0 \\ * & * & * & * & * & -h_2\overline{R}_4 & 0 \\ * & * & * & * & * & * & -h_{12}\overline{R}_5 \end{bmatrix}$$

$$\hat{\Pi}_1 = \begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & XN_3\overline{R}_1 & 0 & \tfrac{1}{1-d_2}(I + XN_1)A_2\overline{R}_6 & 0 & A_3\overline{R}_8 & B_1 \\ * & \hat{\Pi}_{22} & \overline{R}_3 N_4 \overline{R}_1 & 0 & \tfrac{1}{1-d_2}\overline{R}_3 N_2 A_2 \overline{R}_6 & 0 & 0 & 0 \\ * & * & -\overline{R}_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\overline{R}_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -(1-d_2)\overline{R}_6 & 0 & 0 & 0 \\ * & * & * & * & * & -(1-d_2)\overline{R}_7 & 0 & 0 \\ * & * & * & * & * & * & -(1-\tau_2)\overline{R}_8 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\hat{\Pi}_2 = [\omega_1, 0, 0, 0, \omega_2, 0, 0, 0]^T,$$

$$\hat{\Pi}_3 = diag\{-\overline{R}_3, -h_2\overline{R}_4, -I, -\overline{R}_1, -\overline{R}_2, -\overline{R}_3, -\overline{R}_6, -\tau_1\overline{R}_8\},$$

$$\Psi_1 = diag\{\psi_1, \psi_2, \psi_3\},$$

$$\Psi_2 = \begin{bmatrix} \varphi_1^T & 0 & \varphi_2^T & h_2\varphi_2^T & \varphi_3^T & 0 & h_2\varphi_4^T & h_{12}\varphi_4^T & \varphi_4^T & 0 & 0 \end{bmatrix}^T,$$

with

$$\hat{\Pi}_{11} = sym\{(AX + BKX + XN_1X\},$$

$$\hat{\Pi}_{12} = A_1\overline{R}_3 + X(-N_1 + N_2^T - N_3)\overline{R}_3,$$

$$\hat{\Pi}_{22} = -(1 - h_3)\overline{R}_3 - \overline{R}_3\left(sym\{N_4 + N_2\}\right)\overline{R}_3,$$

$$\widetilde{A} = [AX + BKX, A_1\overline{R}_3, 0, 0, \tfrac{1}{1-d_2}AA_2\overline{R}_6, A_2\overline{R}_7, A_3\overline{R}_8, B_1],$$

$$\overline{M}_1 = col\{XN_1\overline{R}_4, \overline{R}_3 N_2 \overline{R}_4, 0\}, \overline{M}_2 = col\{XN_3\overline{R}_5, \overline{R}_3 N_4 \overline{R}_5, 0\},$$

$$\omega_1 := col\{BKX, h_2 BKX, (C + DK)X, X, X, X, X, \tau_1 X\},$$

$$\omega_2 := col\{0, \tfrac{1}{1-d_2}CA_2\overline{R}_6, \tfrac{1}{1-d_2}A_2\overline{R}_6, \tfrac{1}{1-d_2}A_2\overline{R}_6, \tfrac{1}{1-d_2}A_2\overline{R}_6, \tfrac{1}{1-d_2}A_2\overline{R}_6, \tfrac{\tau_1}{1-d_2}A_2\overline{R}_6\}$$

$$\psi_1 := col\{H_1, 0\}, \ \psi_2 := col\{H_1, H_1, H_2, 0\},$$

$$\psi_3 := diag\{H_1, H_1, H_1, 0, 0\},$$

$$\varphi_1 = \begin{bmatrix} EX + E_3KX & E_1\overline{R}_3 & 0 & 0 & 0 & 0 & E_2\overline{R}_8 & E_4 & 0 \end{bmatrix},$$

$$\varphi_2 = \begin{bmatrix} E_3KX & 0 \end{bmatrix}, \ \varphi_3 = \begin{bmatrix} E_5X + E_6KX & 0 & 0 & 0 & \tfrac{1}{1-d_2}E_5A_2\overline{R}_6 & 0 \end{bmatrix},$$

$$\varphi_4 = \begin{bmatrix} EX + E_3KX & E_1\overline{R}_3 & 0 & 0 & \tfrac{1}{1-d_2}EA_2\overline{R}_6 & 0 & E_2\overline{R}_8 & E_4 & 0 \end{bmatrix}$$

On the other hand, by applying the same Lyapunov-Krasovskii functional candidate (8) for the uncertain neutral system (1), under $w(t) \equiv 0$, for the index J_2 in (4a) we get

$$\begin{aligned} J_2 &\le \int_0^\infty \chi^T(x(t)) S_1 \chi(x(t)) + \chi^T(x(t)) K^T S_2 K \chi(x(t)) + \dot{V}(t) \quad dt \\ &\le \int_0^\infty \hat{\vartheta}^T(t) \hat{\Sigma} \hat{\vartheta}(t)\, dt \end{aligned} \qquad (25)$$

where $\hat{\vartheta}(t) := col\{\chi(x(t)), x(t - h(t)), x(t - h_1), x(t - h_2), x(t - d(t)), \dot{x}(t - d(t)), \int_{t-\tau(t)}^{t} x(s)\, ds\}$ and the matrix $\hat{\Sigma}$ is given by

$$\hat{\Sigma} = \widetilde{\Pi} + \widetilde{A}^T (h_2 R_4 + h_{12} R_5 + R_7)\widetilde{A} + h_2 M_1 R_4^{-1} M_1^T + h_{12} M_2 R_5^{-1} M_2^T \qquad (26)$$

where

$$\widetilde{\Pi}=\begin{bmatrix}\widetilde{\Pi}_{11} & \Pi_{12} & N_3 & 0 & \widetilde{\Pi}_{15} & 0 & P(A_3+\Delta A_2(t)) \\ * & \Pi_{22} & N_4 & 0 & \frac{1}{1-d_2}N_2A_2 & 0 & 0 \\ * & * & -R_1 & 0 & 0 & 0 & 0 \\ * & * & * & -R_2 & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Pi}_{55} & 0 & 0 \\ * & * & * & * & * & -(1-d_2)R_7 & 0 \\ * & * & * & * & * & * & -(1-\tau_2)R_8\end{bmatrix}$$

with

$$\widetilde{A}=[A+\Delta A(t)+(B+\Delta B(t))K, A_1+\Delta A_1(t),\, 0,0,\tfrac{1}{1-d_2}(A+\Delta A(t))A_2,\; A_2, A_3+\Delta A_2(t)]$$

$$\widetilde{\Pi}_{11}=sym\{P((A+\Delta A(t))+(B+\Delta B(t))K)+N_1\}+K^T(B+\Delta B(t))^T \times(R_3+h_2\ R_4)(B+\Delta B(t))K+K^TS_2K+S_1+\sum_{i=1}^{3}R_i+R_6+\tau_1 R_8,$$

$$\widetilde{\Pi}_{15}=\tfrac{1}{1-d_2}(\sum_{i=1}^{3}R_i+R_6+\tau_1R_8)A_2+\tfrac{1}{1-d_2}(P+N_1)A_2,$$

$$\Pi_{55}=-(1-d_2)R_6+\sum_{i=1}^{3}R_i+R_6+\tau_1R_8)A_2.$$

Therefore, the condition $\hat{\Sigma}<0$ in (25) implies

$$\begin{aligned}&\int_0^\infty \dot{V}(t)\,dt=\lim_{t\to\infty}V(t)-V(0)\\ &\le-\int_0^\infty \chi^T(x(t))S_1\,\chi(x(t))+\chi^T(x(t))K^T\ S_2\ K\,\chi(x(t))\,dt\end{aligned}\tag{27}$$

By Theorem 1.6 of the reference Kolmanovskii and Myshkis (1992), we conclude that the system (1) with $w(t)\equiv 0$ is asymptotically stabilizabe by (6). Now, by considering the asymptotically stability of the system (1) by (6) the H_2 performance measure for the system is established as

$$\int_0^\infty \chi^T(x(t))S_1\,\chi(x(t))+\chi^T(x(t))K^T\ S_2\ K\,\chi(x(t))\,dt\le V(0)=J_0$$

where

$$\begin{aligned}J_0&=(\phi(0)-\tfrac{1}{1-d_2}A_2\,\phi(-d(0)))^T\,P\,(\phi(0)-\tfrac{1}{1-d_2}A_2\,\phi(-d(0)))\\ &+\sum_{i=1}^{2}\int_{-h_i}^{0}\phi(s)^T\,R_i\,\phi(s)\ ds+\int_{-h(0)}^{0}\phi(s)^T\,R_3\,\phi(s)\ ds\\ &+\int_{-h_2}^{0}\int_{\theta}^{0}\dot{\phi}(s)^T R_4\,\dot{\phi}(s)\ ds\,d\theta+\int_{-h_2}^{-h_1}\int_{\theta}^{0}\dot{\phi}(s)^T R_5\,\dot{\phi}(s)\ ds\,d\theta\\ &+\int_{-d(0)}^{0}\phi(s)^T\,R_6\,\phi(s)\ ds+\int_{-d(0)}^{0}\dot{\phi}(s)^T\,R_7\,\dot{\phi}(s)\ ds+\int_{-\tau(0)}^{0}\int_{\beta}^{0}\phi(s)^T\,R_8\,\phi(s)\ ds\ d\beta\end{aligned}\tag{28}$$

Similar to the case of H_∞ performance measure, after applying some matrix manipulations to the inequality $\hat{\Sigma}<0$ we obtain for any scalar $\varepsilon_2>0$,

$$\begin{bmatrix}\overleftrightarrow{\Pi} & \varepsilon_2\widetilde{\Psi}_1 & \widetilde{\Psi}_2^T \\ * & -\varepsilon_2 I & 0 \\ * & * & -\varepsilon_2 I\end{bmatrix}<0 \tag{29}$$

where

$$\overleftrightarrow{\Pi}=\begin{bmatrix}\widetilde{\Pi}_1 & \widetilde{\Pi}_2 & h_2\overleftrightarrow{A}^T & h_{12}\overleftrightarrow{A}^T & \overleftrightarrow{A}^T & h_2\overleftrightarrow{M}_1 & h_{12}\overleftrightarrow{M}_2 \\ * & \widetilde{\Pi}_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & -h_2\overline{R}_4 & 0 & 0 & 0 & 0 \\ * & * & * & -h_{12}\overline{R}_5 & 0 & 0 & 0 \\ * & * & * & * & -\overline{R}_7 & 0 & 0 \\ * & * & * & * & * & -h_2\overline{R}_4 & 0 \\ * & * & * & * & * & * & -h_{12}\overline{R}_5\end{bmatrix}$$

$$\widetilde{\Pi}_1=\begin{bmatrix}\hat{\Pi}_{11} & \hat{\Pi}_{12} & XN_3\overline{R}_1 & 0 & \frac{1}{1-d_2}(I+XN_1)A_2\overline{R}_6 & 0 & A_3\overline{R}_8 \\ * & \hat{\Pi}_{22} & \overline{R}_3N_4\overline{R}_1 & 0 & 0 & 0 & 0 \\ * & * & -\overline{R}_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\overline{R}_2 & 0 & 0 & 0 \\ * & * & * & * & -(1-d_2)\overline{R}_6 & 0 & 0 \\ * & * & * & * & * & -(1-d_2)\overline{R}_7 & 0 \\ * & * & * & * & * & * & -(1-\tau_2)\overline{R}_8\end{bmatrix}$$

$$\widetilde{\Pi}_2=[\vec{\omega}_1,0,0,0,\vec{\omega}_2,0,0,0,0]^T$$

$$\widetilde{\Pi}_3=diag\{-\overline{R}_3,-h_2\overline{R}_4,-S_1,-S_2,-\overline{R}_1,-\overline{R}_2,-\overline{R}_3,-\overline{R}_6,-\tau_1\overline{R}_8\},$$

$$\widetilde{\Psi}_1=diag\{\widetilde{\psi}_1,\widetilde{\psi}_2,\widetilde{\psi}_3\},$$

$$\widetilde{\Psi}_2=\begin{bmatrix}\widetilde{\varphi}_1^T & 0 & \widetilde{\varphi}_2^T & h_2\widetilde{\varphi}_2^T & 0 & h_2\widetilde{\varphi}_4^T & h_{12}\widetilde{\varphi}_4^T & \widetilde{\varphi}_4^T & 0\end{bmatrix}^T,$$

with

$$\overleftrightarrow{A}=[AX+BKX, A_1\overline{R}_3,\, 0,0,\tfrac{1}{1-d_2}AA_2\overline{R}_6, A_2\overline{R}_7, A_3\overline{R}_8],$$

$$\vec{\omega}_1:=col\{BKX,h_2BKX,S_1X,S_2KX,X,X,X,X,\tau_1X\},$$

$$\vec{\omega}_2:=col\{0,\tfrac{1}{1-d_2}A_2\overline{R}_6,\tfrac{1}{1-d_2}A_2\overline{R}_6,\tfrac{1}{1-d_2}A_2\overline{R}_6,\tfrac{1}{1-d_2}A_2\overline{R}_6,\tfrac{\tau_1}{1-d_2}A_2\overline{R}_6\},$$

$$\overleftrightarrow{M}_1=col\{XN_1\overline{R}_4,\overline{R}_3N_2\overline{R}_4,0\},$$

$$\overleftrightarrow{M}_2=col\{XN_3\overline{R}_5,\overline{R}_3N_4\overline{R}_5,0\},\ \widetilde{\psi}_1:=col\{H_1,0\},$$

$$\widetilde{\psi}_2:=col\{H_1,H_1,0\},\widetilde{\psi}_3:=diag\{H_1,H_1,H_1,0\},$$

$$\widetilde{\varphi}_1=\begin{bmatrix}EX+E_3KX & E_1\overline{R}_3 & 0 & 0 & 0 & 0 & E_2\overline{R}_8 & 0\end{bmatrix},$$

$$\widetilde{\varphi}_2=\begin{bmatrix}E_3KX & 0\end{bmatrix},$$

$$\widetilde{\varphi}_4=\begin{bmatrix}EX+E_3KX & E_1\overline{R}_3 & 0 & 0 & \frac{1}{1-d_2}EA_2\overline{R}_6 & 0 & E_2\overline{R}_8 & 0\end{bmatrix}.$$

Theorem 1: Consider the system (1)-(3) and let $\gamma>0$ be a given scalar. If there exists scalars $\{\varepsilon_i\}_{i=1}^{3}>0$, a matrix Y, and positive definite matrices X and $\{\overline{R}_i\}_{i=1}^{8}$, satisfying the following LMIs,

$$\begin{bmatrix}\hat{\Pi} & \varepsilon_1\Psi_1 & \overline{\Psi}_2^T \\ * & -\varepsilon_1 I & 0 \\ * & * & -\varepsilon_1 I\end{bmatrix}<0 \tag{30a}$$

$$\begin{bmatrix}\overleftrightarrow{\Pi} & \varepsilon_2\widetilde{\Psi}_1 & \overleftrightarrow{\Psi}_2^T \\ * & -\varepsilon_2 I & 0 \\ * & * & -\varepsilon_2 I\end{bmatrix}<0 \tag{30b}$$

where

$$\hat{\Pi}=\begin{bmatrix}\overline{\Pi}_1 & \overline{\Pi}_2 & h_2\vec{A}^T & h_{12}\vec{A}^T & \vec{A}^T & h_2\widetilde{M}_1 & h_{12}\widetilde{M}_2 \\ * & \hat{\Pi}_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & -h_2\overline{R}_4 & 0 & 0 & 0 & 0 \\ * & * & * & -h_{12}\overline{R}_5 & 0 & 0 & 0 \\ * & * & * & * & -\overline{R}_7 & 0 & 0 \\ * & * & * & * & * & -h_2\overline{R}_4 & 0 \\ * & * & * & * & * & * & -h_{12}\overline{R}_5\end{bmatrix},$$

$$\overline{\Pi}_1=\begin{bmatrix}\widetilde{\Pi}_1 & \xi_b \\ * & -\gamma^2 I\end{bmatrix},$$

$$\widetilde{\Pi}_1 = \begin{bmatrix} sym\{(A+I)X+BY\} & (A_1-2I)\overline{R}_3+\varepsilon_3 X & \overline{R}_1 & 0 & \frac{2}{1-d_2}A_2\overline{R}_6 & 0 & A_3\overline{R}_8 \\ * & -2\varepsilon_3\overline{R}_3 & -\frac{1-h_3}{2}\overline{R}_1 & 0 & \frac{1}{1-d_2}\varepsilon_3 A_2\overline{R}_6 & 0 & 0 \\ * & * & -\overline{R}_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\overline{R}_2 & 0 & 0 & 0 \\ * & * & * & * & -(1-d_2)\overline{R}_6 & 0 & 0 \\ * & * & * & * & * & -(1-d_2)\overline{R}_7 & 0 \\ * & * & * & * & * & * & -(1-\tau_2)\overline{R}_8 \end{bmatrix}$$

$$\overline{\Pi}_2 = [\overline{\omega}_1, 0, 0, 0, \omega_2, 0, 0, 0]^T ,$$

$$\overline{\Psi}_2 = \begin{bmatrix} \overline{\varphi}_1^T & 0 & \overline{\varphi}_2^T & h_2\overline{\varphi}_2^T & \overline{\varphi}_3^T & 0 & h_2\overline{\varphi}_4^T & h_{12}\overline{\varphi}_4^T & \overline{\varphi}_4^T & 0 & 0 \end{bmatrix}^T ,$$

$$\overleftrightarrow{\Pi} = \begin{bmatrix} \widetilde{\Pi}_1 & \overleftrightarrow{\Pi}_2 & h_2\overleftarrow{A}^T & h_{12}\overleftarrow{A}^T & \overleftarrow{A}^T & h_2\overleftrightarrow{M}_1 & h_{12}\overleftrightarrow{M}_2 \\ * & \widetilde{\Pi}_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & -h_2\overline{R}_4 & 0 & 0 & 0 & 0 \\ * & * & * & -h_{12}\overline{R}_5 & 0 & 0 & 0 \\ * & * & * & * & -\overline{R}_7 & 0 & 0 \\ * & * & * & * & * & -h_2\overline{R}_4 & 0 \\ * & * & * & * & * & * & -h_{12}\overline{R}_5 \end{bmatrix},$$

$$\overleftrightarrow{\Pi}_2 = [\overleftrightarrow{\omega}_1, 0, 0, 0, \overleftrightarrow{\omega}_2, 0, 0, 0, 0]^T ,$$

$$\overleftrightarrow{\Psi}_2 = \begin{bmatrix} \overleftrightarrow{\varphi}_1^T & 0 & \overleftrightarrow{\varphi}_2^T & h_2\overleftrightarrow{\varphi}_2^T & 0 & h_2\overleftrightarrow{\varphi}_4^T & h_{12}\overleftrightarrow{\varphi}_4^T & \overleftrightarrow{\varphi}_4^T & 0 \end{bmatrix}^T ,$$

$$\xi_b := col\{B_1, 0\} ,$$

$$\vec{A} = [AX + BY, A_1\overline{R}_3 , 0, 0, \tfrac{1}{1-d_2} AA_2\overline{R}_6, A_2\overline{R}_7, A_3\overline{R}_8, B_1] ,$$

$$\widetilde{M}_1 = col\{\overline{R}_4, \varepsilon_3\overline{R}_4, 0\} , \ \widetilde{M}_2 = col\{\overline{R}_5, -\tfrac{1-h_3}{2}\overline{R}_5, 0\} ,$$

$$\overline{\omega}_1 := col\{BY, h_2BY, CX + DY, X, X, X, X, \tau_1 X\} ,$$

$$\overleftrightarrow{\omega}_1 := col\{BY, h_2BY, S_1X, S_2Y, X, X, X, X, \tau_1 X\} ,$$

$$\overline{\varphi}_1 = \begin{bmatrix} EX + E_3Y & E_1\overline{R}_3 & 0 & 0 & 0 & 0 & E_2\overline{R}_8 & E_4 & 0 \end{bmatrix},$$

$$\overline{\varphi}_2 = \begin{bmatrix} E_3Y & 0 \end{bmatrix}, \ \overline{\varphi}_3 = \begin{bmatrix} E_5X + E_6Y & 0 & 0 & 0 & \tfrac{1}{1-d_2}E_5A_2\overline{R}_6 & 0 \end{bmatrix},$$

$$\overline{\varphi}_4 = \begin{bmatrix} EX + E_3Y & E_1\overline{R}_3 & 0 & 0 & \tfrac{1}{1-d_2}EA_2\overline{R}_6 & 0 & E_2\overline{R}_8 & E_4 & 0 \end{bmatrix},$$

$$\overleftarrow{A} = [AX + BY, A_1\overline{R}_3 , 0, 0, \tfrac{1}{1-d_2} AA_2\overline{R}_6, A_2\overline{R}_7, A_3\overline{R}_8] ,$$

$$\overleftrightarrow{\varphi}_1 = \begin{bmatrix} EX + E_3Y & E_1\overline{R}_3 & 0 & 0 & 0 & 0 & E_2\overline{R}_8 & 0 \end{bmatrix}, \ \overleftrightarrow{\varphi}_2 = \begin{bmatrix} E_3Y & 0 \end{bmatrix},$$

$$\overleftrightarrow{\varphi}_4 = \begin{bmatrix} EX + E_3Y & E_1\overline{R}_3 & 0 & 0 & \tfrac{1}{1-d_2}EA_2\overline{R}_6 & 0 & E_2\overline{R}_8 & 0 \end{bmatrix}$$

then, the robust mixed H_2/H_∞ delayed state feedback control gain in (6) is given by $K = YX^{-1}$ and an upper bound of the H_2 performance measure is obtained by

$$\begin{aligned} J_0 = {} & (\phi(0) - \tfrac{1}{1-d_2} A_2\,\phi(-d(0)))^T X^{-1} (\phi(0) - \tfrac{1}{1-d_2} A_2\,\phi(-d(0))) \\ & + \sum_{i=1}^{2} \int_{-h_i}^{0} \phi(s)^T \overline{R}_i^{-1} \phi(s)\ ds + \int_{-h(0)}^{0} \phi(s)^T \overline{R}_3^{-1} \phi(s)\ ds \\ & + \int_{-h_2}^{0}\int_{\theta}^{0} \dot{\phi}(s)^T \overline{R}_4^{-1} \dot{\phi}(s)\ ds\, d\theta + \int_{-h_2}^{-h_1}\int_{\theta}^{0} \dot{\phi}(s)^T \overline{R}_5^{-1} \dot{\phi}(s)\ ds\, d\theta \\ & + \int_{-d(0)}^{0} \phi(s)^T \overline{R}_6^{-1} \phi(s)\ ds + \int_{-d(0)}^{0} \dot{\phi}(s)^T \overline{R}_7^{-1} \dot{\phi}(s)\ ds \\ & + \int_{-\tau(0)}^{0}\int_{\beta}^{0} \phi(s)^T \overline{R}_8^{-1} \phi(s)\ ds\, d\beta \end{aligned} \quad (31)$$

Proof: Omitted due to space constraints. ■

4. SIMULATION RESULTS

Consider the system (1) with the following matrices

$$A = \begin{bmatrix} -1 & 0 \\ 0.2 & -1.2 \end{bmatrix};\ A_1 = \begin{bmatrix} 0.01 & -0.04 \\ 0.02 & 0.01 \end{bmatrix};\ A_2 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.1 \end{bmatrix};$$

$$A_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix};\ B = \begin{bmatrix} 2 \\ 1 \end{bmatrix};\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix};$$

$$D = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ S_2 = 1;\ \phi(t) = \begin{bmatrix} 0.5 \\ -0.3 \end{bmatrix};$$

$$H_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix};\ H_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ E = \begin{bmatrix} 1 & 0.1 \end{bmatrix};\ E_1 = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix};$$

$$E_2 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix};\ E_3 = 0.3;\ E_4 = 0;\ E_5 = \begin{bmatrix} 0.1 & 0 \end{bmatrix};\ E_6 = 0.1 .$$

Using Theorem 1 and solving LMIs (30a, b) with parameters $h_1 = 0.1, h_2 = 0.8, h_3 = 0.7, d_1 = d_2 = 0.3,$ $\tau_1 = 0.3,$ and $\tau_2 = 0$, the corresponding suboptimal H_2 performance measure of the resulting closed-loop system is given by $J_0 = 1.5539$ and the minimum value of the parameter γ in optimal H_∞ performance measure is obtained as 0.578 . Hence, according to Theorem 1, a robust mixed H_2/H_∞ delayed state feedback control law is given by

$$u(t) = \begin{bmatrix} -0.5835 & -0.0008 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0.0835 \end{bmatrix} x(t - 0.3\sin(t)^2) . \quad (32)$$

For simulation purpose, we simply choose a unit step in the time interval $[1, 2]$ as the disturbance, $\Delta(t) = \sin(t)$ as the norm-bounded uncertainty and select time delays as $h(t) = 0.1 + 0.7\sin(t)^2$, $d(t) = 0.3\sin(t)^2$ and $\tau(t) = 0.3$. The simulation results are shown in Figures 1 and 2. Responses of two states, i.e., $x_1(t), x_2(t)$, of the closed-loop system are depicted in Figure 1 and compared with the corresponding state trajectories in the open-loop system under the initial condition $x(0) = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}^T$. It is seen from Figure 1 that the closed-loop system is asymptotically stable. The corresponding control signal (32) is also shown in Figure 2.

5. CONCLUSION

The problem of robust mixed H_2/H_∞ delayed state feedback control was proposed for a class of uncertain neutral systems with time-varying discrete and distributed delays. Based on the Lyapunov-Krasovskii functional theory, new required sufficient conditions were established in terms of delay-range-dependent LMIs for the stability and stabilization of the considered system with considering a mixed H_2/H_∞ performance measure.

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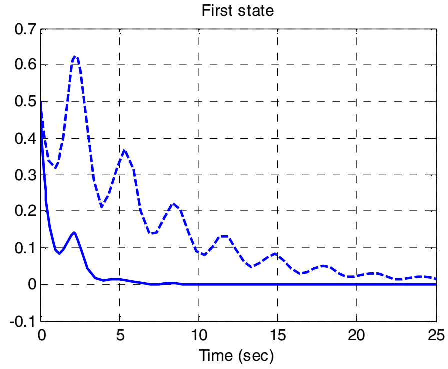
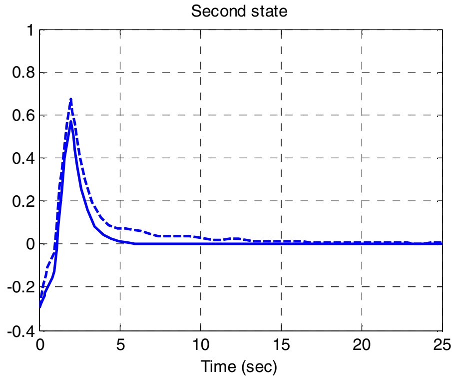


(a)



(b)

Fig 1. Response of the states $x_1(t)$ and $x_2(t)$: closed-loop system (solid line) and b) open-loop system (dashed line).

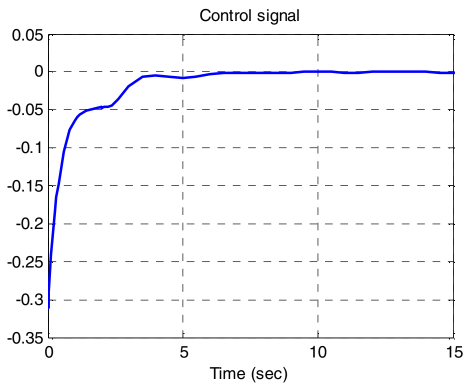


Fig. 2. Control law for system.