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Supply Chain Planning under Uncertainty: A Chance Constrained Programming Approach

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Abstract: Uncertainty issues associated with a multi-site, multi-product supply chain planning problem has been analyzed in this paper using the chance constraint programming approach. In literature, such problems have been addressed using the two stage stochastic programming approach. While this approach has merits in terms of decomposition, computational complexity even for small size planning problem is large. This problem is overcome in our paper by adopting the chance constraint programming approach for solving the mid term planning problem. It is seen that this approach is generic, relatively simple to use, and can be adapted for bigger size planning problems as well. We demonstrate the proposed approach on a relatively moderate size planning problem taken from the work of McDonald and Karimi (1997) and discuss various aspects of uncertainty in context of this problem.

1. INTRODUCTION

The primary objective of any supply chain planning is effective coordination and integration of the key business activities undertaken by an enterprise, starting from the procurement of raw materials to the distribution of the final products to the customers. The competitive pressures of the global economy motivate manufacturing and service enterprises to focus on supply chain planning on a priority basis (Shapiro, 2001). In the volatile market situations where enterprises have to meet customer satisfaction under changing market conditions, it is more realistic to consider the effect of uncertainties on supply chain planning so as to minimize their impact; deterministic models are unable to capture the demands and trade-off between various cost components such as inventory costs and demand satisfaction realistically in the presence of uncertainty.

Based on representation of the uncertain parameters three prevailing methods of handling uncertainties are *Stochastic programming* (*programming with recourse*), *Fuzzy mathematical programming* (*flexible and possibilistic programming)* and *Probabilistic programming* are the three most popular approaches of handling uncertainties (Prekopa, 1995; Birge and Louveaux, 1994).

Programming with recourse under stochastic programming uses the standard two-stage approach: first stage variables are to be decided before the realization of uncertain parameters ("here and now" decisions) whereas the second stage variables are chosen as a corrective measure against any infeasibility arising due to a particular realization of uncertainty. The objective here is to choose the first stage variables in such a way that the sum of the first stage costs and the expected value of the random second stage costs is minimized. The main challenge in solving the two-stage stochastic problem is the calculation of expectation term for the inner recourse problem. Whether the uncertainty be expressed in *scenario-based or distribution based* methodologies, these methods have drawbacks like (i) number of scenarios to be analyzed increases exponentially with increase in number of uncertain parameters which requires Dantzig-Wolfe or Benders decomposition based approaches to solve a large scale formulation, (ii) making the problem nonlinear to make the problem size in control etc.

The recourse based approach to stochastic programming requires the decision maker to assign a cost to recourse activities that are taken to ensure feasibility of the second stage problem. But these compensating second stage actions often either lead to impracticality (actions difficult to carry out practically) or the cost associated with the decision is hard to specify. In such circumstances, emphasis is shifted towards the reliability of a system by requiring a decision to be feasible with high probability. In the *probabilistic programming* or *chance-constraint programming* approach (Charnes and Cooper, 1958), the focus is on the system's ability to meet feasibility in an uncertain environment. The system reliability is expressed in terms of probability of satisfying the constraints.

In this paper, the uncertainty issues associated with a multi-site, multi-product supply chain mid term planning problem, using chance constrained programming approach, has been analyzed in details which is not reported in the literature so far to the best of our knowledge. While the two-stage stochastic programming approach has merits in terms of decomposition, the associated computational complexity even for small size planning problem is large. This problem is overcome by adopting the chance constraint programming approach for solving the mid term planning problem. The mid term planning model of McDonald and Karimi (1997) forms the basis of this work on which various impacts of uncertainty have been shown in this work.

The rest of the paper is organized as follows. In the next section, a brief overview of chance constrained programming is presented. Section 3 contains the adaptation of the McDonald and Karimi (1997) model into the chance constrained programming framework. Results are presented in section 4 for the first case study of the work of McDonald and Karimi (1997). Finally the work is summarized and concluding remarks are provided in section 5.

2. CHANCE CONSTRAINED PROGRAMMING

In chance constrained programming, constraints that are associated with random parameters are expressed in terms of certain probability of getting satisfied. In this framework, a standard optimization formulation with uncertainty

$$\mathrm{Min}\{f(x) \mid h(x,\xi) \geq 0\} \tag{1}$$

can be expressed as

$$\mathrm{Min}\{f(x) \mid P(h_k(x,\xi) \geq 0) \geq p\} \quad (k = 1,\ldots,u) \tag{2}$$

where f(x) is the objective function, x is the set of decision variables, ξ is the set of random parameters, P is the probability measure of the given probability space of the uncertain parameter and $p \in (0,1]$ is the probability level with which each of the u constraints $h_k(x, \xi) \geq 0$ of the entire constraint set $h(x, \xi) \geq 0$ needs to be satisfied. The higher the value of p, the more reliable is the modelled system. On the other hand, the set of feasible x is more and more shrunk with value of p close to unity. Assuming (i) a normal distribution for the random parameter, ξ and (ii) random parameters are separable from the decision variables, the constraints in Equation 2 can be transformed into an equivalent deterministic form:

$$\begin{aligned} &P(h_k(x,\xi) \geq 0) \geq p \Leftrightarrow P(\tilde{h}_k(x) \geq \xi_k) \geq p \\ &\Leftrightarrow \tilde{h}_k(x) \geq \hat{\xi}_k := \bar{\xi}_k + q_p \sigma_{\xi_k} \end{aligned} \tag{3}$$

where ξ_k is the random parameter associated with k^{th} constraint, $\bar{\xi}_k$ and σ_{ξ_k} are the mean and standard deviation values of the corresponding random parameters and q_p is the p-quantile of the standard normal distribution with zero mean and unit standard deviation (e.g. 97% probability corresponds to $q_p = q_{0.97} = 2.0$). In case the random parameters are not separable, similar treatment has to be done with the decision variable terms in the Equation 3. The term quantile times standard deviation corrects the nominal requirement and provides robustness of the generated optimal operating conditions under uncertain situations.

Based on the requirements of several constraints getting satisfied either individually or together, the methodology is called individual or joint chance constraint programming respectively. These two different concepts can be represented as Equation (2) and (4) respectively.

$$\mathrm{Min}\{f(x) \mid P[(h_k(x,\xi) \geq 0) \quad (k = 1,\ldots,u)] \geq p\} \tag{4}$$

It is seen that feasibility in the joint chance constraint case entails feasibility in the individual chance constraint case but the reverse is not true. In the joint chance constraint case, the deterministic equivalent form incorporates the quantile form on the multivariate probability distribution considering all the random parameters under consideration. Passing from joint to individual chance constraints may appear as a complication as that transforms single inequality into a multiple number (u) of inequalities. As the numerical treatment of probability functions involving high dimensional random vectors is much more difficult than in one dimension, the increase in number of inequalities is more than compensated by a much simpler implementation. Unlike the two stage stochastic programming approach, the advantage of these methods is that they are relatively easy to formulate and the problem size of the resultant deterministic equivalent does not blow up even for large number of random parameters.

3. MID TERM PLANNING MODEL

There are several entities in the supply chain namely, raw material supplier, production unit, retailer and customer. Given the topology of these entities, the purpose of the planning model is to determine (i) the amount to be procured as raw material at production sites, (ii) amount to be produced at the production units, (iii) the amount to be shipped from production unit to suppliers, suppliers to markets or among various production units (for production of interdependent products), (iv) amount of inventory to be kept at various locations (safety stock) to meet the stochastic demand prevailing in the market for the optimal operation of the supply chain over a relatively moderate time period (1-2 years). The generic (MILP) midterm planning model of McDonald and Karimi (1997) is adapted here for the discussion related to the demand uncertainty.

Min Cost:

$$\begin{aligned} \mathrm{Cost} = &\sum_{f,j,s,t} FC_{f,j,s,t} Y_{f,j,s,t} + \sum_{i,j,s,t} \nu_{i,j,s} P_{i,j,s,t} + \sum_{i,s,t} p_{i,s} C_{i,s,t} + \sum_{i,s,t} h_{i,s,t} I_{i,s,t} + \\ &\sum_{i,s,c,t} t_{s,c} S_{i,s,c,t} + \sum_{i,s,s',t} t_{s,s'} \sigma_{i,s,s',t} + \sum_{i,s,t} \zeta_{i,s} I^{\Delta}_{i,s,t} + \sum_{i,c,t} \mu_{i,c} I^{-}_{i,c,t} \end{aligned} \tag{5}$$

Manufacturing Constraints:

$$P_{i,j,s,t} = R_{i,j,s,t} RL_{i,j,s,t} \qquad \forall i \in I \setminus I^{RM} \tag{6}$$

$$\sum_{f} RL_{f,j,s,t} - H_{j,s,t} \leq 0 \tag{7}$$

$$RL_{f,j,s,t} = \sum_{i \in \Phi_{i,f}} RL_{i,j,s,t} \tag{8}$$

$$C_{i,s,t} = \sum_{i' \ni \beta_{i',i,s} \neq 0} \beta_{i',i,s} \sum_{j} P_{i',j,s,t} \qquad \forall i \in I \setminus I^{FP} \tag{9}$$

$$C_{i,s,t} = \sum_{s'} \sigma_{i,s',s,t} \qquad \forall i \in I^{IP} \tag{10}$$

Supply Chain Constraints:

$$I_{i,s,t} = I_{i,s,t-1} + \sum_{j} P_{i,j,s,t} - \sum_{s'} \sigma_{i,s',s,t} - \sum_{c} S_{i,s,ct} \forall i \in I \setminus I^{RM} \tag{11}$$

$$I^{-}_{i,c,t} \geq I^{-}_{i,c,t-1} + d_{i,c,t} - \sum_{s} S_{i,s,c,t} \qquad \forall i \in I^{FP} \tag{12}$$

$$\sum_{s,t' \leq t} S_{i,s,c,t'} \leq \sum_{t' \leq t} d_{i,c,t'} \qquad \forall t \in T \tag{13}$$

$$I^{\Delta}_{i,s,t} \geq I^{L}_{i,s,t} - I_{i,s,t} \qquad \forall i \in I^{FP} \tag{14}$$

Lower Bound Constraints:

$$P_{i,j,s,t},\ RL_{i,j,s,t},\ RL_{f,j,s,t}, C_{i,s,t},\ S_{i,s,c,t},\ I_{i,s,t},\ I^{-}_{i,c,t},\ \sigma_{i,s,s',t}, I^{\Delta}_{i,s,t} \geq 0,\ Y_{f,j,s,t} \in \{0,1\} \tag{15}$$

Upper Bound Constraints:

$$P_{i,j,s,t} \leq R_{i,j,s,t} H_{j,s,t} \tag{16}$$

$$RL_{f,j,s,t} - H_{j,s,t} Y_{f,j,s,t} \leq 0 \tag{17}$$

$$RL_{f,j,s,t} - MRL_{f,j,s,t} Y_{f,j,s,t} \geq 0 \tag{18}$$

$$I^{\Delta}_{i,s,t} \leq I^{L}_{i,s,t} \tag{19}$$

$$S_{i,s,c,t} \leq \sum_{t' \leq t} d_{i,c,t'} \tag{20}$$

$$I^{-}_{i,c,t} \leq \sum_{t' \leq t} d_{i,c,t'} \tag{21}$$

where $P_{i,j,s,t}$, $RL_{i,j,s,t}$, $RL_{f,j,s,t}, C_{i,s,t}$, $S_{i,s,c,t}$, $I_{i,s,t}$, $I^{-}_{i,c,t}$, $\sigma_{i,s,s',t}, I^{\Delta}_{i,s,t}$ represents the production, run length for each product, run length for each product family, consumption, supply to the market, inventory at production site, missing demand at market, intermediate product, inventory below safety level for product i or family f of several products to be produced at facility j at site s or customer c or market m at time period t respectively. Here $Y_{f,j,s,t}$ represents the binary variable to decide a product family f to be produced at machine j, site s and time period t or not. Few other important parameters are demand ($d_{i,c,t}$), machine uptime ($H_{j,s,t}$), minimum run length for the product family f ($MRL_{f,j,,s,t}$), Safety stock target for product ($I^{L}_{i,s,t}$), effective rate of production (R_{ijst}) whereas various unit cost parameters are inventory holding cost (h_{ist}), revenue (μ_{ic}), raw material price (p_{is}), penalty for dipping below safety level (ζ_{is}), fixed production cost ($FC_{f,j,s,t}$) and variable production cost(ν_{ijs}) and transportation costs ($t_{ss'}$, t_{sc}).

The set of products in the system is denoted by the index set I ≡ {i}. This set can be classified into three categories, (1) raw materials (IRM), (2) intermediate products (IIP) (3) Finished products (IFP), so that I = {IRM∪IIP∪IFP}. An intermediate product can also belong to the set of finished products. The set of machines is denoted as J ≡ {j} and the set facilities where these machines are located as S ≡ {s}. The set of customers is denoted as C ≡ {c}, while the set of time period is denoted as T ≡ {t}. F≡{f} represents the set of families, and Φi,f is defined as the cross-set indicating that product i is a member of family f. The readers are referred to the work of McDonald and Karimi (1997) for further modeling details.

In the above planning model, uncertain demands appear as terms independent of the decision variables and equations 12, 13, 20 and 21 are, therefore, modified accordingly (into equations 22 - 25). All four demand related constraints are solved with probability "p" and therefore a corresponding quantile value of q_d is used, assuming normal distribution for all product demands. Equations having uncertain demand terms are modified to the following form where different $\overline{d}_{i,c,t}$ values are the nominal values of different product i and $\sigma_{d_{i,c,t}}$ are their corresponding standard deviation figures. Similarly the equations concerning uncertainty related to machine uptime (7, 16, 17) are also accordingly modified into equations 26 – 28 where $\overline{H}_{j,s,t}$ and $\sigma_{m_{j,s,t}}$, q_m are their corresponding nominal, standard deviation and quantile value figures assuming normal distribution for this case as well.

$$I^{-}_{i,c,t} \geq I^{-}_{i,c,t-1} + (\overline{d}_{i,c,t} + q_d \sigma_{d_{i,c,t}}) - \sum_{s} S_{i,s,c,t} \ \forall i \in I^{FP} \tag{22}$$

$$\sum_{s,t' \leq t} S_{i,s,c,t'} \leq \sum_{t' \leq t} (\overline{d}_{i,c,t} + q_d \sigma_{d_{i,c,t}}) \qquad \forall t \in T \tag{23}$$

$$S_{i,s,c,t} \leq \sum_{t' \leq t} (\overline{d}_{i,c,t} + q_d \sigma_{d_{i,c,t}}) \tag{24}$$

$$I^{-}_{i,c,t} \leq \sum_{t' \leq t} (\overline{d}_{i,c,t} + q_d \sigma_{d_{i,c,t}}) \tag{25}$$

$$\sum_{f} RL_{f,j,s,t} \leq (\overline{H}_{j,s,t} - q_m \sigma_{m_{j,s,t}}) \tag{26}$$

$$P_{i,j,s,t} \leq R_{i,j,s,t} (\overline{H}_{j,s,t} - q_m \sigma_{m_{j,s,t}}) \tag{27}$$

$$RL_{f,j,s,t} - (\overline{H}_{j,s,t} - q_m \sigma_{m_{j,s,t}}) Y_{f,j,s,t} \leq 0 \tag{28}$$

Max Reliability: q_k (k = d or m) (29)

As reliability of the model (equation 29) has an inherent Pareto trade-off with the total cost of the model, we finally obtain the formulation for the bi-objective optimization using the planning model. Equations 5 – 29 (except equations 7, 12, 13, 16, 17, 20 and 21) form the complete set of equations describing the multi-objective planning model under demand as well as machine uptime uncertainty which is solved here by ε-constraint approach.

4. RESULTS AND DISCUSSION

The motivating example considered here is taken from the first case study of McDonald and Karimi (1997). There are two production locations (S1 and S2) having one unit in each of them. Each of this production unit has a single raw material supplier. Production units S1 and S2 are connected to market M1 and M2 respectively. There are 34 products that are circulated in this supply chain. Unit S1 manufactures products P1-P23 whereas the products P24-P34 are manufactured at S2. Products at S2 are produced from a set of products that are produced at S1 e.g. product P24 is produced from product P1 and so on (see McDonald and Karimi, 1997). There are eleven product families F1 – F11 that are composed by taking the products of the production site S1 e.g. products P1, P2, P3 form product family F1, product P4, P5 form product family F2 and so on (see McDonald and Karimi, 1997). Market M1 has a set of customers who have demands for products P1-P23 and market M2 has customers having demands for products P24-P34. We assume that the demand uncertainty for all 34 products can be reasonably modeled by the normal distribution. Nominal demands for all these products are taken as the deterministic demand values given in the original problem for a 1 year planning horizon (each time period of duration 1 month and therefore 12 such time periods are considered). To see the effect of sudden demands, the demand values of the original problem for time periods 6 and 12 are changed to 300% of the demand values given in the original work while all other demand values for the rest of the 10 time periods are considered to be 20% of the demand values reported in the original problem and the nominal demands for the uncertain case are updated accordingly. Based on whether we solve the multi-objective midterm planning product formulation without any minimum run length (Model 1 consisting of equations 5-29, expect 7,8,12,13,17,18,20,21,26-28, fixed charge terms in objective function and binary variables) or the multi-objective midterm planning product family formulation with minimum run length (Model 2 consisting of equations 5-29, expect 12,13,20,21,26-28), our results change accordingly. The first problem results in an LP formulation whereas the second problem is an MILP problem. The complete formulation has been coded in the modeling environment of GAMS® (Brooke et al., 1998) and solved using BDMLP solver. A typical optimization run for model 1 (LP) includes 4379 single equations, 2723 single variables and is solved in 0.5 CPU seconds (on an average) where similar formulation in model 2 (MILP) includes 4788 single equations, 2988 single variables, 132 binary variables and is solved on an average in 300 CPU seconds (a maximum limit on node to be checked is kept as 5000) on a Pentium 4, 512 MB RAM IBM PC. Model 2 converges to 4% of the best possible value with an average absolute gap of 300 spanning over 25 different runs.

Pareto Optimal (PO) points for model 1 and model 2 with demand variations of 10% and 15% of the nominal values respectively are presented in Fig 1. Different instantiations of reliability for demand uncertainty model, henceforth called as demand reliability index, Dr, can be obtained by changing the quantile value, q_d, of the standard normal distribution curve of demands for different products A value of Dr closer to 1 represents demand reliability of 84.1345%; this would imply that the demands would lie within the first quantile 84.13 % of the time and therefore inclusion of such a description in the demand satisfaction constraint would represent the demand uncertainty to the above extent. Likewise, Dr values of 2 and 5 represent demand reliability of 97.725% and 99.9999% respectively. On each of these PO fronts, one extreme point gives the most demand-reliable system (higher Dr value side e.g. 6) whereas the other extreme point gives the least expensive solution (low Dr value side e.g. 1). The value of Dr is not restricted to any integer value. A decision maker needs to make a single choice amongst all PO points as the operating point based on the demand satisfaction requirement. As uncertainty increases (more variation and thereby standard deviation), it is seen from Fig 1 that the

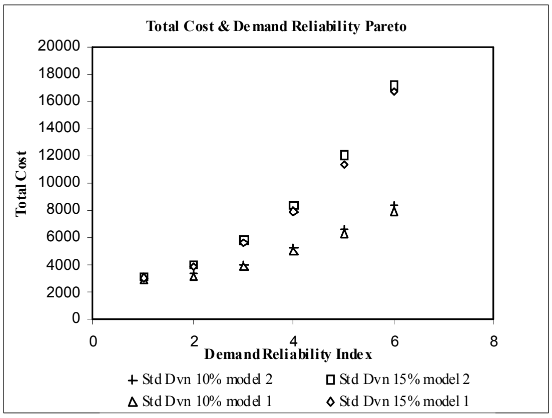

Fig. 1. Total cost vs. demand reliability index (Dr) Pareto fronts for different standard deviation values of uncertain demands for model 1 and model 2 respectively

same value of demand reliability (Dr) is obtained at higher cost; hence the PO front of higher standard deviation case lies above the relatively smaller standard deviation case. PO front of model 2 (MILP), being a more restrictive case of model 1 (LP) and hence leading to more cost, therefore, lies marginally above the PO front of model 1. As the total cost increases with increase in Dr, the penalty cost for inability to maintain safety level and cost for missing customer demands also increase proportionately. With increase in reliability of the model, the model tends to meet more demands, if required, at the cost of safety stocks as the unit cost value of the product revenue is more than that of the penalty values. So, the safety stocks are allowed to deplete first in those cases followed by demand miss, if the productions are at their respective full capacities. However, similar trends are not obvious for other cost components in the objective function.

Next, we focus on a few points of the model 2 PO front (Fig 1): specifically we focus on the two points corresponding to Dr = 2, one for the 10% demand variation case and the other for the 15% demand variation case. These two cases are compared with the results of planning model run in a deterministic fashion for the nominal values of the demands (deterministic benchmark case). The demand, production and inventory storage patterns for product P1 can be seen from the results presented in Fig 2. As compared to the nominal case, the plan for the uncertainty cases shows a trend of higher production to handle uncertainty in future. The starting of production only around the sixth time period can be attributed to the demand till that time period being met by the already existing initial inventory at the production site.

More accumulation of inventory for future uncertainty is not visible as there is a cost associated with it. Actually, the unit cost component of the McDonald and Karimi (1997) model is defined in such a way that the model gives higher preference for maintaining inventory at the safety level as long as there is no demand miss and that happens during the time periods of 6 - 11. At the time period 12, the model allows its safety level to get depleted because the missing demand at market is more expensive than keeping the inventory at safety level.

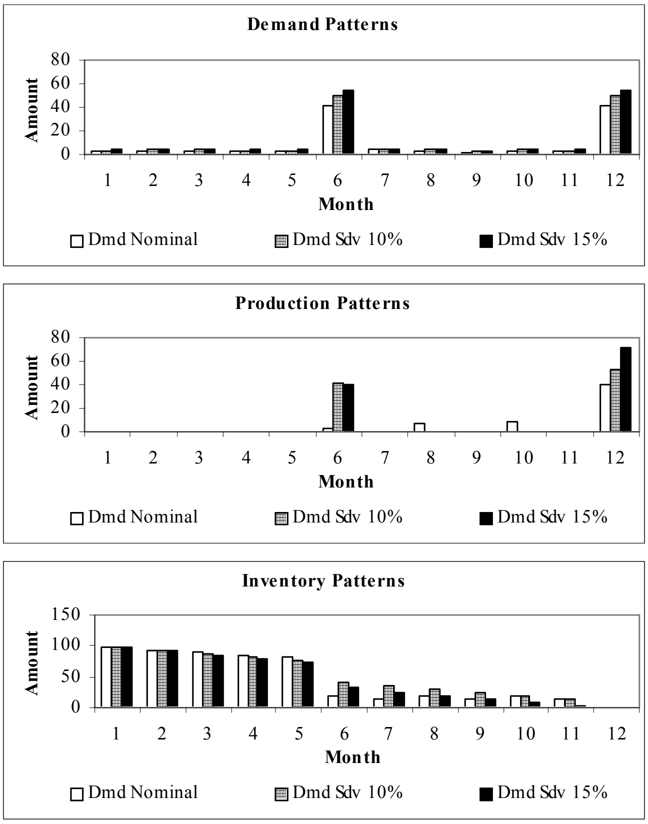

Fig. 2. Demand, Production and Inventory patterns for product P1 under different demand variations over the entire planning horizon for model 2

The case is more aggravated when the variance of the product uncertainty is even higher (15%). As all the 34 products are to be produced over 12 time periods with interdependence between various products (with finite production capacity), a straight forward correlation among the patterns and the variance is not obvious. But in general, as the uncertainty grows, the equivalent deterministic demand increases leading to higher production rates as compared to nominal conditions. Similar plots for other 33 products have been generated but are not presented here for the sake of brevity. The nominal case presented here shows why uncertainty can not be handled using the mean values of the demands and leads to either misses of potential opportunity or unnecessary and untimely accumulation of inventory leading to very high inventory cost. The corresponding model 1 production results are presented in Fig 3. The significant difference in these two cases is because of the absence of the minimum run length constraint in the case of model 1; products are made at all possible time nodes, if required. This ensures that no demand is missed unless machines are operated at their maximum capacity. But in case of model 2, even for the deterministic benchmark case where machines are not operated at their maximum capacity, there is a marginal demand miss (0.824% on an average - 0.676% for site S1 and 1.47% for site S2).

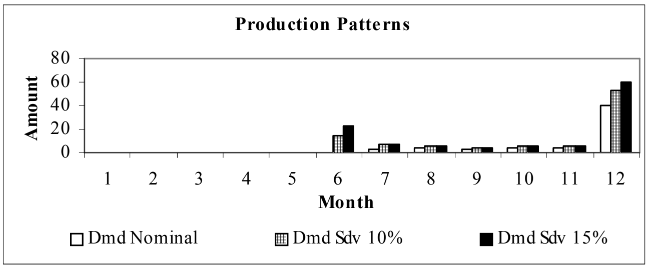

Fig. 3. Production patterns for product P1 under different demand variations over the planning horizon for model 1

This happens due to the presence of minimum run length constraint and fixed charge component in the objective function.

As one increases the value of Dr, for a fixed value of demand standard deviation, the model becomes more reliable and there is an increased emphasis on meeting higher demands. Three scenarios have been investigated: first when demands are exactly known (deterministic benchmark case, Dr= 0) while the other two cases are when demands have to be satisfied with 97% (Dr = 2) and 99.9999% (Dr = 5) probability. As this probability value increases, more demands for the products appear and products are made in higher amounts to combat uncertainty associated with demand (not shown here for the sake of brevity).

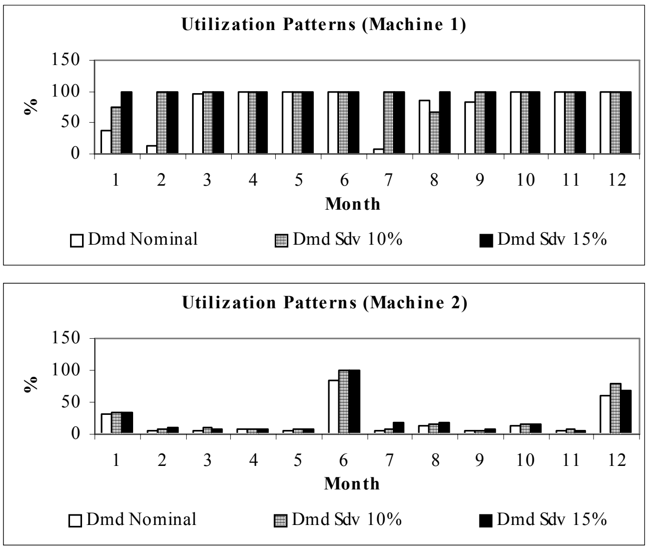

Fig. 4. Machine uptime for machine at site S1 and S2 under different demand variation cases over the entire planning horizon for model 2

Machine utilization for machine 1 at production site 1 improves from an average value of 77% for the nominal case to an average value of 95% for the case of 10% standard deviation case and 100% for the case of 15% standard deviation for Dr = 2 (model 2). During the study, machine 2

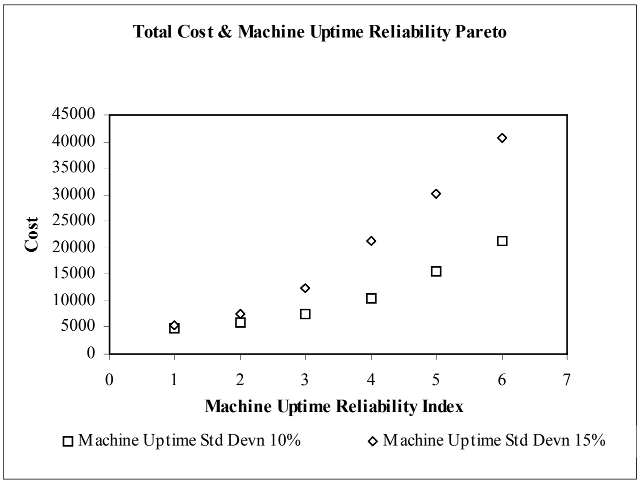

Fig. 5. Total cost vs. machine uptime reliability index (Ur) Pareto front for different values of machine uptime reliability index for model 2 (Dr = 2 and standard deviation values of uncertain demands kept fixed at 15% but the same for machine uptime kept at 10% and 15% of their nominal values)

was never observed as a resource limiting one and was, therefore, having average uptime values of 20%, 24% and 24% for the nominal, 10% standard deviation and 15% standard deviation case for the same value of Dr (Fig 4). As the site S2 machine is not capacity limiting, we analyze the site S1 machine further. When site S1 machine is fully utilized (e.g. Dr = 2 and 15% demand standard deviation), we introduce an upstream uncertainty in the machine availability.

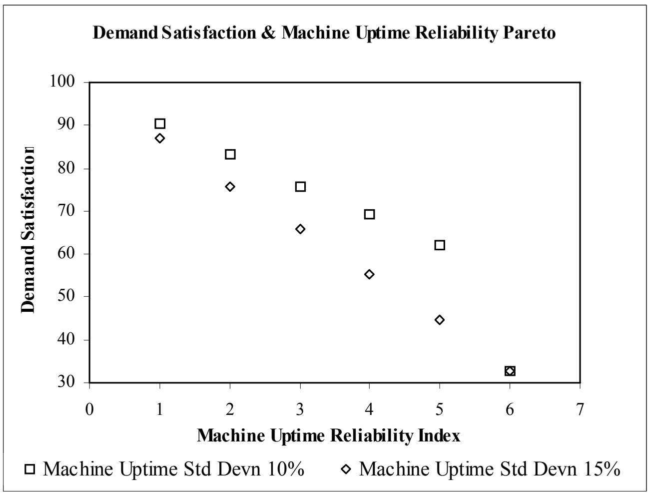

Fig. 6. Total cost vs. machine uptime reliability index (Ur) Pareto fronts for different standard deviation values of uncertain machine uptime for model 2

So far we have seen that the demands sought from the market are met with reasonable degree of demand satisfaction. But practically there may be various instances of uncertain situations that can lead to uncertainty in the upstream processes for which demands are difficult to meet to the fullest extent. Machine stoppage due to power failures or some maintenance problem in machines etc. can be some examples for such upstream uncertainties. If machine uptime reliability index (Ur) is defined in the same way as was defined for demand reliability index (Dr), a similar Pareto trade off (Fig 5) can be shown between the total cost and machine uptime reliability index for a fixed value of Dr. This total cost versus machine uptime reliability Pareto has been generated for different instances of Ur values, keeping Dr = 2 and demand and machine uptime standard deviation 15% of the corresponding nominal values. As we go for more reliable uptime model for a fixed value of Dr, we tend to incur a higher cost. In this case, the reliability considers both demand and machine uptime uncertainty and therefore incurs a higher cost. Hence the Ur Pareto front lies above the corresponding Dr Pareto front (Fig 1). As there is an increase in the value of Ur, less effective machine time is available for production leading to deteriorating demand satisfaction relationship as shown in Fig 6. Total cost, reliability index (demand or machine uptime) and demand satisfaction form a triangular Pareto trade-off combination whose boundaries are presented by lines joining the corresponding PO points in Figs 5 and 6.

5. CONCLUSION

In this paper, the multi-objective mid term supply chain planning problem is solved using chance constrained programming approach. The slot based planning model of McDonald and Karimi (1997) is adopted under chance constrained programming paradigm and solved for various uncertain scenarios to see the effect of variation in uncertainty on planning model. The problem solved here is relatively difficult to solve using the popular conventional scenario based two-stage recourse based programming approach as considering five scenarios for each of the thirty four products in that approach leads to a very large problem ($5^{34\times12\times12}$ scenarios) to solve. In our approach, the problem formulation is quite generic and easy to model, and the time involved in solving the problem is relatively small. Due to these strong points, the chance constrained optimization promises a great potential in handling problems under uncertainty.

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