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Whether the Spreaded Good Opinion About Fuzzy Controllers is Justified ⋆

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Abstract: In the paper, using some MATLAB fuzzy logic toolbox Demos, in which the fuzzy controllers are compared with the classical PID ones, it is shown that the well tuned classical PID are significantly better than those fuzzy presented in the Demos. It is shown, that using fuzzy approach, it is very difficult to shape the input-output nonlinearity, describing the so called fuzzy block of the fuzzy controller. It is also shown, that the linear fuzzy block (created to obtain comparable results with the classical PID controllers) is not justified at all, because it may be replaced by the usual summing junction connection, which is significantly simpler. The considerations of the paper do not support the idea of fuzzy controllers.

1. INTRODUCTION

The most important reason of creation of the idea of fuzzy controllers was the possibility of utilization of the knowledge of experts concerning the rules of control. The possibility of utilization of this knowledge seemed to be interesting especially then, when the models of the plant was not known and when it was difficult to built these models. It was accepted that the control rules are expressed by the experts in a linguistic form (Lee, 1990). The linguistic formulations like "large positive", "medium positive" etc., determining the value of a variable, must then be completed by some membership functions, explaining the meaning of these formulations by means of the fuzzy logic language (Driankov *et al.*,1996; Pedrycz, 1993).

And here there arises the problem, because the responsible experts from control, usually do not wont to choose the appropriate membership functions from a large set of them. Therefore these functions, as well as the other operations used in fuzzy controllers like: aggregation, activation, accumulation and defuzzyfication (Jantzen, 1998), which are also not uniquely determined must be chosen by the controller designer. In this manner the designer has a larger influence on the shape of the input-output characteristic of the nonlinear static element (called in the present paper the fuzzy block) of the fuzzy controller than the linguistic rules of the expert have.

Therefore nowadays, all the component parts of the process of the fuzzy controller design, together with the rules, membership functions and mentioned above operations are chosen by the designer to obtain reasonable input-output characteristic of the fuzzy block (Jantzen, 1998). One may note that this method of design (based on fuzzy approach), of the nonlinear static element with a demanded nonlinear input-output characteristic is very complicated. In connection with this there arises the question whether this method of design may be justified at all.

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The second cause of undertaking the considerations of the present paper is the spreading conviction (especially in the fuzzy logic literature) about superiority of fuzzy controllers. This convictions may be met also in industry. Lastly, the author of the present paper heard from the specialist who designed the automation of power block, that the people from industry had questioned about possibility of applying the fuzzy controllers as modern and better solution. Since similar convictions are not isolated and the conviction of the author of the present paper is different, then it is worthwhile to feed the discussion concerning the matter (see Athans – Zadeh debate).

In the present paper, using three MATLAB Fuzzy Logic Demos, in which the fuzzy controllers are compared with classical PID ones, it is shown that the well tuned classical PID controllers are significantly better than those fuzzy presented in the Demos. It is noted, that it is very difficult to shape the input-output nonlinearity of the fuzzy block using fuzzy approach. It is also shown, that the linear fuzzy block created e.g. in (Mizumoto, 1992; Jantzen, 1998) to obtain comparable results of the fuzzy and classical PID controllers is not justified at all, because it may be replaced by usual and significantly simpler summing junction connection.

2. THE PLANT IN THE MATLAB DEMOS

The fuzzy logic toolbox of the MATLAB contains several Demos in which a fuzzy controller is compared with the classical PID one (see Programs of MATLAB fuzzy logic toolbox). We will focus our attention on three of them, namely sltank.mdl, sltankrule.mdl and sltank2.mdl. The plant simulated in these Demos is the tank shown in Fig. 1a, with a liquid inflow q_1, outflow q_2 and level h.

The case of the free gravity outflow $q_2 = c\sqrt{h}$ is considered so that the tank is described by the equation

$$P\frac{dh}{dt} + c\sqrt{h} = q_1 \qquad (1)$$

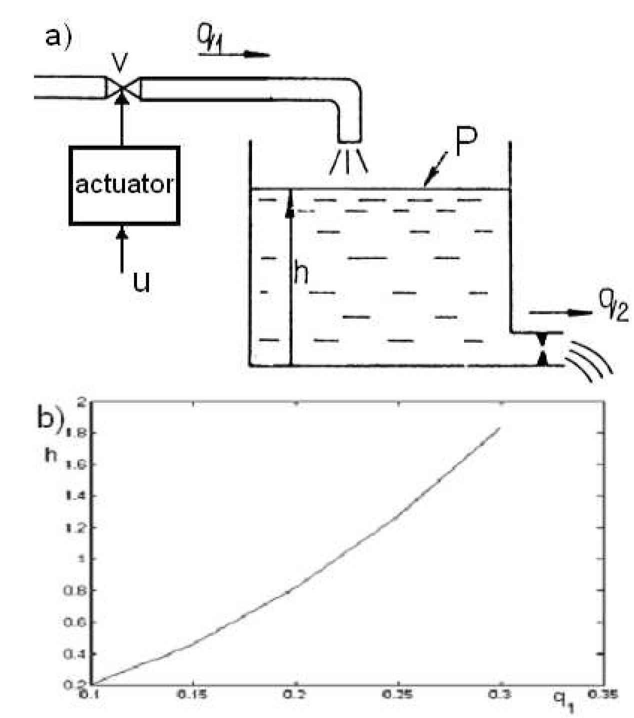

Fig. 1. a) tank with free outflow b) steady state characteristic

where P is the area of the liquid surface and c is a constant coefficient. The simulated model of the tank takes into account the constraint $0 \leq h \leq 2$. Thus the steady state characteristic for $0 \leq h \leq 2$ is described by

$$h = \frac{1}{c} q_1^2 \tag{2}$$

The steady state characteristic with the readings taken from the simulated tank is shown in Fig.1b. Thus the plant is nonlinear, as for $h \approx 0.3$ the gain $\Delta h / \Delta q_1 \approx 5.1$, while for $h \approx 1.5$ we have $\Delta h / \Delta q_1 \approx 11.2$.

In the linearity range $0 \leq q_1 \leq 0.313$ the model of the valve v together with the actuator is described by

$$q_1(t) = c_1 \int_0^t u(\tau) d\tau \tag{3}$$

where c_1 is a constant coefficient and u is the control signal. Thus dependence (3) describes the model of the integrator.

3. THE CLASSICAL PID CONTROLLERS

The transfer function of the PID controller

$$C(s) = k_1 + k_2 \frac{1}{s} + k_3 \frac{Ns}{s + N} \tag{4}$$

in the Demos sltank.mdl and sltankrule.mdl has the parameters $k_1 = 2$, $k_2 = 0$, $k_3 = 1$, $N = 100$, which are not well tuned to the plant. In connection with this the response of the systems with the applied classical PD controller to the rectangular stepwise reference value is at the first look somewhat worse than for the systems with the applied fuzzy controller. The fuzzy controller, which is the same in both the mentioned Demos, will be described in the next section.

In the sltank2.mdl not an approximation of derivative is used (as in (4)), but the strict formula $k_3 s$, which causes some troubles in numerical calculation of derivative. Also for this Demo the parameters of classical PID controller (taking the values $k_1 = 2$, $k_2 = 0$, $k_3 = 1$) are not well tuned, which causes that the system with the applied classical PD controller at the first look is also somewhat worse than the system with the fuzzy controller. The fuzzy controller applied here is different from that applied in the first two Demos.

4. DESCRIPTION OF THE FUZZY CONTROLLERS

4.1 sltank.mdl and sltankrule.mdl

Since in the closed loop it appears integrator (actuator controlling the valve) a fuzzy PD controller is applied in the Demos. Strictly speaking the parts P and D of the fuzzy controller are realized outside of the fuzzy block, which is a nonlinear element with two inputs: error $e = r - h$ (r is the reference value) and the derivative of level dh and one output u, determining the control signal. It should be stressed that the fuzzy block is a static, nonlinear element described by the input-output characteristic, corresponding to some function $u = f(e, dh)$, the mathematical formula of which is not known. The input-output characteristic may be obtained by appropriate numerical calculations (in the two discussed MATLAB Demos using e.g. the command "fuzzy tank"). Only the fuzzy block (corresponding to the function $u = f(e, dh)$) is designed using fuzzy logic approach.

The membership functions of the signals e, dh and u, for the fuzzy block applied in the discussed two Demos are shown in Fig. 2 a, b, c, respectively. The graphical illustration of the applied rules, obtained from the MATLAB FIS editor is shown in Fig. 3. The principle of operation of the fuzzy block is also illustrated, with accounting activation, accumulation and defuzzyfication. For the exemplary inputs $e = -0.349$, $dh = -0.0609$, the determination of the output $u = -0.236$ is shown.

The surface described by the function $u = f(e, dh)$ and the intersections $u = f(e, 0)$ and $u = f(0, dh)$ are shown in Fig. 4 a, b, c, respectively.

The time responses of the Demos to the rectangular change of the reference value r are shown in Fig. 5a. At the first look they look well. However in the zoomd responses (around the higher and lower reference values), shown in Fig.5 b and c, respectively, some overshots and steady state errors appear.

Note that the nonsymmetrical placement of the membership functions for u (in Fig. 2c the membership function for "open low" is shifted left) has been applied to suite the controller characteristic to the nonlinearity of the plant. It causes the stronger reaction of the derivative D around the higher reference value (e positive) due to asymmetry of the function $u = f(0, dh)$ shown in Fig. 4c. Owing to this the overshots appearing around the higher and lower reference values are comparable. However it appears additionally a non demanded effect in the form of the steady state errors,

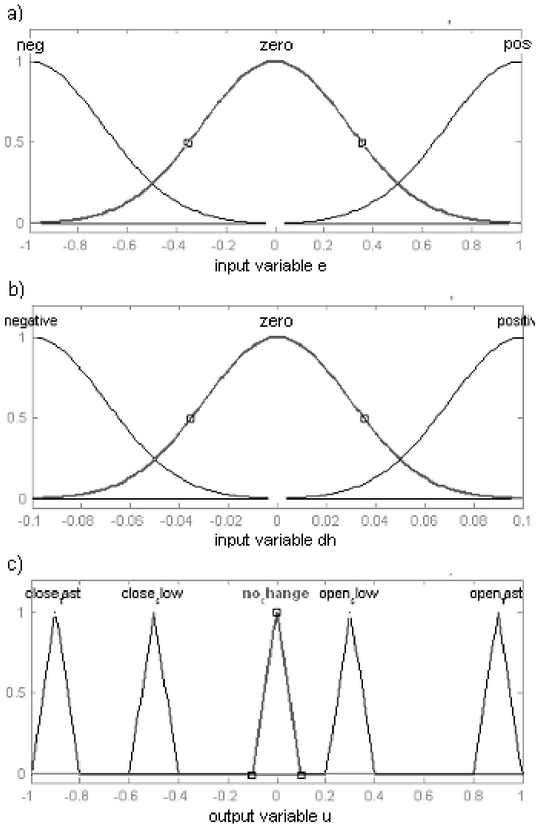

Fig. 2. The membership functions for the signals a) e, b) dh, c) u

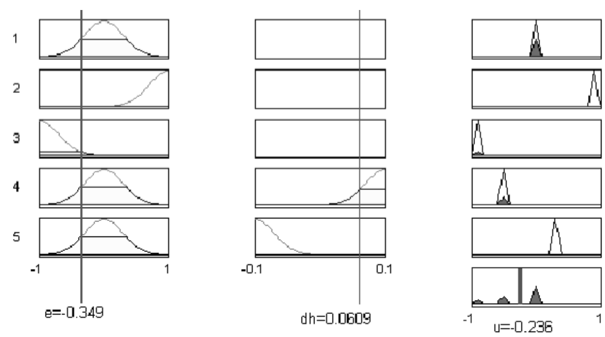

Fig. 3. The rules used for the "fuzzy tank" in sltank.mdl

though in the loop it appears integrator. This may be justified by the fact that due to asymmetry of the plot from Fig. 4c the function $u = f(e, dh)$ takes a nonzero value for $e = 0$ and $dh = 0$. Really, from accurate readings taken from simulations, it results that for $e = 0$ and $dh = 0$ it is $u = -0.000761$ while for $e = 0.01$ and $dh = 0$ it is $u = 0$. Therefore the steady state error $e = r - h = 0.01$ appears both for higher and lower reference value (see Fig. 5b and 5c).

It is worthwhile to stress that if we would apply a symmetrical placement of the membership functions for u (i.e. the membership function "open low" would be shifted by 0.2 to the right) then the steady state errors for higher and lower reference value would be zero, but due to nonlinearity of the plant the overshot around the higher reference value would be higher and for the lower reference value it would be no overshot. These considerations illustrate

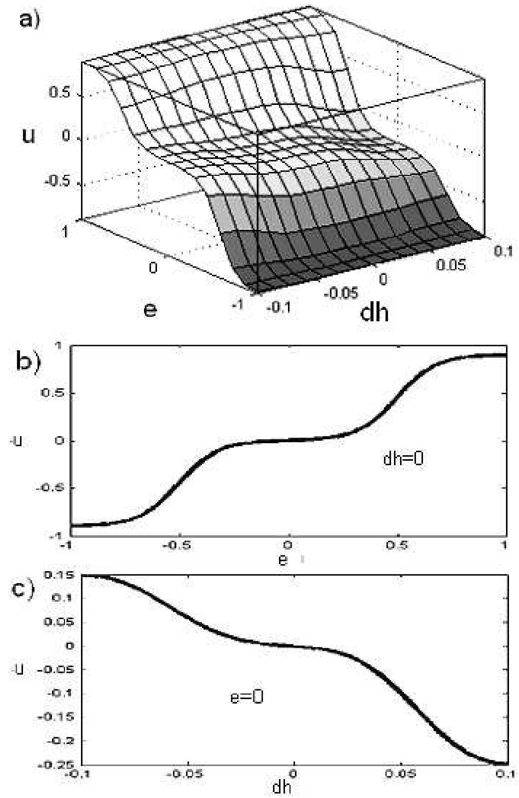

Fig. 4. a) The surface $u = f(e, dh)$, and the intersections b) $u = f(e, 0)$ and c) $u = f(0, dh)$

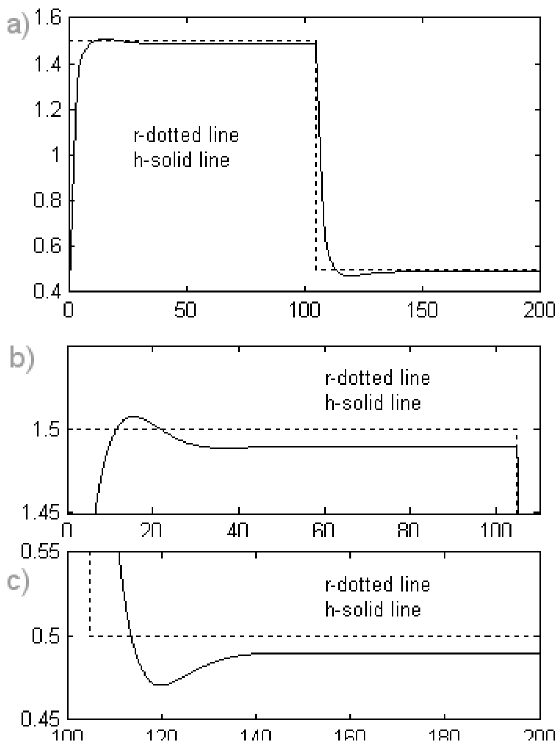

Fig. 5. a) Time responses of the sltank,mdl with fuzzy controller, b) and c) the zoomd responses

the difficulties in shaping nonlinearity of the characteristic $u = f(e, dh)$ of the fuzzy block, using fuzzy approach.

4.2 *sltank2,mdl*

To omit repetitions in considerations, the fuzzy controller for the sltank2.mdl will not be described in detail. However it is worthwhile to note, that in this case the trial of the suiting the fuzzy block characteristic $u = f(e, dh)$ to the nonlinearity of the plant was made by choosing the nonsymmetrically placed membership functions for the variable e shown in Fig. 6, which causes an increase of the P regulator gain for negative error e. The membership functions for dh and u are placed symmetrically. Also in this case in the closed loop system there appear steady state errors resulting from the same reason as in the first two Demos previously discussed.

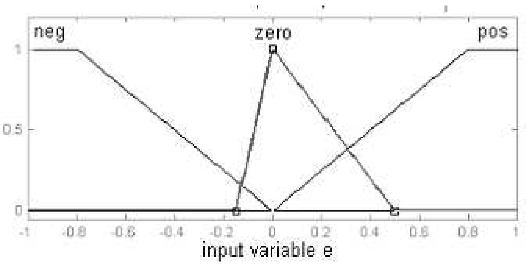

Fig. 6. The membership functions of e for sltank2.mdl

5. THE COMPARISON WITH THE WELL TUNED CLASSICAL PID CONTROLLER

To make an objective comparison of the systems with fuzzy and classical PID controllers some modification of the later has been introduced in the Demos. Fist, in all the three Demos the saturations of the classical PID control signal u was introduced so that $-0.9 \leq u \leq 0.9$, which is comparable with the saturations appearing in fuzzy implementation of the control signal u in the Demos. Second the parameters of the classical PID controller have been tuned to the values $k_1 = 16$, $k_2 = 0$, $k_3 = 10.2$, $N = 100$.

The responses of the sltank.mdl and sltankrule.mdl Demos with modified in this manner the classical PD controller, to rectangular stepwise change of the reference value, are shown in Fig. 7. It is seen that they are significantly better than those of the fuzzy controller (compare Fig. 5). Also after zooming no steady state errors and no overshots are shown. A similar improvement has been obtained for the sltank2.mdl.

More exactly the difference between the quality of the fuzzy and classical PID controllers is seen when the sinusoidal reference value $r = 1 - 0.5sin(0.03t)$ is applied. In Fig. 8a it is seen the time response of the sltank.mdl Demo with fuzzy controller, while in Fig. 8b – with well tuned classical PID one. From these responses the superiority of the classical PID over the fuzzy controller is exactly seen.

One may note that in this case the large error of the sltank.mdl with fuzzy controller results from the shape of the characteristic $u = f(e, dh)$ shown in Fig. 4b. Really, the slope of this characteristic in the vicinity of $e = 0$ is equal to 0.0761 and this slope determines the gain of the P part of the fuzzy controller at the vicinity of $e = 0$. Under sinusoidal reference value the appearing errors lay in the region with small gain of the P part, which gives the relatively large errors. Thus the relatively large errors

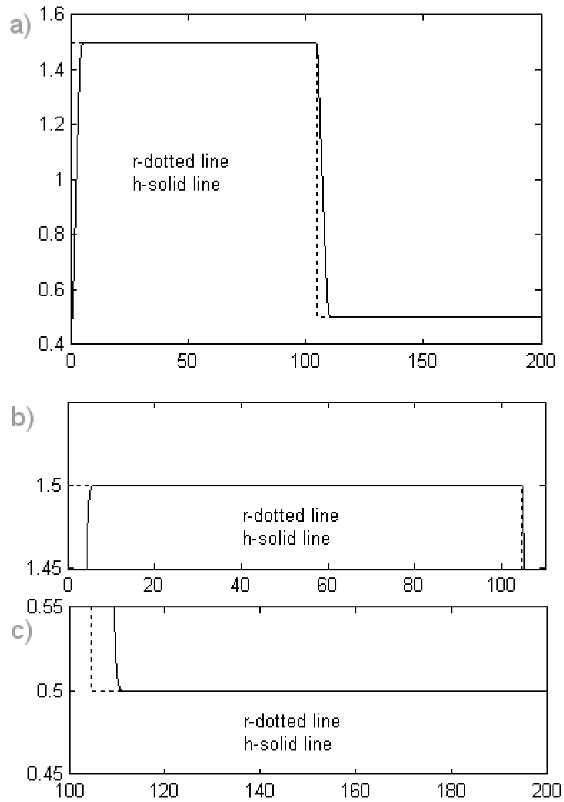

Fig. 7. a) Time responses of the sltank.mdl with classical well tuned PID controller, b) and c) the zoomd responses

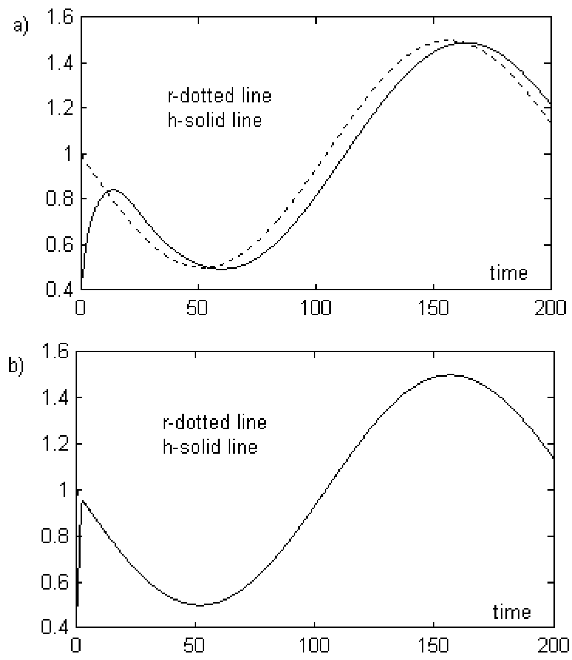

Fig. 8. a) Time responses of the sltank.mdl with a) fuzzy controller, b) classical well tuned PID controller (for $r = 1 - 0.5sin(0.03t)$)

result among others from the shape of the P characteristic shown in Fig. 4b.

6. SIMPLER IMPLEMENTATION OF NONLINEARITY OF THE FUZZY BLOCK

The rules used for designing the fuzzy block (nonlinear element) may be described in a tabular linguistic form shown in Fig. 9. Here, as the inputs the error e and the error derivative de are used. The input variables are laid out along the axis, while the output variable u is inside the table. The used acronyms for output are: PB – positive big, PM – positive medium, NB – negative big, NM – negative medium. From this table it results that e.g. if e is Pos and de is Zero then u is PM. Of course the linguistic statements like Pos, Neg, PB, PM should be completed by the corresponding membership functions for appropriate variable.

error e \ derivative de	Neg	Zero	Pos
Neg	NB	NM	Zero
Zero	NM	Zero	PM
Pos	Zero	PM	PB

Fig. 9. Tabular linguistic form of the rules

Sometimes, in the place of the linguistic determination of the output u, like PB, PM some appropriate definite values 200, 100, etc. called singleton outputs are used, which is shown in Fig. 10. Singleton outputs simplifies design of the fuzzy block, significantly.

error e \ derivative de	Neg	Zero	Pos
Neg	-200	-100	0
Zero	-100	0	100
Pos	0	100	200

Fig. 10. Tabular linguistic form of the rules with singleton outputs

One step ahead is the usage of singletons (or determined values) for all the variables e, de, u. In this manner there arise the lookup table (Jantzen,1998) shown in Fig. 11. It is easy to note that the lookup table determines the nonlinearity $u = f(e, de)$ in discrete points. The lookup table may be accomplished by some appropriate interpolation, giving the determination of the nonlinearity in any needed point $[e, de]$ not appearing in the table. In comparison to the fuzzy, approach the lookup table is the significantly easier way of implementation of nonlinearity $u = f(e, de)$, which may be shaped locally, using some appropriate interpolation.

Another method of implementation of nonlinearity, which may be locally shaped, is its approximation using appropriate polynomials.

One may say that fuzzy approach is the most complicated and less appropriate way of designing the nonlinearity. Moreover the assumption that the rules are taken from experts is unrealistic, First, the linguistic rules itself

error e \ derivative de	-100	-50	0	50	100
-100	-200	-160	-100	-40	0
-50	-160	-121	-61	0	40
0	-100	-61	0	61	100
50	-40	0	61	121	160
100	0	40	100	160	200

Fig. 11. Example of a lookup table

without the membership functions have little information. Second, the responsible experts from control are not able to choose the membership functions from a large set of them. Third, the shape of the nonlinearity $u = f(e, de)$ are dependent from many freely chosen design components, not dependent from experts such as: the shape of the membership functions, as well as the formulas for aggregation, accumulation and defuzzyfication. In this manner we return to design of the fuzzy block by appropriate choice of the mentioned design components to obtain a reasonable nonlinearity. But this way is ineffective. It is easier to shape the needed nonlinearity using other e.g. mentioned above methods.

7. THE CASE OF LINEAR FUZZY BLOCK

A special kind of the fuzzy block is when the function $u = f(e, de)$ is linear (Mizumoto, 1992; Jantzen, 1998). One may suppose that the goal of designing of the linear fuzzy block (in the domain of its determination, or using fuzzy logic language –in the universe of discourse) was to obtain a comparable performances of the fuzzy and classical PID controllers. The membership functions of the signals e and de are shown in Fig. 12a, while of the singleton outputs are shown in Fig. 12b .

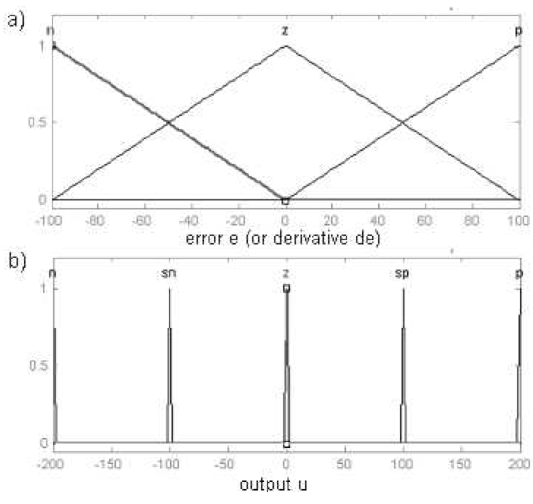

Fig. 12. The membership functions of a linear fuzzy block a) the same for e and de, b) singletons for u

If for the aggregation, accumulation and defuzzyfication the operations min, max and "centre of gravity" are used, respectively, the function $u = f(e, de)$ describes the plain shown in Fig. 13a. The plots of the intersections $u = f(e, 0)$ and $u = f(0, de)$ with the axis (e, u) and (de, u), respectively, are the same and they are shown in Fig. 13b. Therefore we have $f(e, 0) = e$ and $f(0, de) = de$ from which it results that the function $u = f(e, de)$ takes the form $u = e + de$. Thus in this case the function of the fuzzy block may be implemented as usual summing junction connection and it is rather a strange thing to propose then the very complicated implementation using fuzzy approach. The advantage of the simple summing junction implementation in comparison to the very complicated fuzzy implementation is evident.

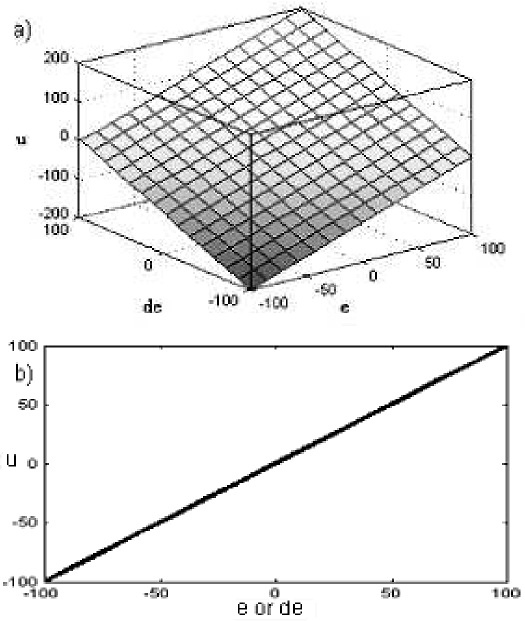

Fig. 13. a) The surface $u = f(e, de)$ and b) the plots $u = f(e, 0)$ and $u = f(0, de)$

One may note that also any nonlinear function $u = f(e, de)$ may be significantly easier implemented using other e.g. mentioned in section 6 methods than using fuzzy approach. By the way, the fuzzy block described by nonlinear function $u = f(e, de)$ may be interpreted as "nonlinear summing junction" accumulating the influence of the P and D part of the controller.

8. FINAL CONCLUSIONS

As it results from the analyzed Demos the "fuzzy controllers" are worse than the classical PID ones. There arises the question whether this observation concerns also other systems with fuzzy controllers and therefore whether it has some more general character. One may suppose that yes and this view is supported by the following consideration.

The fuzzy block or nonlinear static element described by the function $u = f(e, de)$, theoretically gives some limited possibility of improving controller. However first, the same possibility gives the nonlinear element described by the function $u = f(e, de)$, which may be easier implemented using other methods e.g. lookup tables or polynomial approximation; the latter methods gives the possibility of local shaping of the nonlinearity. Second, the local shaping of a demanded nonlinearity by means of the fuzzy approach is a very difficult if at all implementable way. Third, the problem of demanded, static nonlinearity with two or more inputs and one output which improves controller is weakly recognized in nonlinear control theory; one may suppose that generally a dynamic nonlinear element may create better possibilities of improving controller, but this problem goes beyond the scope of fuzzy approach. Fourth, the essential disadvantage is non analytical description of the nonlinearity $u = f(e, de)$ using fuzzy approach, which creates additional difficulties in analyzing the stability and operation of the system (Passino, 1998); only methods based on simulations are available.

One may realize, that the fuzzy controllers applied in the discussed Demos are not the best ones and probably the better fuzzy controllers may be found. However from the above considerations it results how difficult task it is. The difficulty results from the fact, that they are no elaborated methods of design of fuzzy controllers, therefore only the intuitive seeking of the solution by using simulations may be applied.

Of course, it is possible to built the fuzzy blocks with more than two inputs, however then the problem becomes more complicated, because more rules must be used to design the controller. The above remarks concern also this case.

From above considerations it results that they are no advantages speaking for fuzzy controllers.

To summarize, the good opinion about fuzzy controllers is completely not justified, especially for the control system for which the speed of decaying of the transients is interesting for users. In this case the information about dynamics of the plant must be accounted during design of the controller.

On the other hand, if the plant is stable and a very slow control is acceptable, then the dynamics of the plant plays a negligible role. One may suppose, that in this case the linguistic rules of experts and fuzzy approach may solve the problem. However even then, the fuzzy way of design is to difficult to be accepted. Really, for this case which is relatively easy for solving, some other simpler methods may be applied (e.g. the classical integral I controller with small gain).

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Programs of MATLAB fuzzy logic toolbox Demos.