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Passive actuators' fault tolerant control for affine nonlinear systems ⋆

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Abstract: In this paper we consider the problem of passive fault tolerant control for nonlinear affine systems with actuators faults. We treat two types of faults; additive and loss of effectiveness faults. In each case we propose a Lyapunov-based feedback controller that ensures the local uniform asymptotic (exponential) stability of the faulty system, if the safe nominal system is locally uniformly asymptotically (exponentially) stable. We show the effectiveness of the fault tolerant controllers on the autonomous helicopter numerical example.

Keywords: Nonlinear systems, passive fault tolerant control, Lyapunov-based control.

1. INTRODUCTION

Fault tolerant control (FTC) aims to achieve acceptable performance and stability for the safe fault-free system as well as for the faulty system. Many methods have been proposed to deal with this problem. For survey papers on the field of FTC, the reader may refer to Zhang and Jiang [2003]. While the available schemes can be classified into two types, namely passive and active FTC (Zhang and Jiang [2003]), the work presented here falls into the first category of passive FTC. Indeed, active FTC aims to ensure stability and some performance, possibly degraded, for the post-fault model, and this by reconfiguring on-line the controller, based on the fault detection and diagnosis (FDD) block that detects isolates and estimates the current fault (Zhang and Jiang [2003]). Contrary to this active solution, another solution consists in using a unique robust controller that will deal with *all the expected faults*. In this case no on-line control reconfiguration is needed and no FDD block is required. This solution has the drawback of being reliable only for the class of faults expected and taken into account in the design of the passive FTC. However, it has the advantage of avoiding the time delay due to on-line diagnosis of the faults and reconfiguration of the controller, required in active FTC (Zhang and Jiang [2006]), which is very important in practical situations where the time windows during which the system stay stabilizable is very short, e.g. the unstable double inverted pendulum example Nieamann and Stoustrup [2005]. Several passive FTC methods have been proposed, mainly based on robust theory, e.g. QFT method Wu et al. [1999], H_∞ Nieamann and Stoustrup [2005] and nonlinear regulation theory Bonivento et al. [2004].
We follow here the recent work in Bonivento et al. [2004], where actuator faults were modelled as bounded additive periodic unknown signals that were superposed onto the control signal. Nonlinear regulation theory was then used to compensate the effect of all the fault that are generated by a given internal model. We follow here the same idea of modelling actuator faults as additive signals, and consider here any bounded unknown time-varying additive signals. We also consider the case of loss of actuator effectiveness, modelled by a time-varying multiplicative factor that, when multiplied to the control signal, will reduce its effectiveness depending on the value of this factor. The idea used here is based on Lyapunov analysis, in the sense that if we have for the nominal plant a stabilizing closed-loop controller with a corresponding Lyapunov function, we can build a fault tolerant (FT) controller, based on this nominal controller and Lyapunov function, so that it ensures the stability of the faulty system. As this control scheme is a passive fault tolerant control, it does not necessitate any FDD block.

This note is organized as follows: In the second section we introduce the systems we are dealing with here together with the assumptions required. Next, we present the main result in section 3, where we introduce the FT controllers and the stability analysis. We report in section 4, the simulation results obtained on the autonomous helicopter example and conclude the note in section 5.

2. PROBLEM STATEMENT

We consider here nonlinear systems of the form

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m$ represent, respectively, the state and the input vectors. The vector fields f, columns of g are supposed to satisfy the classical smoothness assumptions, with $f(0) = 0$. We also assume the system (1) to be locally reachable (in the sense of definition 5, in Vidyasagar [1993], p. 400). Added to the previous classical assumptions, we need also the following to hold:

Assumption(1): We assume the existence of a nominal closed-loop control $u_{nom}(t, x)$, such that the solutions of the closed-loop system

$$\dot{x} = f(x) + g(x)u_{nom}(t, x) \tag{2}$$

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satisfy [1] $||x(t)|| \leq \beta(||x(t_0)||, t - t_0)$, $\forall x_{t_0} \in D$, $\forall t \geq t_0$, where $D = \{x \in \mathbb{R}^n \mid ||x|| < r_0\}, r_0 > 0$ and β is a class $\mathcal{KL}$ function [2].

Assumption(2): We assume here two types of actuator faults:
- Firstly, we consider faults that enter the system in an additive way, i.e. the faulty model writes as

$$\dot{x} = f(x) + g(x)(u + F(t,x)) \tag{3}$$

where F represents the actuators' fault and is s.t. $||F(t,x)|| \leq b(t,x)$, where $b : [0,\infty) \times D \to \mathbb{R}$ is a nonnegative continuous function.

- Secondly, we consider loss of actuator effectiveness model, representing the actuators' fault by a multiplicative matrix as

$$\dot{x} = f(x) + g(x)\alpha u \tag{4}$$

where, $\alpha \in \mathbb{R}^{m\times m}$ is a diagonal time variant matrix, with the diagonal elements $\alpha_{ii}(t)$, $i = 1, ..., m$ s.t., $0 < \epsilon_1 \leq \alpha_{ii}(t) \leq 1$.

Remarks
- Assumption (1) means simply that the closed-loop system (2) is locally, in D, uniformly asymptotically stable (UAS) (Khalil [2002], p. 150, Lemma 4.5).

- Assumption (2) is not restrictive and can be satisfied in practical applications, e.g. rotor/stator mechanical additive faults occurring in induction motors (Bonivento et al. [2004]), and the loss of effectiveness faults for aircraft actuators (Zhang and Jiang [2000]). ♦

We state now the fault tolerant control problem we want to solve here.

Problem statement:*Having a stabilizing closed-loop control for (1), s.t. Assumption 1 holds, find stabilizing closed-loop controls, for each of the faulty systems (3), and (4).*

We can now present the main results of this note.

3. PROBLEM'S SOLUTION

Firstly, we present two propositions treating separately the stabilization of (3) and (4).

Proposition 1: The control law

$$u(t,x) = u_{nom}(t,x) - sgn((\frac{\partial V}{\partial x}g)^T)(b(t,x)+\epsilon),\ \epsilon > 0 \tag{5}$$

where $u_{nom}(t,x)$ is s.t. Assumption 1 is satisfied, V is the associated Lyapunov function, $b(t,x)$ is defined in Assumption 2 and $sgn(v)$ denotes the vector sign function, s.t. $sgn(v)(i) = sgn(v(i))$; ensures that the equilibrium point $x = 0$ is locally UAS, in D, for the closed-loop system (3) and (5).

Proof: Based on Assumption 1, we can ensure (Khalil [2002], p. 167. Theorem 4.16) the existence of a Lyapunov function $V : [0,\infty) \times D \to \mathbb{R}$, s.t.

$$\begin{gathered} \alpha_1(||x||) \leq V(t,x) \leq \alpha_2(||x||) \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(f + gu_{nom}) \leq -\alpha_3(||x||) \\ \left|\left|\frac{\partial V}{\partial x}\right|\right| \leq \alpha_4(||x||) \end{gathered} \tag{6}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are class $\mathcal{K}$ functions in D. We can then evaluate the derivatives of V along the closed-loop faulty system (3) controlled with (5) [3]:

$$\begin{aligned} \frac{dV}{dt} &= \frac{\partial V}{\partial x}(f + g(u+F)) + \frac{\partial V}{\partial t} \\ &= \frac{\partial V}{\partial x}f + \frac{\partial V}{\partial x}gu + \frac{\partial V}{\partial x}gF + \frac{\partial V}{\partial t} \\ &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(f + gu_{nom}) + (\frac{\partial V}{\partial x}g)sgn((\frac{\partial V}{\partial x}g)^T)(-b-\epsilon) + \frac{\partial V}{\partial x}gF \\ &\leq -\alpha_3(||x||) + \left|\left|\frac{\partial V}{\partial x}g\right|\right|_1 b(t,x) + \left|\left|\frac{\partial V}{\partial x}g\right|\right|_1 (-b-\epsilon) \\ &\leq -\alpha_3(||x||) - \left|\left|\frac{\partial V}{\partial x}g\right|\right|_1 \epsilon \leq -\alpha_3(||x||) \end{aligned}$$

thus we can conclude (Khalil [2002], p. 151, Theorem 4.8 and p. 152, Theorem 4.9), that $x = 0$ is locally UAS, in D, equilibrium point for (3) and (5). □

We have a similar result for the system (4) as follows.

Proposition 2: The control law

$$u(t,x) = u_{nom}(t,x) - sgn((\frac{\partial V}{\partial x}g)^T)(||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1),\ \beta_1 \geq 1 \tag{7}$$

where $u_{nom}(t,x)$ is s.t. Assumption 1 is satisfied, V is the associated Lyapunov function, and $sgn(.)$ denotes the sign function; ensures that the equilibrium point $x = 0$ is locally UAS, in D, for the closed-loop system (4) and (7).

Proof [4]: The proof follows the same steps, since we compute the time derivative of the Lyapunov function V associated with the stable nominal closed-loop (due to Assumption 1). We can write then

$$\begin{aligned} \frac{dV}{dt} &= \frac{\partial V}{\partial x}(f + g\alpha u) + \frac{\partial V}{\partial t} \\ &= \frac{\partial V}{\partial x}f + \frac{\partial V}{\partial x}g\alpha u + \frac{\partial V}{\partial t} \\ &= \frac{\partial V}{\partial x}f + \frac{\partial V}{\partial x}g(I_{m\times m} - \xi)u + \frac{\partial V}{\partial t},\ \xi = I_{m\times m} - \alpha \\ &= \frac{\partial V}{\partial x}f + \frac{\partial V}{\partial x}g(u_{nom} + \tilde{u}) - \frac{\partial V}{\partial x}g\xi(u_{nom} + \tilde{u}) + \frac{\partial V}{\partial t}, \\ &\qquad \tilde{u} = -sgn((\frac{\partial V}{\partial x}g)^T)(||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1) \\ &\leq -\alpha_3(||x||) + \frac{\partial V}{\partial x}g\alpha\tilde{u} - \frac{\partial V}{\partial x}g\xi u_{nom} \end{aligned}$$

[1] $||v||$ denotes the norm of v in its Euclidian space.
[2] A continuous function $\alpha : [0,\ a) \to [0,\ \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.
A continuous function $\beta : [0,\ a) \times [0,\ \infty) \to [0,\ \infty)$ is said to belong to class $\mathcal{KL}$ if for each fixed s the mapping $\beta(r,s)$ belongs to class $\mathcal{K}$ with respect to r and for each fixed r the mapping $\beta(r,s)$ is decreasing with respect to s and $\beta(r,s) \to 0$ as $s \to \infty$ (Khalil [2002], p.144).

[3] $||v||_1 = \sum_{i=1}^{i=n} |v(i)|,\ v \in \mathbb{R}^n$.
[4] Hereafter, $I_{m\times m}$ denotes the diagonal identity matrix, and $||A||_\infty = \max_{i\in\{1,...,m\}} \sum_{j=1}^{j=m} |a_{ij}|,\ A \in \mathbb{R}^{m\times m}$ is the maximum row matrix norm Horn and Johnson [1985].

$$\leq -\alpha_3(||x||) + \left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 ||\xi||_\infty ||u_{nom}|| + \frac{\partial V}{\partial x} g\alpha(-sgn((\frac{\partial V}{\partial x} g)^T)(||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1)), \quad 0 < \epsilon_1 < ||\xi||_\infty \leq 1$$

$$\leq -\alpha_3(||x||) + (1 - ||\alpha||_\infty) \left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 ||u_{nom}|| - \left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 ||u_{nom}||\beta_1 ||\alpha||_\infty/\epsilon_1, \ 0 < \epsilon_1 < ||\alpha||_\infty \leq 1$$

$$\leq -\alpha_3(||x||) + \left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 ||u_{nom}|| - \left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 ||u_{nom}||\beta_1$$

$$\leq -\alpha_3(||x||) + \left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 ||u_{nom}||(1 - \beta_1) \leq -\alpha_3(||x||)$$

Thus the UAS of the equilibrium point of (4) and (7) holds locally in D.□

Remarks
- If the nominal closed-loop system (2), is exponentially asymptotically stable-EAS, locally in D, then the results of propositions 1 and 2, hold equally, except that the UAS is replaced with EAS. The proof is straightforward, substituting $-\alpha_3(||x||)$ by $-c||x||^2, \ c > 0$ due to the EAS (Khalil [2002], pp. 162-163, Theorem 4.14), and going through the same steps in the proofs of the propositions, we can conclude eventually on local EAS in D (Khalil [2002], p. 154, Theorem 4.10).
- The positive term ϵ in (5), can be omitted when dealing with a perfect model. However, in practice a nonzero term is necessary to compensate for inevitable model uncertainties.
- If the nominal closed-loop system is autonomous, i.e. u_{nom} is a pure state feedback, then the results of Propositions 1 and 2 stay unchanged, except that the Lyapunov functions are functions of the state vector only. (Khalil [2002], p. 163, Theorem 4.14 and p. 124, Theorem 4.2).
- We mentioned earlier, in section 2, that the nominal system is assumed to be locally reachable. This property is not guarantee to hold for the faulty systems (3). The only case where the faulty system remains reachable is for a single input system and for a uniquely time dependent fault $F(t)$. The proof is easily obtained by computing the reachability Lie bracket condition (Vidyasagar [1993], p. 409). However, the same condition shows that if the nominal system is locally reachable then the faulty system (4) remains locally reachable if $\alpha_{ii}(t) \neq 0, \ \forall t, \ \forall i = 1, ..., m$.
- We proved in Propositions 1 and 2 that the controllers (5) and (7) ensure the local UAS of (3) and (4) respectively. However, we can easily see that a combined controller

$$u(t, x) = u_{nom}(t, x) - sgn((\frac{\partial V}{\partial x} g)^T)(||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1 + b(t, x) + \epsilon), \ \epsilon > 0, \ \beta_1 > 0 \quad (8)$$

will also ensure the local UAS for both systems (3) and (4). Indeed, if we evaluate the derivative of the Lyapunov function V along (3) and (8), we can see that the derivative will write exactly the same as in the proof of Proposition 1, but with an extra negative term $-\left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 (||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1)$, that will not change the negativeness of $\frac{dV}{dt}$, and thus will not change the stability result. We can have a similar reasoning for (4) and (8), where the extra negative term will be $-\left|\left|\frac{\partial V}{\partial x} g\right|\right|_1 (b(t, x) + \epsilon)$. However, the cost to pay for having a unique controller for both types of faults, will be the control effort needed. Indeed, if we use (8) to compensate for additive faults in (3), the extra term $-sgn((\frac{\partial V}{\partial x} g)^T)(||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1)$ will not be necessary for the fault compensation but will still give an extra amplitude to the control effort. The same can be seen when applying (8) to (4). ♦

Now we have to consider the practical problem that may be caused by the discontinuity of the controls (5) and (7). This problem is well known in sliding mode control schemes (Slotine [1984]), and is usually solved by approximating the discontinuous function sgn by a continuous function. Following this idea, we propose hereafter two propositions showing the effect of such approximation on the stability results obtained before with the discontinuous controllers (5) and (7).

Proposition 3: The control law

$$u(t, x) = u_{nom}(t, x) - sat((\frac{\partial V}{\partial x} g)^T)(b(t, x) + \epsilon), \ \epsilon > 0 \quad (9)$$

ensures that the solutions of the closed-loop system (3) and (9) satisfy; $\forall x(t_0)$ s.t. $||x(t_0)|| \leq \alpha_2^{-1}(\alpha_1(r_0))$, $\exists T \geq 0$, s.t. $\begin{cases} ||x(t)|| \leq \beta(||x(t_0)||, t - t_0), \ \forall t_0 \leq t \leq t_0 + T \\ ||x(t)|| \leq \alpha_2^{-1}(\alpha_1(\tilde{x})), \ \forall t \geq t_0 + T \end{cases}$, where, for a vector v, $sat(v) = \begin{cases} v(i)/\tilde{\epsilon}, \ if \ |v(i)| \leq \tilde{\epsilon} \\ sgn(v(i)), \ if \ |v(i)| > \tilde{\epsilon} \end{cases}$, $\tilde{x} = \alpha_3^{-1}(2m\tilde{\epsilon}b_{max}) \leq \alpha_2^{-1}(\alpha_1(r_0))$, $b(t, x) \leq b_{max}, \forall t, \ \forall x \in D$, and $\alpha_1, \ \alpha_2, \ \alpha_3$ are class $\mathcal{K}$ functions in D defined in (6), β is class $\mathcal{KL}$.

Proof: refer to Benosman and Lum [2007].

We can prove the same results for the faulty system (4).

Proposition 4: The control law

$$u(t, x) = u_{nom}(t, x) - sat((\frac{\partial V}{\partial x} g)^T)(||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1), \beta_1 \geq 1 \quad (10)$$

ensures that the solutions of the closed-loop system (4) and (10) satisfy; $\forall x(t_0)$ s.t. $||x(t_0)|| \leq \alpha_2^{-1}(\alpha_1(r_0))$, $\exists T \geq 0$, s.t. $\begin{cases} ||x(t)|| \leq \beta(||x(t_0)||, t - t_0), \ \forall t_0 \leq t \leq t_0 + T \\ ||x(t)|| \leq \alpha_2^{-1}(\alpha_1(\tilde{x})), \ \forall t \geq t_0 + T \end{cases}$, where, for a vector v, $sat(v) = \begin{cases} v(i)/\tilde{\epsilon}, \ if \ |v(i)| \leq \tilde{\epsilon} \\ sgn(v(i)), \ if \ |v(i)| > \tilde{\epsilon} \end{cases}$, $\tilde{x} = \alpha_3^{-1}(2m\tilde{\epsilon}u_{nom-max}) \leq \alpha_2^{-1}(\alpha_1(r_0))$, $||u_{nom}|| \leq u_{nom-max}, \forall t$, and $\alpha_1, \ \alpha_2, \ \alpha_3$ are class $\mathcal{K}$ functions in D defined in (6), β is class $\mathcal{KL}$.

Proof: refer to Benosman and Lum [2007].

Remarks
- In the case of continuous controllers (9) and (10), we do not guarantee local UAS anymore. However, we guarantee that the closed-loop trajectories are bounded by a class $\mathcal{K}$ function, and that this bound can be made as small as desired by choosing a small $\tilde{\epsilon}$ in the definition of the function sat.
- Similarly as for the discontinuous combined controller (8), we can also easily see that the continuous combined controller

$$u(t,x) = u_{nom}(t,x) - sat((\frac{\partial V}{\partial x}g)^T)(||u_{nom}|| + \frac{||u_{nom}||}{\epsilon_1}\beta_1 + b(t,x) + \epsilon),$$
$$\epsilon > 0,\ \beta_1 > 0 \tag{11}$$

implies the same solutions' boundeness results, stated in Proposition 3 when applied to (3) and the boundeness results stated in Proposition 4 when applied to (4). Again this will be at the cost of a higher control efforts.

- Finally, we can easily see from the proves of the previous propositions that the controllers (5), (7),(8) (9), (10) and (11) if applied to the nominal system (1), will ensure local UAS in D. Thus in practice we do not need to detect the fault and switch from the nominal controller to the FTC, i.e. we can apply the FTCs directly to the safe nominal system, and hence no FDD block is required. ♦

4. THE HELICOPTER EXAMPLE

In this section, we report the numerical results obtained on a scaled model of an autonomous helicopter. We consider here the simplified model used in Mazenc et al. [2003], and given by the Lagrangian equations [5] :

$$m\ddot{\xi} = u\begin{pmatrix} -c(\psi)s(\theta)c(\phi) - s(\psi)s(\phi) \\ -c(\psi)s(\theta)s(\phi) + s(\psi)c(\phi) \\ -c(\psi)c(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} \tag{12}$$
$$J(\eta)\ddot{\eta} = -C(\eta,\dot{\eta})\dot{\eta} + \tau$$

where J is given by equation (13),

$$C(\eta,\dot{\eta}) = \frac{I_x c(\theta)}{2}C_1 + \frac{(I_x - I_y)}{2}C_2 + \frac{(I_y - I_z)}{2}C_3$$
$$C_1 = \begin{pmatrix} 0 & -\dot{\psi} & -\dot{\theta} \\ \dot{\psi} & 0 & \dot{\phi} \\ -\dot{\theta} & -\dot{\phi} & 0 \end{pmatrix},\ C_2 = \begin{pmatrix} -\dot{\theta}s(2\psi) & -\dot{\phi}s(2\theta) & 0 \\ \dot{\phi}s(2\theta) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

C_3 given by equation (14), and $\xi = (x,y,z)^T$ is the vector of the aircraft body inertial positions, in a stationary right-hand side inertial frame (z axis pointing down), $\eta = (\phi,\theta,\psi)^T$, is the vector of the yaw, pitch and roll Euler angles, u is the principle lift force due to the main rotor, $\tau = (\tau_\phi,\tau_\theta,\tau_\psi)^T$ is the vector of the generalized forces applied to the engine, due to the combination of the tail rotor force and the lateral components of the main rotor lift, m, g denotes the engine mass and the gravity field respectively, and finally [6] I_x, I_y, I_z. denote the inertial moments of the aircraft.

In the sequel we consider a normalized model, by dividing the first positions' equations in (12), by mg, thus the engine positions' vector ξ is normalized by a factor g and the main force u is normalized by a factor mg. The angular equations in (12) have not been scaled and thus, hereafter the torques $\tau_\phi,\tau_\theta,\tau_\psi$ are given in $N.m$ and the angles of ϕ,θ,ψ in rad.

This model can be written in the form of equation (1), by defining the state vector as $(x,\ \dot{x},\ y,\ \dot{y},\ z,\ \dot{z},\ \phi,\ \dot{\phi},\ \theta,\ \dot{\theta},\ \psi,\ \dot{\psi})^T$ and the control vector as $(u,\ \tau_\phi,\ \tau_\theta,\ \tau_\psi)^T$.

We recall now the Lyapunov based stabilizing controller presented in Mazenc et al. [2003], where it was proved that the system (12), in closed-loop with the nominal controller (15) is asymptotically stable for all the initial states in

$$D = \{\xi,\dot{\xi},\eta,\dot{\eta}\ |\ |\phi| < \pi/3,\ |\theta| < \pi/3,\ |\psi| < \pi/3,\ |\dot{\phi}| < 1,\ |\dot{\theta}| < 1,\ |\dot{\psi}| < 1\}.$$

Furthermore the associated decreasing Lyapunov function writes as:

$$\begin{aligned} &V = ln(1+V_1) + 11713\ W(\dot{\phi},\phi) + 294913\ W(z,\dot{z}), \\ &W(a,b) = (0.5a^2 + \sqrt{1+b^2} - 1)^2 + 2(0.5a^2 + \sqrt{1+b^2} - 1) \\ &\qquad + \frac{b}{\sqrt{1+b^2}}a,\ \forall (a,b) \in \mathbb{R}^2, \\ &V_1 = 16(\dot{\theta}^2 + \dot{\theta}\theta + 0.5\theta^2 - 16ln(|c(\theta)|) + 4(-\dot{x} + \frac{\theta+\dot{\theta}}{8})^2) \\ &+(1 + (-x - 32\dot{x} + \frac{49}{8}\theta + \dot{\theta}/8)^2)^{1/2} \\ &+16(\dot{\psi}^2 + \dot{\psi}\psi + 0.5\psi^2 - 16ln(|c(\theta)|) + 4(\dot{y} - \dot{x} + \frac{\psi+\dot{\psi}}{8})^2) \\ &+(1 + (y - x + 32\dot{y} - 32\dot{x} + \frac{49}{8}\psi + \dot{\psi}/8)^2)^{1/2} - 2. \end{aligned} \tag{16}$$

We stress here that we do not report the results due to the discontinuous controllers (5) and (7) since they exhibited high chattering effects which is incompatible with this practical application. We also stress that, in all the following tests the FT controller (9) will be tested with the value $b = \epsilon = 1$ and $\tilde{\epsilon} = 0.01$ (in the definition of the sat function). Also, the controller (10) is tested with the coefficients $\epsilon_1 = 0.05,\ \beta_1 = 1$ and $\tilde{\epsilon} = 0.01$.

We report hereafter the following results: Firstly, we consider the additive type of faults modelled by (3), and simulate a fault scenario where a time-varying additive actuators fault occurs at $t = 100\ sec$. We show the simulation results obtained with both the nominal controller (15) and the FT controller (9). Secondly, we consider loss-of-effectiveness faults, and simulate a fault scenario where at $t = 100\ sec$ a loss of effectiveness occurs on all the actuators, with periodic multiplicative coefficients. We show again the results due to the nominal controller and those due to the FT controller (10).

First, we have considered periodic additive fault in model (3), with

$$F(t) = \begin{cases} 0 \times (1,1,1,1)^T,\ t < 100\ sec, \\ (0.2 + 0.05sin(0.2\pi t)) \times (1,1,1,1)^T,\ t \geq 100\ sec. \end{cases}$$

The application of the nominal controller (15) leads to the positions depicted in figures (1) and (2). It is clear that the periodic fault effect propagated to the engine attitude and that the nominal controller, was enable to compensate for this faults. The application of the FT controller (9) leads to better results, as we can see in figures (3) and (4), the FT controller, shown in figure (5), managed to compensate for the periodic faults. Indeed, when the FT controller (9) is applied the periodic fault is immediately rejected from the states trajectories as we can see in figures (3), (4) comparatively to figures (1), (2). Instead, the oscillations appear in closed-loop faulty controller via the extra term $-sat((\frac{\partial V}{\partial x}g)^T)(b(t,x) + \epsilon)$, and eventually the oscillations are completely damped-out from the closed-loop control signals after approximatively 100 sec; see figure (5).

Let's consider now the loss of actuators effectiveness model (4), when considering periodic multiplicative actuators faults

$$\alpha(t) = \begin{cases} I_{2\times 2}, t < 100\ sec, \\ (0.2 + 0.05sin(0.2\pi t)) \times I_{2\times 2},\ t \geq 100\ sec. \end{cases}$$

We see on figure (6), that the lateral positions are still stabilized to their origin. However, the altitude is no longer stable. This is due to the fact that the nominal feedback controller cannot compensate for the loss of effectiveness,

[5] Hereafter we use the notations $c(.) = cos(.),\ s(.) = sin(.)$.

[6] For the numerical simulations we use the same values as in Koo and Sastry [1998]: $I_x = 0.142413\ kg.m^2,\ I_y = 0.271256\ Kg.m^2,\ I_z = 0.271492\ kg.m^2$.

$$J(\eta) = \begin{pmatrix} I_x(1+c(\theta)^2) + I_y c(\theta)^2(c(\psi)^2 - 1) - I_z c(\theta)^2 c(\psi)^2 & (I_y - I_z)c(\theta)s(\psi)c(\psi) & -I_x s(\theta) \\ (I_y - I_z)c(\theta)s(\psi)c(\psi) & I_y(1+s(\psi)^2) - I_z s(\psi)^2 & 0 \\ -I_x s(\theta) & 0 & I_x \end{pmatrix} \tag{13}$$

$$C_3 = \begin{pmatrix} -\dot{\theta}s(2\psi)c(\psi)^2 - \dot{\psi} - c(\theta)^2 s(2\phi)\dot{\phi} & -\dot{\phi}s(2\theta)c(\psi)^2 - \dot{\theta}s(2\psi)s(\theta) - \dot{\psi}c(\theta)^2 s(2\psi) & -\dot{\phi}c(\theta)^2 s(2\psi) + \\ & & \dot{\theta}c(\theta)c(2\psi) \\ \dot{\phi}s(2\theta)c(\psi)^2 + \dot{\psi}c(\theta)c(2\psi) & \dot{\phi}s(2\psi)s(\theta) + \dot{\psi}s(2\psi) & \dot{\phi}c(\theta)c(2\psi) \\ \dot{\phi}c(\theta)^2 s(2\psi) - \dot{\theta}c(\theta)c(2\psi) & -\dot{\phi}c(\theta)c(2\psi) & 0 \end{pmatrix} \tag{14}$$

$$\begin{aligned}
u &= \frac{1}{c(\psi)c(\theta)}\left(1 + \frac{z}{\sqrt{(1+z^2)}} + \frac{\dot{z}}{\sqrt{(1+\dot{z}^2)}}\right), \tau = C(\eta,\dot{\eta})\dot{\eta} + J^{-1}\tilde{\tau},\ \tilde{\tau} = (\tilde{\tau}_\phi, \tilde{\tau}_\theta, \tilde{\tau}_\psi)^T, \tilde{\tau}_\phi = -\frac{\phi}{\sqrt{1+\phi^2}} - \frac{\dot{\phi}}{\sqrt{1+\dot{\phi}^2}}, \\
\tilde{\tau}_\theta &= -\dot{\theta} - 8tan(\theta) + \frac{8\dot{x} - 17\dot{\theta}}{(64 + (8\dot{x} - 17\dot{\theta})^2)^{1/4}} + \frac{8\dot{x} - 9\theta - 17\dot{\theta}}{(1 + (8\dot{x} - 9\theta - 17\dot{\theta})^2)^{1/6}} + \frac{x + 32\dot{x} - \frac{49}{8}\theta - 4\dot{\theta}}{(1 + (8\dot{x} - 9\theta - 17\dot{\theta})^2)^{1/3}(1 + (x + 32\dot{x} - \frac{49}{8}\theta - 4\dot{\theta})^2)^{1/2}}, \\
\tilde{\tau}_\psi &= -\dot{\psi} - 8tan(\psi) - \frac{8\dot{y} - 8\dot{x} + 17\dot{\psi}}{(64 + (8\dot{y} - 8\dot{x} + 17\dot{\psi})^2)^{1/4}} - \frac{8\dot{y} - 8\dot{x} + 9\psi + 17\dot{\psi}}{(1 + (8\dot{y} - 8\dot{x} + 9\psi + 17\dot{\psi})^2)^{1/6}} \\
&- \frac{y - x + 32\dot{y} - 32\dot{x} + \frac{49}{8}\psi + 4\dot{\psi}}{(1 + (8\dot{y} - 8\dot{x} + 9\psi + 17\dot{\psi})^2)^{1/3}\sqrt{1 + (y - x + 32\dot{y} - 32\dot{x} + \frac{49}{8}\psi + 4\dot{\psi})^2}},
\end{aligned} \tag{15}$$

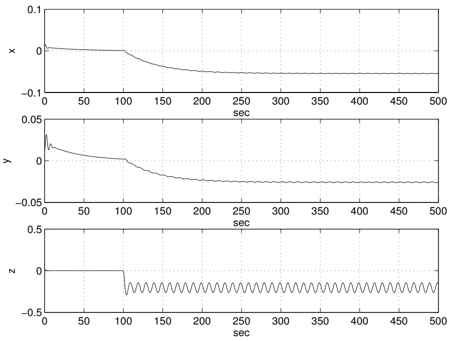

Fig. 1. Linear positions of the helicopter-periodic additive faults at $t = 100$ sec-nominal controller

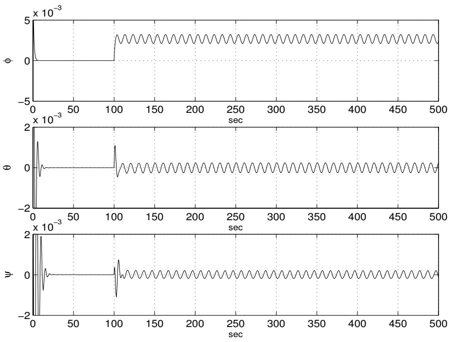

Fig. 2. Angular positions of the helicopter-periodic additive faults at $t = 100$ sec-nominal controller

since the static value of u is not big enough to cancel the loss of effectiveness.

We report now the results due to the FT controller (10). As expected, as seen clearly on figures (7) and (8), that the FT controller managed to compensate for the loss of effectiveness and that all the attitude of the engine is stabilized to the origin. The corresponding faulty controls are given in figure (9), where we can see that the FT controller (10) compensates for the loss of effectiveness by augmenting the static effort of the main lift force u.

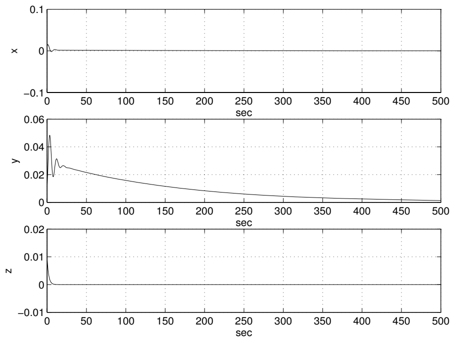

Fig. 3. Linear positions of the helicopter-periodic additive faults at $t = 100$ sec-FTC (9)

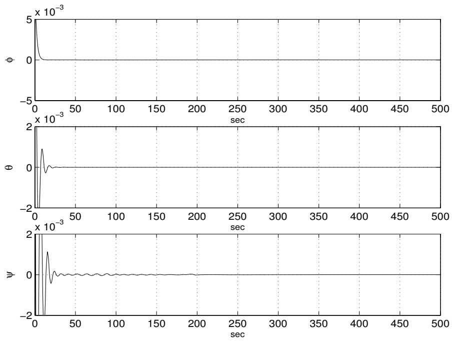

Fig. 4. Angular positions of the helicopter-periodic additive faults at $t = 100$ sec-FTC (9)

5. CONCLUSION

In this work we have modelled actuator's fault as additive unknown bounded time-varying signals that are superposed onto the actuator signals. We have also considered a multiplicative actuator faults model, by multiplying the actuators signal by a positive time-varying coefficient less then one, which is well known as loss of actuator effectiveness fault model. In both cases, we have used Lyapunov based controllers to ensure local uniform asymptotic stability of the equilibrium point in the faulty case, if the controlled nominal fault free system is already locally UAS. The advantage of the FTC presented is that no

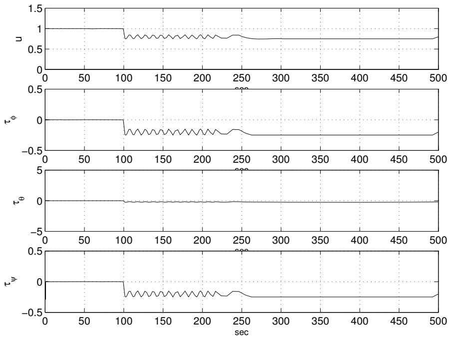

Fig. 5. FT controller (9)-periodic additive faults at $t = 100$ *sec*

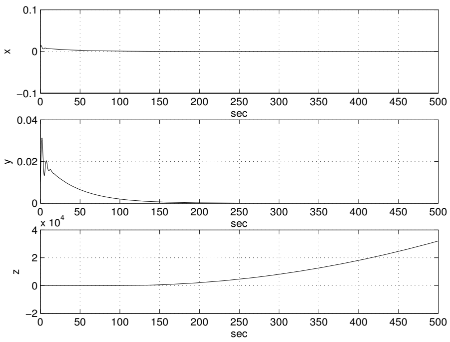

Fig. 6. Linear positions of the helicopter-periodic multiplicative faults at $t = 100$ *sec*-nominal controller

fault detection is needed. The efficiency of the controllers have been shown on the helicopter numerical example. However, a drawback of the scheme remains its dependency on the availability of an explicit Lyapunov-function based stabilizing controller for the safe system. Future work will investigate the extension of this stabilization results to output trajectory tracking for a particular class of nonlinear systems.

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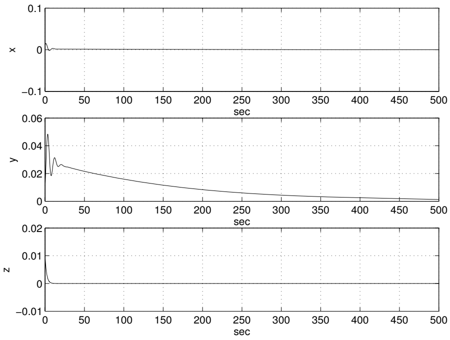

Fig. 7. Linear positions of the helicopter-periodic multiplicative faults at $t = 100$ *sec*-FTC (10)

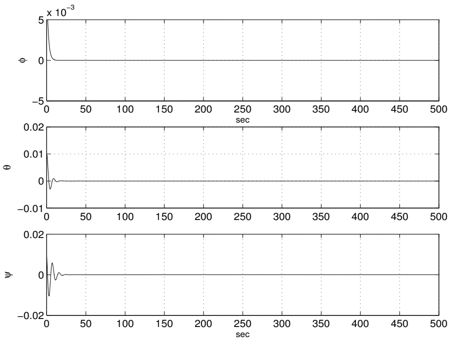

Fig. 8. Angular positions of the helicopter-periodic multiplicative faults at $t = 100$ *sec*-FTC (10)

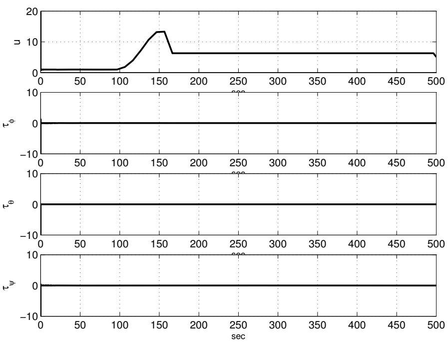

Fig. 9. FT controller (10)-periodic multiplicative faults at $t = 100$ *sec*

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