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A Study on the Effect of Variable Initial State Error in Average Operator-Based Iterative Learning Control ⋆

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Abstract: This paper studies the effect of variable initial state error in iterative learning control (ILC) algorithms for linear time-invariant (LTI) systems. It is first pointed out that the previous result based on an average operator has a restriction due to a specific condition for convergence, even though it shows that the effect of the initial state error can be accurately estimated while the existing algorithms show only the boundness of the error or the convergence from stochastic point of view. To relieve this limitation, a modified ILC algorithm is proposed and a sufficient condition for convergence is presented. In order to show the validity of the proposed algorithm, a numerical example is given.

1. INTRODUCTION

In many studies on iterative learning control (ILC) algorithms, there have been a great deal of efforts to lighten a restriction that the initial state value of the system should be same as that of the desired trajectory at each iteration. Lee and Bien found that the proportional term of the error in PD-type ILC algorithms can be positively utilized in a way that the effect of the initial state error is exponentially reduced when the initial state value is same at each iteration but different from the desired one (Lee and Bien [1996]). Then, Park *et al.* generalized this result to a PID-type ILC algorithm (Park et al. [1999]) and a continuous operator-based ILC algorithm (Park and Bien [2000]) showing that the error reduction can be effectively controlled by using multiple learning gains or an appropriate operator. Sun and Wang also introduced initial rectifying action to address the initial state error problem (Sun and Wang [2002]). However, since it is practically impossible to set the initial state value of the system at the same value perfectly, it is inevitable to have deviation in initialization from the initial state value at the previous iteration even though it may be very small.

To relieve this limitation, there have been various studies to analyze the effect of variable initial state error and find a robust algorithm alleviating the requirement so that the initial state $x_k(0)$ at each iteration k remains in the neighborhood of any fixed point x_0, i.e.,

$$\|x_k(0) - x_0\|_\infty \le \epsilon \tag{1}$$

where $\|\cdot\|_\infty$ is defined as

$$\|x\|_\infty = \sup_{1\le i\le n} |x^i|$$

for an n-dimensional vector $x = \left(x^1, x^2, \cdots, x^n\right)'$, and its induced matrix norm is defined as

$$\|A\|_\infty = \sup_{1\le i\le n} \sum_{j=1}^{r} |a^{ij}|$$

for an $n \times r$ matrix A with components a^{ij}. From the boundness of the initial state error (1), they showed that the output error trajectory $e_k(\cdot)$ between the desired output trajectory $y_d(\cdot)$ and the output trajectory of the system $y_k(\cdot)$, $e_k(\cdot) = y_d(\cdot) - y_k(\cdot)$, is bounded in the sense of λ-norm as shown in the following inequality:

$$\lim_{k\to\infty} \|e_k(\cdot)\|_\lambda \le k_s O\left(\lambda^{-1}\right)\epsilon \tag{2}$$

where k_s depends on the system parameters and the learning gains, $O\left(\lambda^{-1}\right)$ is a function of λ which decreases as λ increases, and $\|\cdot\|_\lambda$ is defined by

$$\|f(\cdot)\|_\lambda = \sup_{0\le t\le T} e^{-\lambda t}\|f(t)\|_\infty$$

for a vector function $f : [0,T] \to \mathcal{R}^n$. However, since the boundness is obtained in the sense of λ-norm, the previous result (2) cannot ensure that a huge magnitude in the actual output error does not appear at a large time instant $t \gg 0$ even when the error bound is very tiny in the sense of λ-norm, since the λ-norm has the property by which the actual error trajectory is exponentially

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weighted. For example, in the previous results, even when $\lim_{k\to\infty} e_k(t) = \epsilon e^{at}$, the result shows a very small error bound in the sense of λ-norm: $\lim_{k\to\infty} \|e_k(\cdot)\|_\lambda \leq \epsilon, \lambda > a = \|A\|_\infty$. This may prevent active applications of the method in real world problems with a long time interval $[0,T], T \gg 0$. Sun and Wang proved the boundness of the output error without using λ-norm (Sun and Wang [2003]). However, the upper bound still increases as time interval increases. Fang and Chow showed that the output error asymptotically converges to zero, i.e., $\lim_{k\to\infty} e_k(t) = 0$, for $0 \ll t \leq T$ (Fang and Chow [2003]). However, the asymptotic convergence doesn't show the effect of the initial state error for whole trajectory. Saab studied the convergence of the ILC algorithms from stochastic point of view assuming zero-mean white initial state error and showed that the input error covariance matrices converge to zero (Saab [2003]). However, since the result is given in the sense of mean and standard deviation, the exact effect of the initial state error cannot be found. Recently, Park proposed an average operator-based algorithm and showed that the effect of the initial state error can be estimated in terms of desired output trajectory, system parameters, initial state values and learning gains assuming the initial state value satisfies the following conditions (Park [2005]):

$$\lim_{k\to\infty} avg\,\{x_i(0)\}_{i=0}^{k} = x_0 \tag{3}$$

and, for some positive constants β and γ,

$$\|avg\,\{x_i(0)\}_{i=0}^{k} - x_0\|_\infty \leq \beta e^{-\gamma k} \tag{4}$$

where $avg\,\{\cdot\}_{i=0}^{k}$ denotes an average operator which is defined as

$$avg\,\{h_i(\cdot)\}_{i=0}^{k} = \frac{1}{k+1}\sum_{i=0}^{k} h_i(\cdot)$$

for a sequence $h_0(\cdot), h_1(\cdot), \cdots, h_k(\cdot)$. However, the condition (4) still has a restriction that the algorithm can be applied only when it is ensured that the property of initialization process satisfies (4) by long periods of observation. Otherwise, we cannot guarantee that the effect of the initial state error is converged and accurately estimated.

The main purpose of this paper is to remove the strict condition (4) and find a new convergence condition to get a similar result by which the effect of the initial state error can be obtained as a time function of initial state values, system parameters and learning parameters. It is remarked that the assumption (3) is acceptable in general since the average of the samples approaches the population mean, the mean of the underlying distribution, as the sample size increases.

2. AVERAGE OPERATOR-BASED ILC

In this section, the effect of variable initial state error is shown for LTI systems. Consider the linear system described by (5) and the ILC algorithm described by (6).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{5}$$

$$\begin{aligned} u_{k+1}(t) &= avg\,\{u_i(t)\}_{i=0}^{k} \\ &\quad + \Gamma\left(avg\,\{\dot{e}_i(t)\}_{i=0}^{k} - R\; avg\,\{e_i(t)\}_{i=0}^{k}\right) \end{aligned} \tag{6}$$

Here, $x \in \mathcal{R}^n$, $u \in \mathcal{R}^r$ and $y \in \mathcal{R}^q$ denote state, input and output, respectively. A, B and C are matrices with appropriate dimensions and it is assumed that CB is a full rank matrix. Suppose that the desired output trajectory $y_d(\cdot)$ is continuously differentiable on $[0,T]$ and the algorithm starts with a bounded control input $u_0(\cdot)$. As commented in Park [2005], the algorithm (6) can be implemented by incremental update formulas to prevent computational burden:

$$\begin{aligned} v_{k+1}(t) &= avg\,\{u_i(t)\}_{i=0}^{k+1} = \frac{k+1}{k+2}v_k(t) + \frac{1}{k+2}u_{k+1}(t) \\ v_0(t) &= u_0(t) \\ s_{k+1}(t) &= \sum_{i=0}^{k+1}\left(\dot{e}_i(t) - Re_i(t)\right) = s_k + \dot{e}_{k+1}(t) - Re_{k+1}(t) \\ s_0(t) &= \dot{e}_0(t) - Re_0(t) \\ u_{k+1} &= v_k(t) + \frac{1}{k+1}\Gamma s_k(t). \end{aligned}$$

This implementation requires memory only for $v_k(t)$, $s_k(t)$ and k, and only the small computation for each iteration.

Before showing the effect of the initial state error, we need the following lemmas, whose results are utilized in the proof of the main result on convergence.

Lemma 1. Let a_k be a nonnegative real value for every integer $k \geq 0$. Assume that a_0 is bounded and suppose that ϵ and ρ be real values such that $\epsilon > 0$ and $0 \leq \rho < \sqrt{2}-1$. Then, the inequality

$$a_{k+1} < \rho\; avg\,\{a_i\}_{i=0}^{k} + \epsilon$$

implies

$$a_k < \left(\sqrt{k+1} - \sqrt{k}\right)a_0 + \frac{\epsilon}{1-\rho}, \forall k \geq 0. \tag{7}$$

Proof. For the proof, we employ the method of mathematical induction. For each $m \geq 0$, let P_m be the statement that

$$a_m < \left(\sqrt{m+1} - \sqrt{m}\right)a_0 + \frac{\epsilon}{1-\rho}.$$

From the assumptions, we can easily obtain that the statement P_0 is true. That is

$$a_0 < \left(\sqrt{1} - \sqrt{0}\right)a_0 + \frac{\epsilon}{1-\rho}.$$

Now, suppose that statement P_n is true for every integer n with $0 \leq n \leq k$. Then, it can be shown that

$$\begin{aligned} a_{k+1} &< \rho\; avg\,\{a_i\}_{i=0}^{k} + \epsilon \\ &< \frac{\rho}{k+1}\left\{\left(\sqrt{1}-\sqrt{0}\right)a_0 + \frac{\epsilon}{1-\rho} + \left(\sqrt{2}-\sqrt{1}\right)a_0 \right. \\ &\quad + \frac{\epsilon}{1-\rho} + \left(\sqrt{3}-\sqrt{2}\right)a_0 + \frac{\epsilon}{1-\rho} + \cdots \\ &\quad \left. + \left(\sqrt{k+1}-\sqrt{k}\right)a_0 + \frac{\epsilon}{1-\rho}\right\} + \epsilon \end{aligned}$$

$$= \frac{\rho}{\sqrt{k+1}} a_0 + \frac{\epsilon}{1-\rho}. \tag{8}$$

Since $0 \le \rho < \sqrt{2} - 1$, we can find that

$$k + 1 + \rho < \sqrt{(k+2)(k+1)}, \forall k \ge 0. \tag{9}$$

From (8) and (9), we can conclude that

$$a_{k+1} < \left(\sqrt{k+2} - \sqrt{k+1}\right) a_0 + \frac{\epsilon}{1-\rho}$$

which establishes the truth of the statement P_{k+1}. By mathematical induction, (7) is true. This completes the proof.

Lemma 2. Let a_k and b_k be nonnegative real values for every integer $k \ge 0$. Assume that a_0 is bounded and suppose that $\lim_{k\to\infty} b_k = 0$ and $0 \le \rho < \sqrt{2} - 1$. Then, the inequality

$$a_{k+1} \le \rho \; avg \left\{a_i\right\}_{i=0}^{k} + b_k$$

implies

$$\lim_{k\to\infty} a_k = 0.$$

Proof. Since $\lim_{k\to\infty} b_k = 0$ and $\lim_{k\to\infty} \frac{a_0}{\sqrt{k+1}+\sqrt{k}} = 0$ from the assumptions, for any $\epsilon > 0$, there exists a positive K such that

$$b_k < \frac{\epsilon}{2}(1-\rho), \forall k \ge K$$

and

$$\frac{a_0}{\sqrt{k+1}+\sqrt{k}} < \frac{\epsilon}{2}, \forall k \ge K. \tag{10}$$

This gives

$$a_{k+1} < \rho \; avg \left\{a_i\right\}_{i=0}^{k} + \frac{\epsilon}{2}(1-\rho), \forall k \ge K.$$

From Lemma 1 and (10), we can obtain that

$$a_k < \left(\sqrt{k+1} - \sqrt{k}\right) a_0 + \frac{1}{1-\rho}\frac{\epsilon}{2}(1-\rho)$$
$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon, \forall k \ge K,$$

and we can conclude that

$$\lim_{k\to\infty} a_k = 0.$$

This completes the proof.

Now, the effect of the initial state error for the ILC law (6) will be shown.

Theorem 3. Suppose that the update law (6) is applied to the system (5) and the initial state value at each iteration satisfies the condition (3). If there exists ρ satisfying

$$\|I - \Gamma CB\|_\infty \le \rho < \sqrt{2} - 1$$

then

$$\lim_{k\to\infty} \left(y_k(t) + Ce^{At}(x_0 - x_k(0))\right) = y_d(t) - e^{Rt}C(x_d(0) - x_0), \forall t \ge 0.$$

Proof. Let $u_a(t)$ and $x_a(t)$ be the control input and the state that satisfy (11)

$$x_a(t) = x_0 + \int_0^t (Ax_a(\tau) + Bu_a(\tau))\, d\tau$$
$$y_d(t) - e^{Rt}C(x_d(0) - x_0) = Cx_a(t). \tag{11}$$

Let

$$\Delta u_k(t) = u_a(t) - u_k(t)$$
$$\Delta x_k(t) = x_a(t) - x_k(t).$$

It follows from (6) and (11) that

$$\begin{aligned}\Delta u_{k+1}(t) &= avg \left\{\Delta u_i(t)\right\}_{i=0}^{k} \\ &\quad - \Gamma\left(avg\left\{\dot{e}_i(t)\right\}_{i=0}^{k} - R\; avg\left\{e_i(\cdot)\right\}_{i=0}^{k}\right) \\ &= avg\left\{\Delta u_i(t)\right\}_{i=0}^{k} \\ &\quad - \Gamma\left(C\; avg\left\{\Delta \dot{x}_i(t)\right\}_{i=0}^{k} - RC\; avg\left\{\Delta x_i(\cdot)\right\}_{i=0}^{k}\right) \\ &= (I - \Gamma CB)\, avg\left\{\Delta u_i(t)\right\}_{i=0}^{k} \\ &\quad - \Gamma(CA - RC)\, avg\left\{\Delta x_i(t)\right\}_{i=0}^{k}. \end{aligned} \tag{12}$$

Taking the norm $\|\cdot\|_\lambda$ on both sides of (12), we have

$$\|\Delta u_{k+1}(\cdot)\|_\lambda \le \rho \|avg\left\{\Delta u_i(\cdot)\right\}_{i=0}^{k}\|_\lambda + \|\Gamma(CA - RC)\|_\infty \|avg\left\{\Delta x_i(\cdot)\right\}_{i=0}^{k}\|_\lambda. \tag{13}$$

From (11), we can obtain

$$\Delta x_k(t) = e^{At}(x_0 - x_k(0)) + \int_0^t e^{A(t-\tau)} B \Delta u_k(\tau) d\tau. \tag{14}$$

Applying an average operator and taking the norm $\|\cdot\|_\lambda$ on both sides of (14), we find that

$$\|avg\left\{\Delta x_i(\cdot)\right\}_{i=0}^{k}\|_\lambda \le \|x_0 - avg\left\{x_i(0)\right\}_{i=0}^{k}\|_\infty + \frac{1 - e^{-(\lambda - a)T}}{\lambda - a}\|B\|_\infty \|avg\left\{\Delta u_i(\cdot)\right\}_{i=0}^{k}\|_\lambda \tag{15}$$

where

$$\lambda > a = \|A\|_\infty.$$

Substituting (15) into (13), we further find that

$$\begin{aligned}&\|\Delta u_{k+1}(\cdot)\|_\lambda \\ &\quad \le \left(\rho + c_1 \frac{1 - e^{-(\lambda - a)T}}{\lambda - a}\|B\|_\infty\right)\|avg\left\{\Delta u_i(\cdot)\right\}_{i=0}^{k}\|_\lambda \\ &\quad\quad + c_1\|x_0 - avg\left\{x_i(0)\right\}_{i=0}^{k}\|_\infty \end{aligned} \tag{16}$$

where

$$c_1 = \|\Gamma(CA - RC)\|_\infty.$$

Since $0 \le \rho < \sqrt{2} - 1$ by assumption, it is possible to choose λ sufficiently large so that

$$\rho_0 = \rho + c_1 \frac{1 - e^{-(\lambda - a)T}}{\lambda - a}\|B\|_\infty < \sqrt{2} - 1.$$

From (16) and Lemma 2, we can obtain that

$$\lim_{k\to\infty} \Delta u_k(t) = 0. \tag{17}$$

From (11), (14) and (17), we can finally conclude that

$$\lim_{k\to\infty} \left(y_k(t) + Ce^{At}\left(x_0 - x_k(0)\right)\right) = y_d(t) - e^{Rt}C\left(x_d(0) - x_0\right).$$

This completes the proof.

Theorem 3 implies that if the initial state value satisfies the condition (3), the effect of the initial state error can be exactly determined by the information about the desired output trajectory, the system parameters, the population mean x_0, the initial state value at the current iteration $x_k(0)$ and the learning gain R. From the result of Theorem 3, we can easily find that the effect of the deviation in the desired initial state value and the population mean, $x_d(0) - x_0$, can be controlled by the learning gain R, and the effect of the error between the population mean and the initial state value of the system, $x_0 - x_k(0)$, is affected by the system matrix A. Thus, if the system is stable and we choose R so that all the eigenvalues of R are negative, the effect of the initial state error is asymptotically reduced as time increases.

If we can get the uncertainty bounds for each component of A and C, the bounds of C and Ce^{At} can be obtained as $\|C\|_\infty \le \eta_1$ and $\|Ce^{At}\|_\infty \le \eta_2(t)$, respectively, and this gives more accurate bound of the output error:

$$\lim_{k\to\infty} \|y_d(t) - y_k(t)\|_\infty \le \eta_1 \|e^{Rt}\|_\infty \|x_d(0) - x_0\|_\infty + \eta_2(t)\epsilon,$$

while the previous result (Park and Bien [2000]) only shows

$$\lim_{k\to\infty} \|y_d(\cdot) - y_k(\cdot)\|_\lambda \le \eta_1 \|e^{R\cdot}\|_\lambda \|x_d(0) - x_0\|_\infty + \eta_1\left(1 + \frac{1 - e^{-(\lambda - a)T}}{(\lambda - a)(1-\rho_0)}\|B\|_\infty \|\Gamma\left(CA - RC\right)\|_\infty\right)\epsilon$$

in the sense of λ-norm, where $\|x_0 - x_k(0)\|_\infty \le \epsilon$ and $a = \|A\|_\infty$.

It is remarked that, if the initial state value is the same as x_0 at each iteration, the effect of the initial state error in Theorem 3 becomes

$$\lim_{k\to\infty} y_k(t) = y_d(t) - e^{Rt}C\left(x_d(0) - x_0\right)$$

and, by this observation, the proposed ILC algorithm (6) can be considered as an extension of the previous result (Lee and Bien [1996]).

If we adopt a continuous operator P as follows:

$$u_{k+1}(t) = avg\left\{u_i(t)\right\}_{i=0}^k + \Gamma\left(avg\left\{\dot{e}_i(t)\right\}_{i=0}^k + \left(P\ avg\left\{e_i(\cdot)\right\}_{i=0}^k\right)(t)\right),$$

it can be easily shown that, based on the previous result in Park and Bien [2000], the effect of the initial state error can be controlled in a variety of ways:

$$\lim_{k\to\infty} \left(y_k(t) + Ce^{At}\left(x_0 - x_k(0)\right)\right) = y_d(t) - \tilde{e}(t), \forall t \ge 0$$

where $\tilde{e}(t)$ is the solution of

$$\dot{\tilde{e}}(t) + \left(P\tilde{e}(\cdot)\right)(t) = 0$$
$$\tilde{e}(0) = C\left(x_d(0) - x_0\right).$$

It is also remarked that, when the system (5) has relative degree $(\mu_1, \cdots, \mu_q)$, we can apply the following algorithm with a slight modification:

$$u_{k+1}(t) = avg\left\{u_i(t)\right\}_{i=0}^k + \Gamma\left(avg\left\{e_i^*(t)\right\}_{i=0}^k - R^* avg\left\{e_i(t)\right\}_{i=0}^k\right)$$

where

$$e_i^*(t) = \begin{bmatrix} \frac{d^{\mu_1}}{dt^{\mu_1}} e_i^1(t) \\ \vdots \\ \frac{d^{\mu_q}}{dt^{\mu_q}} e_i^q(t) \end{bmatrix}, R^* = diag\left\{r_1^{\mu_1}, \cdots, r_q^{\mu_q}\right\},$$

and $e_i^j(t)$ denotes the j-th component of $e_i(t)$. Then, the effect of the initial state error in Theorem 3 is changed as follows:

if there exists ρ satisfying

$$\|I - \Gamma D^*\|_\infty \le \rho < \sqrt{2} - 1$$

then

$$\lim_{k\to\infty} \left(y_k(t) + Ce^{At}\left(x_0 - x_k(0)\right)\right) = y_d(t) - e^{R't}C\left(x_d(0) - x_0\right), \forall t \ge 0$$

where

$$D^* = \begin{bmatrix} C^1 A^{\mu_1 - 1} B \\ \vdots \\ C^q A^{\mu_q - 1} B \end{bmatrix}, R' = diag\left\{r_1, \cdots, r_q\right\},$$

and C^j denotes the j-th row of C.

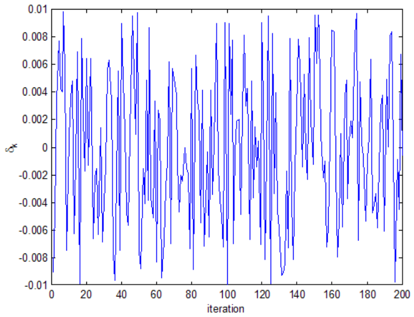
From the implementational point of view, it is rather natural and effective that the learning algorithm is given in discrete-time domain. For this end, we can consider LTI system described by (18) and the ILC algorithm (19) which utilizes one step ahead of the error giving a discrete-time equivalent of a derivative:

$$x_k(m+1) = Ax_k(m) + Bu_k(m), m = 0, 1, \cdots, M-1$$
$$y_k(m) = Cx_k(m), m = 0, 1, \cdots, M \tag{18}$$

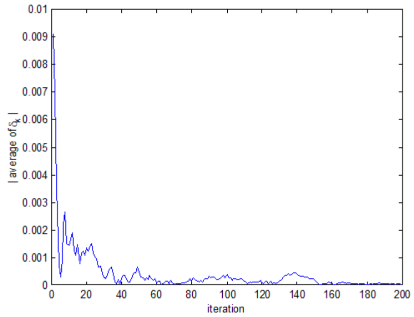
$$u_{k+1}(m) = avg\left\{u_i(m)\right\}_{i=0}^k + \Gamma\left(avg\left\{e_i(m+1)\right\}_{i=0}^k - R\ avg\left\{e_i(m)\right\}_{i=0}^k\right). \tag{19}$$

Then, the effect of the initial state error can be equivalently obtained:

$$\lim_{k\to\infty} \left(y_k(m) + CA^m\left(x_0 - x_k(0)\right)\right) = y_d(m) - R^m C\left(x_d(0) - x_0\right), m \in \{0, 1, \cdots, M\}.$$



(a) Iteration-varying number δ_k



(b) $\|avg\,\{x_i(0)\}_{i=0}^{k} - x_0\|_\infty = \frac{1}{k+1}\sum_{i=0}^{k}\delta_i$

Fig. 1. Random number in the initial state value

3. NUMERICAL EXAMPLE

The following example is given to illustrate the validity of the proposed algorithm.

Example 1: Consider the following linear time-invariant system (Lee and Bien [1996]).

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = [\,0\;\;1\,]\,x(t)$$

Let the desired output trajectory be given as follows:

$$y_d(t) = 4t(1-t), 0 \le t \le 1.$$

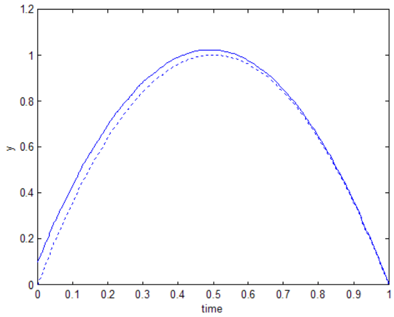
Assume that the initial state value is varying and satisfies the condition (3), which can be modeled as follows:

$$x_k(0) = \begin{bmatrix} 0 \\ 0.1 + \delta_k \end{bmatrix}.$$

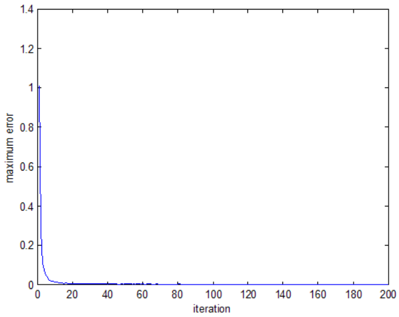
Here, δ_k varies in $[\,-0.01\;0.01\,]$ as shown in Fig. 1. Γ is chosen as 1.3 so that $\|I - \Gamma CB\|_\infty = 0.3 < \sqrt{2} - 1$. Suppose that the following ILC algorithm is applied:

$$u_{k+1}(t) = avg\,\{u_i(t)\}_{i=0}^{k} + 1.3\left(avg\,\{\dot{e}_i(t)\}_{i=0}^{k} + 3\;avg\,\{e_i(t)\}_{i=0}^{k}\right).$$

Fig. 2 (a) shows the output trajectory of the system $y_k(\cdot)$(solid line) and the desired output trajectory $y_d(\cdot)$(dashed line) at 200th iteration. Fig. 2 (b) shows the maximum absolute error between $y_k(t) + Ce^{At}(x_0 - x_k(0))$ and $y_d(t) - e^{Rt}C(x_d(0) - x_0)$. As shown in Fig. 2, we can observe that the output trajectory of the system, $y_k(t)$,



(a) Output trajectory at 200th iteration



(b) $sup_{0 \le t \le 200}\|y_k(t) + Ce^{At}(x_0 - x_k(0)) - y_d(t) + e^{-3t}C(x_d(0) - x_0)\|_\infty$

Fig. 2. Output trajectory and the maximum error

approaches a time function $y_d(t) - e^{Rt}C(x_d(0) - x_0) - Ce^{At}(x_0 - x_k(0))$.

4. CONCLUDING REMARK

In this paper, the robustness of the ILC algorithm against variable initial state error was investigated and a modified ILC algorithm was proposed based on an average operator to remove a strict condition in the previous result. A sufficient condition for convergence was given and it was proved that the effect of the initial state error can be exactly obtained by the information about the desired output trajectory, the system parameters, the initial state values and the learning parameters. From the result of Theorem 3, we can conclude that the effect of the variable initial state error can be rapidly removed as time increases by adjusting the learning gain R when the system is stable. The robustness against variable initial state error for nonlinear systems is open to further investigation.

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