

# The Autocovariance Least Squares Technique for GPS Interference/Jamming Detection

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**Abstract:** In this paper, the Autocovariance Least Squares (ALS) technique is proposed for detecting the Global Positioning System (GPS) measurements' interference or jamming. The interference is modeled by a moderate increase in the measurements noise covariance matrix while jamming is modeled by a larger increase in the measurements noise covariance matrix. The method makes use of the dynamics of the system measured by an inertial measurement unit (IMU) and the propagated residual of an ultra-tightly coupled GPS/IMU Extended Kalman filter (EKF) to form a bank of statistics used to estimate the GPS measurement-noise covariance. Simulated scenarios of different levels of noise magnitude are applied and the proposed method is used to estimate the GPS pseudorange noise covariance matrix. Results are presented at the end of the paper to show the accuracy of the proposed algorithm. The algorithm presented in this paper is vital for high-integrity operation of autonomous navigation systems.

Keywords: Estimation and filtering; Statistical data analysis; Fault detection and identification.

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## 1. INTRODUCTION

With the increase in the number of integrated sensors used in current engineering applications, sensor fusion research is ongoing to increase the accuracy of obtaining high-accuracy estimates from sensors' measurements. Systems that are expected to work autonomously need to be designed carefully to ensure accurate operation under changing operational environments. Also, sensors might degrade in performance over time. Sensors' quality becomes an important issue for such systems. The designer has the option of selecting sophisticated and accurate sensors on the expense of cost. Alternatively, he can adapt less expensive sensors and design his fusion algorithm to account for the less-accurate measurements [1].

One of the widely used sensors for autonomous applications is the GPS and IMU sensors, [2, 3, 4]. In this paper, the problem of detecting interference or jamming on the GPS pseudorange measurements is addressed. Interference is modeled by a medium increase in the GPS measurements-noise covariance matrix. Similarly, GPS measurements jamming is modeled by a large increase in the measurements-noise covariance matrix. Therefore, the problem of detecting signal interference/jamming and correcting for it is addressed through estimating the GPS measurements-noise covariance matrix and using the estimate in the GPS/IMU fusion algorithm. An adaptive algorithm will be presented to estimate the GPS measurements-noise covariance matrix. The algorithm is based on the autocovariance least squares technique [5, 6]. This algorithm will take account for the time varying system dynamics and measurement matrices. The intrinsic feature of this algorithm is that it accounts for

the time correlation between measurement residuals. This correlation exists because of the initial use of incorrect measurements-noise covariance matrix, which is not known *a priori*.

In reference [7], the authors approach the detection of interference/jamming and spoofing in a DGPS-aided inertial system. Interference and jamming are modeled as increase in GPS noise covariance. Spoofing is modeled as a bias in the GPS measurement. A multiple model adaptive estimator (MMAE) is used to detect the covariance of the GPS measurements from a set of assumed failure hypotheses. On the other hand, a moving-bank pseudoresidual MMAE is used to detect and identify spoofing. While their work gave good results for the simulated failures, the need to hypothesize measurement faults requires some knowledge on the expected level of noise magnitude. If non of the chosen hypotheses is close to the true measurement noise covariance, then the needed time to converge on the true covariance will be considerable. Added to that is the computational burden in having enough hypotheses to cover satisfactory range of possible measurement degradation levels.

In this study, a baseband ultra-tightly coupled GPS/IMU filter structure is implemented to check the performance of the proposed algorithm in detecting and estimating different Interference/Jamming levels, Figure 1. In the figure,  $h_{\Delta}()$  is the discriminator function acting on the timing error between the received code signal and its locally generated replica,  $\Delta$  is the time spacing between early and late paths in number of chips,  $T_c$  is the chip period,  $t_d$  is the true time,  $\hat{t}_d$  is the estimated time,  $k$  is the filter gain,  $p$  is the signal amplitude, and  $n'$  is

the additive white Gaussian noise. The output of the navigation filter is used to control a voltage controlled oscillator (VCO) or a numerically controlled oscillator (NCO), which in turn controls the local Pseudo Noise (PN) code generator. In the ultra-tight filter structure, the IMU measures the receiver motion and helps isolate the GPS measurement noise from the true measurement. To estimate the state of the receiver accurately, there is a need to know the covariance matrix of the GPS measurements that may be caused by interference, jamming, spoofing, or signal multipath. The estimated GPS measurements' noise covariance matrix is used to tune the bandwidth of the baseband tracking loop. This motivated the need for estimation methods applied to GPS measurements-noise statistics estimation.

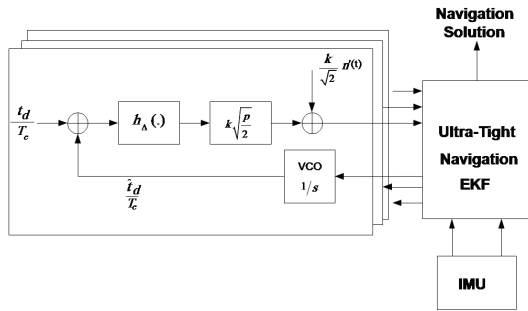


Fig. 1. Ultra-Tightly Coupled GPS/IMU fusion Structure

The GPS/IMU measurements are described next.

## 2. GPS/IMU MEASUREMENTS

The GPS measurements include the C/A code measurements, the carrier phase measurements, and the range rate measurements. These measurements can be acquired on two wave frequencies  $L_1$  and  $L_2$ . The proposed algorithm can be used to estimate the measurement noise statistics of any of these measurements. Nevertheless, in this study, the C/A pseudorange measurements are used and their measurement noise statistics are estimated.

The IMU measures the angular velocity,  $\omega_{EB}^B$ , and the linear acceleration,  $f^B$ , of the platform in three perpendicular directions. The IMU is used to construct the dynamics equation that defines the time propagation of the platform state. By measuring the dynamics of the vehicle, the IMU assist the code-tracking loop in keeping track of the GPS code signal.

The state of the vehicle at any time is defined in terms of its position, velocity, and attitude. The position,  $P^E$ , and velocity,  $V^E$ , are represented in terms of the Earth Centered, Earth Fixed (ECEF) coordinate system while the attitude,  $Q_B^E$  is represented in quaternion format and describes the body frame orientation relative to the ECEF frame. The dynamics of this state is obtained by differentiating the state variables and is represented as:

$$\dot{P}^E = V^E \quad (1)$$

$$\dot{V}^E = C_B^E f^B - 2\omega_{IE}^E \times V^E + G^E \quad (2)$$

$$\dot{Q}_B^E = \frac{1}{2}\Omega_{EB}^B Q_B^E \quad (3)$$

where,  $C_B^E$  is the cosine rotation matrix from the body frame to the ECEF frame,  $\omega_{IE}^E$  is the angular velocity of earth relative to the inertial frame represented in the ECEF frame, and  $G^E$  is the gravity vector in the ECEF frame.  $f^B$  is the specific force of the vehicle represented in the body frame and  $\omega_{EB}^B$  is the angular velocity of the body frame relative to the ECEF frame represented in the body frame. These two vectors define the evolution of the state of the vehicle.  $\Omega_{EB}^B$  is defined as:

$$\Omega_{EB}^B = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}$$

where:  $\omega_{EB}^B = [\omega_x \ \omega_y \ \omega_z]$

Equations 1 to 3 are linearized to obtain the dynamic equation of the vehicle [8, 9, 10].

## 3. ADAPTIVE NOISE ESTIMATION

In this section, the Autocovariance Least Squares technique is proposed for estimating the GPS pseudorange measurements-noise covariance matrix. The formulation of the method for the time-varying GPS/IMU system is described. Given that the GPS/IMU system is represented in the time-varying system model [8, 9, 10]:

$$x_{k+1} = \phi_k x_k + B_k w_k \quad (4)$$

$$z_k = H_k x_k + v_k \quad (5)$$

The linear state estimator is given as:

$$\bar{x}_{k+1} = \phi_k \hat{x}_k$$

$$\hat{x}_k = \bar{x}_k + K_k [z_k - h(\bar{x}_k)]$$

where  $\bar{x}_{k+1}$  denotes the estimate of the error state given all the measurements up to time  $k$ ,  $\hat{x}_k$  is the posteriori error state estimate given all the measurements up to time  $k$ ,  $K_k$  is the filter gain,  $h(\bar{x}_k) = \|\bar{P}^E - P^K\|$ . Defining the estimation error as:  $\epsilon_k = x_k - \bar{x}_k$ , the evolution process of this error can be shown to be:

$$\epsilon_{k+1} = \underbrace{(\phi_k - \phi_k K_k H_k)}_{\bar{\phi}_k} \epsilon_k + \underbrace{[B_k - \phi_k K_k]}_{\bar{B}_k} \underbrace{\begin{bmatrix} \omega_k \\ v_k \end{bmatrix}}_{\bar{\omega}_k} \quad (6)$$

Or:  $\epsilon_{k+1} = \bar{\phi}_k \epsilon_k + \bar{B}_k \bar{\omega}_k$ . And the residual is defined as:

$$r_k = z_k - H_k \bar{x}_k = H_k \epsilon_k + v_k \quad (7)$$

It is assumed that  $(\phi_k, H_k)$  is detectable and that  $\bar{\phi}_k$  is stable. This is valid for our application as will be discussed in the next section. It can be seen that the dynamics noise and the GPS measurements noise are correlated.

$$E[\bar{\omega}_k \bar{\omega}_k^T] = \bar{Q} \omega = \begin{bmatrix} Q_\omega & 0 \\ 0 & R_v \end{bmatrix}$$

$$E[\bar{\omega}_k v_k^T] = [0 \ R_v]^T$$

Any initial estimation error will damp out because of the assumption that  $\bar{\phi}_k$  is stable. The initial error covariance,

$cov(\epsilon_k) = \mathbf{P}_0^-$  will converge to a steady state error value satisfying the equation:

$$\mathbf{P}^- = \bar{\phi}_j \mathbf{P}^- \bar{\phi}_j^T + \bar{\mathbf{B}}_j \bar{\mathbf{Q}}_\omega \bar{\mathbf{B}}_j^T, \quad j = 1, \dots, k \quad (8)$$

To account for the correlation between the state and measurements noise error, the autocovariance defined as  $\mathbf{l}_{k,k+j} = \mathbf{E}[\mathbf{r}_k \mathbf{r}_{k+j}^T]$  is exploited. It can be seen that:

$$\mathbf{E}(\mathbf{r}_k \mathbf{r}_k^T) = \mathbf{l}_{k,k} = \mathbf{H}_k \mathbf{P}^- \mathbf{H}_k^T + \mathbf{R}_v$$

Also,

$$\mathbf{E}[\mathbf{r}_{k+1} \mathbf{r}_k^T] = \mathbf{l}_{k+1,k} = \mathbf{H}_{k+1} \bar{\phi}_k \mathbf{P}^- \mathbf{H}_k^T - \mathbf{H}_{k+1} \phi_k \mathbf{K}_k \mathbf{R}_v$$

Furthermore,

$$\mathbf{E}[\mathbf{r}_{k+2} \mathbf{r}_k^T] = \mathbf{l}_{k+2,k} = \mathbf{H}_{k+2} \bar{\phi}_{k+1} \bar{\phi}_k \mathbf{P}^- \mathbf{H}_k^T - \mathbf{H}_{k+2} \bar{\phi}_{k+1} \phi_k \mathbf{K}_k \mathbf{R}_v$$

And in general for  $j \geq 2$

$$\mathbf{E}[\mathbf{r}_{k+j} \mathbf{r}_k] = \mathbf{l}_{k+j,k} = \mathbf{H}_{k+j} \prod_{n=k}^{k+j-1} \bar{\phi}_n \mathbf{P}^- \mathbf{H}_k^T - \mathbf{H}_{k+j} \prod_{n=k+1}^{k+j-1} \bar{\phi}_n \phi_k \mathbf{K}_k \mathbf{R}_v \quad (9)$$

where  $\prod_{n=k}^{k+j-1} \bar{\phi}_n = \bar{\phi}_{k+j-1} \bar{\phi}_{k+j-2} \dots \bar{\phi}_k$ .

The autocovariance matrix (ACM) is defined as:

$$\mathcal{R}(N) = \begin{bmatrix} \mathbf{l}_{k,k} & \dots & \mathbf{l}_{k+N-1,k} \\ \vdots & & \\ \mathbf{l}_{k+N-1,k}^T & \dots & \mathbf{l}_{k+N-1,k+N-1} \end{bmatrix} \quad (10)$$

The off-diagonal autocovariances are not assumed zero because we do not process the data with the optimal filter, which is not known. The number of lags,  $N$ , needed depends on the desired accuracy of the estimate and on the computational power of the system.

The autocovariance matrix of innovations can be written as:

$$\mathcal{R}(N) = \mathcal{O} \mathbf{P}^- \mathcal{O}^T + \Gamma \left( \bigoplus_{i=1}^N \bar{\mathbf{B}}_i \bar{\mathbf{Q}}_\omega \bar{\mathbf{B}}_i^T \right) \Gamma^T + \Psi \left( \bigoplus_{i=1}^N \mathbf{R}_v \right) + \left( \bigoplus_{i=1}^N \mathbf{R}_v \right) \Psi^T + \bigoplus_{i=1}^N \mathbf{R}_v \quad (11)$$

where:

$$\mathcal{O} = \begin{bmatrix} \mathbf{H}_k \\ \mathbf{H}_{k+1} \bar{\phi}_k \\ \mathbf{H}_{k+2} \bar{\phi}_{k+1} \bar{\phi}_k \\ \vdots \\ \mathbf{H}_{k+N-1} \prod_{i=k}^{k+N-2} \bar{\phi}_i \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} \mathbf{0} & \mathbf{0} \dots \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_{k+1} & \mathbf{0} \dots \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & & & \\ \mathbf{H}_{k+N-1} \prod_{i=k}^{k+N-3} \bar{\phi}_i & \dots & \mathbf{H}_{k+1} & \mathbf{0} \end{bmatrix},$$

$$\Psi = \Gamma \left[ \bigoplus_{j=1}^N (-\phi_k \mathbf{K}_k) \right],$$

and  $\bigoplus_{j=1}^N$  denotes the matrix direct sum. Utilizing the vector operation defined as

$$vec(\mathbf{A}) = \mathbf{A}_s = [\mathbf{a}_1^T \dots \mathbf{a}_k^T]^T$$

where  $\mathbf{a}_k$  is the  $k^{th}$  column of the  $\mathbf{A}$  matrix. The vector operator is applied to equations 8 and 11 to get the following:

$$\mathbf{P}_s^- = (\bar{\phi}_j \otimes \bar{\phi}_j) \mathbf{P}_s^- + (\bar{\mathbf{B}}_j \bar{\mathbf{Q}}_\omega \bar{\mathbf{B}}_j^T)_s, \quad j = 1, \dots, k \quad (12)$$

which yields:

$$\begin{aligned} [\mathcal{R}(N)]_s &= [(\mathcal{O} \otimes \mathcal{O})(\mathbf{I}_{n^2} - \bar{\phi}_{N-1} \otimes \bar{\phi}_{N-1})^{-1} \\ &+ (\Gamma \otimes \Gamma) \mathcal{J}_{n,N} (\mathbf{B}_{N-1} \otimes \mathbf{B}_{N-1}) (\mathbf{Q}_\omega)_s \\ &+ [(\mathcal{O} \otimes \mathcal{O})(\mathbf{I}_{n^2} - \bar{\phi}_{N-1} \otimes \bar{\phi}_{N-1})^{-1} \\ &+ (\Gamma \otimes \Gamma) \mathcal{J}_{n,N} (\phi_{N-1} \mathbf{K}_{N-1} \otimes \phi_{N-1} \mathbf{K}_{N-1}) \\ &+ [\Psi \bigoplus \Psi + \mathbf{I}_{p^2 N^2}] \mathcal{J}_{p,N}] (\mathbf{R}_v)_s \end{aligned} \quad (13)$$

in which  $\mathcal{J}_{p,N}$  is a permutation matrix used to convert the direct sum to a vector, i.e.  $\mathcal{J}_{p,N}$  is a  $(pN)^2 \times p^2$  matrix of zeros and ones satisfying:

$$\left[ \bigoplus_{i=1}^N \mathbf{R}_v \right]_s = \mathcal{J}_{p,N} (\mathbf{R}_v)_s \quad (14)$$

The autocovariance is estimated assuming ergodic process and using the sampling theory as:

$$\hat{\mathbf{l}}_{k,k+j} = \frac{1}{N_d - j} \sum_{i=1}^{N_d - j} \mathbf{r}_i \mathbf{r}_{i+j}^T \quad (15)$$

These estimates are used to construct the autocovariance matrix,  $\hat{\mathcal{R}}(N)$ . The least squares technique can now be used to estimate the measurements covariance matrix,  $\mathbf{R}_v$ . Equation 13 can be rewritten as:

$$\begin{aligned} [\mathcal{R}(N)]_s &- [(\mathcal{O} \otimes \mathcal{O})(\mathbf{I}_{n^2} - \bar{\phi}_{N-1} \otimes \bar{\phi}_{N-1})^{-1} \\ &+ (\Gamma \otimes \Gamma) \mathcal{J}_{n,N} (\mathbf{B}_{N-1} \otimes \mathbf{B}_{N-1}) (\mathbf{Q}_\omega)_s = \\ &[[ (\mathcal{O} \otimes \mathcal{O})(\mathbf{I}_{n^2} - \bar{\phi}_{N-1} \otimes \bar{\phi}_{N-1})^{-1} \\ &+ (\Gamma \otimes \Gamma) \mathcal{J}_{n,N} ] (\phi_{N-1} \mathbf{K}_{N-1} \otimes \phi_{N-1} \mathbf{K}_{N-1}) \\ &+ [\Psi \bigoplus \Psi + \mathbf{I}_{p^2 N^2}] \mathcal{J}_{p,N}] (\mathbf{R}_v)_s \end{aligned} \quad (16)$$

The left hand side of this equation is now known and the unknown is  $\mathbf{R}_v$ . This problem can be formulated as a least squares problem in the following manner:

$$\hat{\mathbf{q}} = \mathcal{A}^\dagger \hat{\mathbf{b}}, \quad \mathcal{A}^\dagger = (\mathcal{A}^T \mathcal{A})^{-1} \mathcal{A}^T \quad (17)$$

where

$$\hat{\mathbf{q}} = (\hat{\mathbf{R}}_v)_s, \quad \hat{\mathbf{b}} = \hat{\mathcal{R}}(N)_s - \mathbf{D} (\mathbf{B}_{N-1} \otimes \mathbf{B}_{N-1}) (\mathbf{Q}_\omega)_s$$

and  $\mathcal{A}$  and  $\mathbf{b}$  are defined so that equation 16 can be written as:  $\mathcal{A} \mathbf{q} = \mathbf{b}$ . Therefore,

$$\begin{aligned} \mathbf{A} &= \mathbf{D}(\phi_{N-1} \mathbf{K}_{N-1} \otimes \phi_{N-1} \mathbf{K}_{N-1}) \\ &\quad + [\Psi \oplus \Psi + \mathbf{I}_{p^2 N^2}] \mathbf{J}_{p,N}, \\ \mathbf{D} &= [(\mathbf{O} \otimes \mathbf{O})(\mathbf{I}_{n^2} - \bar{\phi}_{N-1} \otimes \bar{\phi}_{N-1})^{-1} + (\mathbf{\Gamma} \otimes \mathbf{\Gamma}) \mathbf{J}_{n,N}] \\ \mathbf{q} &= (\mathbf{R}_v)_s, \quad \mathbf{b} = \mathcal{R}(N)_s - \mathbf{D}(\mathbf{B}_{N-1} \otimes \mathbf{B}_{N-1})(\mathbf{Q}_\omega)_s \end{aligned}$$

This algorithm is used to estimate the GPS pseudorange measurement-noise covariance matrix. The GPS/IMU EKF is given enough time to allow the state covariance matrix to converge; in this application this time is 10 seconds. After that, the algorithm stores a set of measurement residuals,  $N_d$ . The correlation matrix is estimated from the stored residual bank. This allows for an estimate of the measurement covariance matrix through a least squares estimator as in Equation 17. Once the GPS measurement-noise covariance is estimated, the acquired measurement-noise covariance is used in the EKF to obtain a high-integrity state estimate. The algorithm should be repeated periodically to account for any change in the GPS receiver environment or the instruments performance.

#### 4. SIMULATION RESULTS

A simulation environment was built to test the algorithms presented in this paper, see Figure 1. The trajectory of the vehicle was set to begin at a certain position, then input linear acceleration and angular velocity were integrated to obtain the true vehicle position, velocity, and attitude. Based on the position of the vehicle, GPS satellites' C/A code measurements were simulated. These measurements were used as input to our noise-estimation algorithm. The input linear acceleration is shown in Figure 2. As seen in the figure, the vehicle was given high acceleration to simulate a harsh environment. The vehicle was simulated to have small angular velocity. This trajectory profile guarantees an observable GPS/IMU system, references [9, 11]. The ultra-tightly coupled GPS/IMU filter structure should be able to operate in this high-dynamics environment since the dynamics of the system are fed back to the code-tracking loop. By estimating the GPS measurement noise, the band width of the code-tracking filter will be adjusted to aid in tracking the C/A code signal.

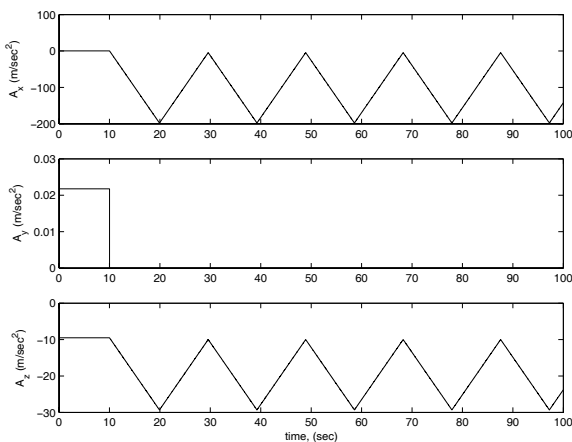


Fig. 2. Vehicle Acceleration Profile

The algorithm proposed was tested for two GPS pseudorange measurements degradation levels. The first test was

associated with a simulated pseudorange measurements-noise standard deviation of 10 meters on each satellite measurement, Figures 3 to 6. There is no correlation between satellite measurements, and therefore, the measurement-noise covariance matrix is a diagonal matrix. This testing scenario simulates a GPS receiver signal in an interference environment.

The second test was associated with a simulated pseudorange measurements-noise standard deviation of 30 meters on each satellite measurement, Figures 7 to 10. Again the true measurement-noise covariance matrix is a diagonal matrix. This testing scenario simulates a GPS receiver signal in a jamming environment. The EKF starts with the use of the ideal pseudorange measurement-noise standard deviation of 1 m on each satellite.

Figures 3 and 4 show the estimation results for the 10 m standard deviation case with  $N = 1$ . By choosing  $N = 1$ , the filter takes no account for the correlation between the measurement residuals due to the incorrect choice of the initial measurements-noise covariance matrix. 7 satellites are in view. Figure 3 shows the square root of the estimate of the diagonal elements of the estimated covariance matrix. It can be seen that the estimate is very close to the true standard deviation, with a small offset. Figure 4 shows the estimates of some off-diagonal elements of the pseudorange measurements-noise covariance matrix. It can be seen that the estimates are close to zero in mean.

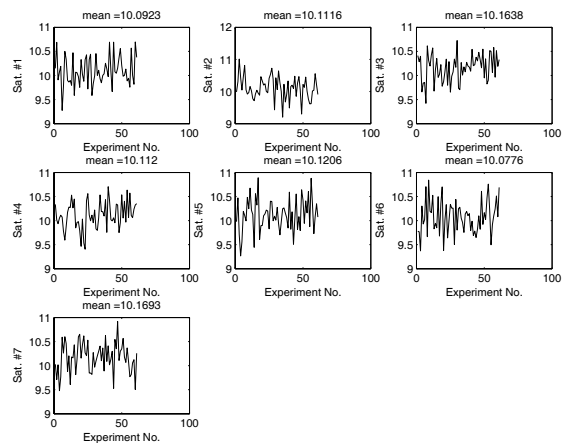


Fig. 3. Covariance Noise Estimate, 10m-std,  $N = 1$

Figures 5 and 6 show the estimation results for the 10 m pseudorange measurement noise case but with  $N = 2$ . This case takes into account the correlation between the measurement residuals at time  $k$  and time  $k - 1$  due to the use of an incorrect measurement-noise covariance matrix. Figure 5 shows the square root of the estimates of the diagonal terms of the covariance matrix. It can be seen that the estimates are right on the true simulated standard deviation. Figure 6 shows the off-diagonal terms of the estimated covariance matrix. It can be seen that the estimates are much closer to being zero in mean than for the case with  $N = 1$ .

Figures 7 and 8 show the estimation results of the pseudorange measurements covariance matrix for the 30 meters standard deviation measurements-noise case.  $N$  was set to 1 in this case. Thus, there the filter does not account for

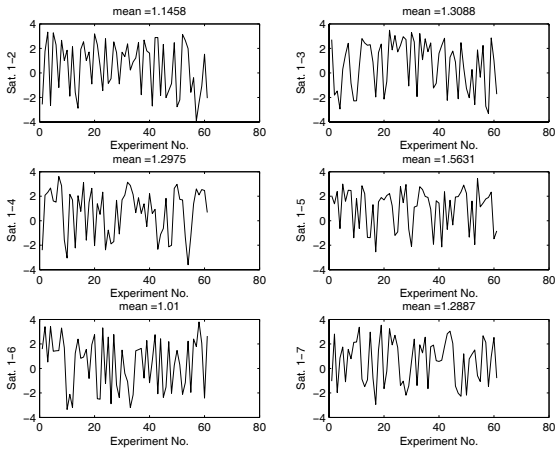


Fig. 4. Off-Diagonal Noise Estimate, 10m-std,  $N = 1$

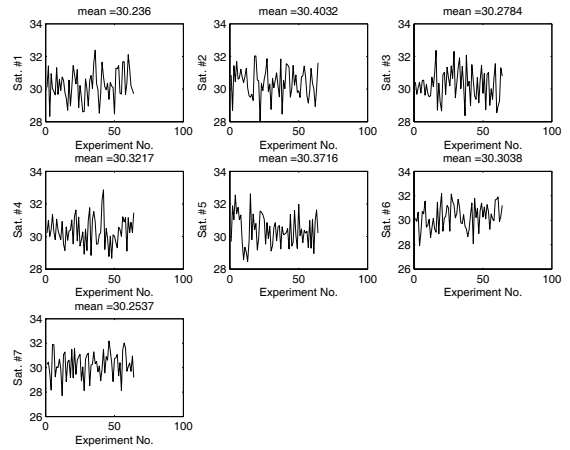


Fig. 7. Covariance Noise Estimate, 30m-std,  $N = 1$

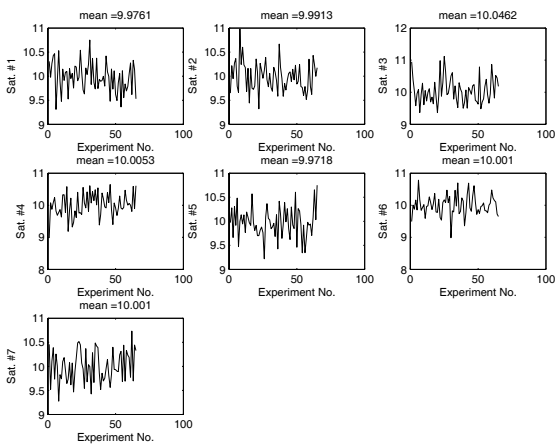


Fig. 5. Covariance Noise Estimate, 10m-std,  $N = 2$

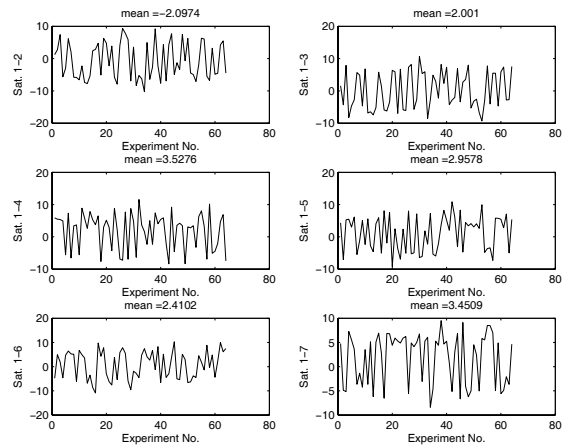


Fig. 8. Off-Diagonal Noise Estimate, 30m-std,  $N = 1$

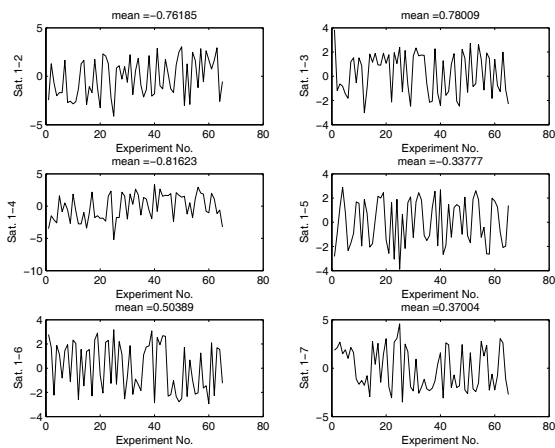


Fig. 6. Off-Diagonal Noise Estimate, 10m-std,  $N = 2$

the time correlation between the measurement residuals. Figure 7 shows the estimates of the standard deviation of satellites pseudorange measurements-noise. It can be seen that the estimates are very close to the true simulated standard deviation but with a little offset. Figure 8 shows the estimates of some of the estimated off-diagonal covariance matrix elements. It can be seen that these estimates are close to zero in mean.

Figures 9 and 10 show the estimation results of the pseudorange covariance matrix for the 30 meter true measurement noise covariance with  $N = 2$ . Therefore, the time correlation between the residuals due to the use of an inaccurate measurements-noise covariance matrix is accounted for. Figure 9 shows the estimates of the standard deviation of the pseudorange measurements-noise. It can be seen that the estimates, in mean, are much closer to the true simulated standard deviation than in the  $N = 1$  case. Figure 10 shows some of the off-diagonal covariance matrix estimates. It can be seen that the estimates are closer to being zero, in mean, than in the  $N = 1$  case.

## 5. CONCLUSIONS

In this paper, an algorithm for estimating GPS measurements noise covariance matrix is proposed. The algorithm is based on the Autocovariance Least Squares Technique. The formulation of the method for the time-varying GPS/IMU problem is described. The method was used along with an ultra-tightly coupled GPS/IMU fusion structure to estimate the possible degradation in the GPS pseudorange measurements which may be due to signal interference or jamming. The measurements-noise covariance is estimated and used in the EKF to allow for a high-integrity estimate of the state of the vehicle. Simulation results of measurements-noise standard deviation

of 10 meters and 30 meters were presented. Estimation results of the pseudorange measurements-noise covariance matrix were presented. The two case of no compensation for the time-correlation between measurement residuals and the compensation for the time correlation were shown and compared. It is seen that when time-correlation is taken into account, the estimated measurements-noise covariance matrix is closer to the true measurements-noise covariance matrix than for the case of no account for time-correlation. The presented work is important to insure high-integrity and high-accuracy operation of GPS/IMU systems in hostile environments.

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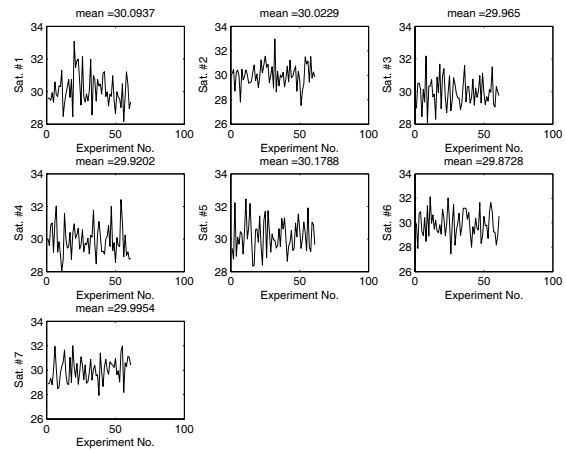


Fig. 9. Covariance Noise Estimate, 30m-std, N = 2

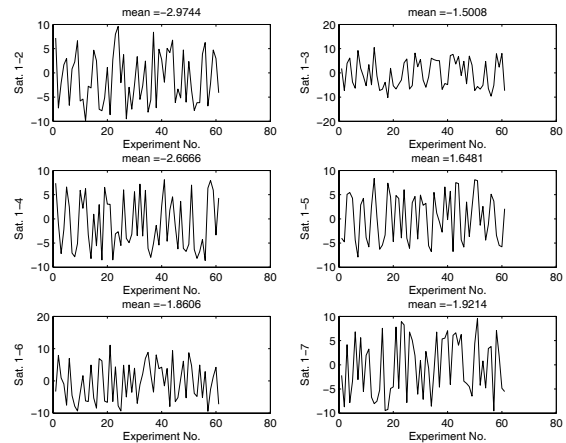


Fig. 10. Off-Diagonal Noise Estimate, 30m-std, N = 2