Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A New Design for Chattering Reduction in Sliding Mode Control

Chi-Che Chen * **Min-Shin Chen** **

* *Department of Mechanical Engineering, National Taiwan University, Taipei, Taiwan, ROC (e-mail: d93522014@ntu.edu.tw)*
** *Department of Mechanical Engineering, National Taiwan University, Taipei, Taiwan, ROC (e-mail: mschen@ntu.edu.tw)*

Abstract: This paper discusses a new design of chattering reduction for sliding mode control. Conventionally, a boundary layer around the sliding surface is used to achieve smooth control signals. However, the boundary layer design become in-effective in chattering reduction when there is high-level measurement noise. To solve this problem, this paper proposes a dynamic sliding mode control, which, with the help of an LTR observer for uncertainty estimation, achieves chattering reduction even in very noisy environments.

1. INTRODUCTION

Sliding mode control is robust with respect to system uncertainties through the use of switching control or variable structure control (Hung *et al.*, 1993). However, chattering of the control signal has become the major obstacle to its applications in the real world. In the literature, the first approach to chattering reduction is the boundary layer control BLC (Slotine and Sastry, 1983; Burton and Zinober, 1986). In the BLC design, the boundary layer width plays two contradicting roles: on one hand, it has to be large to reduce the control chattering; on the other hand, it has to be small to achieve good control accuracy. In cases when the requirement on the control accuracy is high, the boundary layer control becomes ineffective in reducing the control chattering. This is especially true when the state measurements are corrupted with measurement noises. When the measurement noise is of a level that is larger than the boundary layer width, the high-frequency oscillations in the noise will be reflected and amplified in the control signal (see simulation examples below).

The second approach to chattering reduction is the dynamic sliding mode control DSMC (Sira-Ramirez, 1992-1993; Bartolini, 1989; Bartolini and Pydynowski, 1996; Bartolini *et al.*, 1998), where an integrator (or any other strictly proper low-pass filter) is placed in front of the system to be controlled, as shown in Figure 1. The time derivative of control input, $w = \dot{u}$, is treated as the control variable for the augmented system (the system plus the integrator). A switching sliding mode control w is then designed for the augmented system. Fortunately, the switching signal w is contained in the controller, which is normally implemented within a computer, and hence will not do any damage to the real system. The control input to the real system $u = \int w dt$ becomes chattering free since the low-pass integrator in Figure 1 will filter out the high-frequency chattering in w. The advantage of such DSMC design is that control chattering is reduced by *low pass filtering*, not by sacrificing the control accuracy since no boundary layer is used in the design of w. Hence, the mechanism of chattering reduction and that of accuracy control are decoupled in the DSMC design. Another advantage of DSMC is that it is better immune to the measurement noise since the low-pass filter $(1/s)$ in Figure 1 can to some extent filter out the noise contained in the signal w.

Despite its superiority to the BLC, the design of DSMC is challenging for the following reason. In the DSMC design, the sliding variable is different from that in the BLC design since the augmented system in Figure 1 is one dimensional larger than the original system. As a result of this, the resultant new sliding variable in DSMC contains an uncertainty term due to the external disturbance and/or parametric uncertainty. Evaluation of the new sliding variable in DSMC becomes difficult. This issue is not addressed in the early works (Sira-Ramirez, 1993), but has been discussed by Bartolini in his series of works. In Bartolini (1989), a variable structure estimator is proposed to estimate the sliding variable in DSMC, but it must assume *a priori* that the system state is uniformly bounded before proving the system stability. In Bartolini and Pydynowski (1996), a one-dimensional observer is proposed to estimate the sliding variable, but stability is guaranteed only if a differential inequality with bounded coefficients is satisfied. The most recent approach (Bartolini *et al.*, 1998) attempts to solve the problem based on a bang-bang optimal control without using any observer. But it is implementable only if one can detect the sign change of the derivative of an accessible signal. Finally, note that the sliding differentiator (Levant, 1998) approach for estimation of the sliding variable in DSMC also has to assume *a priori* a bounded trajectory.

To overcome the problem of sliding variable estimation in DSMC, this paper proposes using a two-dimensional LTR observer (Doyle and Stein, 1979) for estimation of the potentially unbounded sliding variable in DSMC. The LTR observer, which was originally proposed for gain and phase margins recovery for observer-based LQ control, finds new applications in this paper for the estimation of state-dependent uncertainties in a dynamic system. The method proposed in this paper is better than existing methods since it does not need the bounded trajectory assumption

(Bartolini, 1989), nor the differential inequality condition (Bartolini and Pydynowski, 1996), nor the detection of the sign change of the derivative of an accessible signal (Bartolini *et al.*, 1998).

To simplify the writing of complex mathematical expressions, the following notations will be used in this paper. First, the big $O(\cdot)$: one writes $s(t) = O(v(t))$ if after some finite time $|s(t)| \leq M|v(t)|$ for some finite constant $M > 0$. Second, the small $o(\cdot)$: one writes $s(t) = o(v(t))$ if after some finite time $|s(t)| \leq \epsilon|v(t)|$ where ϵ is a positive number that approaches zero as π approaches infinity in the observer Riccati equation (8) in Section 2. Third, given a time signal $u(t)$, $|u|_t = sup_{\tau \leq t}|u(\tau)|$. These notations are standard in the study of adaptive system stability (Narendra and Annaswamy, 1989).

2. SLIDING VARIABLE DESIGN IN DSMC

Consider the dynamic sliding mode control design for a linear system with uncertainty :

$$\begin{aligned} \dot{x} &= Ax + B(u+e), \quad e = \Delta a x + d \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$ is the accessible system state, $u \in R^1$ is the scalar control input, and $e = \Delta a x + d$ is system uncertainty, in which d is an unknown disturbance satisfying $|d| \leq \bar{d}$, $|\dot{d}| \leq \bar{d}_1$, and $\Delta a \in R^{1\times n}$ contains uncertain parameters with known upper bounds: $\|\Delta a\| \leq \bar{a}$. (A, B) is controllable. The row vector $C \in R^{1\times n}$ is a design parameter chosen such that (A, C) observable and all zeros of the system (A, B, C) are in the open left-half plane.

The structure of dynamic sliding mode control is depicted in Figure 1. Basically, one puts an integrator in front of the system, and $w = \dot{u}$ is treated as the control variable to suppress the uncertainty e. This strategy is sometimes called the first-order *dynamic extension* of the control input u. One then designs a switching sliding mode control for w to eliminate the uncertainty's effects. Even though w is chattering, the control input u to the system will be chattering free because the high-frequency chattering is filtered out by the integrator, which acts as a low-pass filter.

In the dynamic sliding mode control in Figure 1, the augmented system is one dimensional higher than the original system due to the inclusion of the integrator. As a result, the relative degree of the extended system becomes $r+1$, where r is the relative degree of the original system (A, B, C). Hence, a sliding variable for the augmented system is chosen as

$$s = y^{(r)} + \lambda_{r-1}y^{(r-1)} + \cdots + \lambda_1\dot{y} + \lambda_0 y \tag{2}$$

where λ_i's are chosen such that $s = 0$ defines a stable r'th-order ODE of y. The task of the control variable $w = \dot{u}$ is then to drive the new sliding variable s to (almost) zero in the face of uncertainties. However, the new sliding variable s is difficult to evaluate because its first term $y^{(r)}$ contains uncertainties $\Delta a x + d$

$$y^{(r)} = CA^r x + CA^{r-1}B(u + \Delta a x + d). \tag{3}$$

The other terms $y^{(i)} = CA^i x,\ i = 0, \cdots, r-1$ in (2) have no such problems because they are decoupled from the uncertainties.

Since $y^{(r)}$ cannot be evaluated correctly based on x and u, a robust LTR observer (Doyle and Stein, 1979) is suggested below to estimate $y^{(r)}$, and hence s. Define a two-dimensional state

$$q = \begin{bmatrix} y^{(r-1)} \\ y^{(r)} \end{bmatrix} \in R^2, \tag{4}$$

where the first component $y^{(r-1)} = CA^{r-1}x$ is accessible for evaluation given x, but the second component $y^{(r)}$ is *not* accessible due to the uncertainties in (3). In the sequel, an LTR observer is employed to estimate q and hence the inaccessible second component $y^{(r)}$. By taking the time derivative of (3), one can show that q satisfies the dynamic model,

$$\begin{aligned} \text{state :} \quad \dot{q} &= A_2 q + B_2(CA^{r+1}x + CA^r B u \\ &\quad + CA^{r-1}Bw + \Delta p) \qquad (5) \\ \text{output :} \quad & y^{(r-1)}(= CA^{r-1}x) = C_2 q \end{aligned}$$

where Δp is an unknown uncertainty

$$\begin{aligned} \Delta p &= CA^r B(\Delta a x + d) + CA^{r-1}B[\Delta a(Ax \\ &\quad + B[\Delta a x + u + d]) + d^{(1)}] \end{aligned} \tag{6}$$

and system matrices

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_2 = [\,1\ 0\,].$$

The following robust LTR observer (Doyle and Stein, 1979) is suggested to estimate the state vector q in (5):

$$\begin{aligned} \dot{\hat{q}} &= A_2\hat{q} + B_2(CA^{r+1}x + CA^r B u + CA^{r-1}Bw) \\ &\quad + L_2(y^{(r-1)} - C_2\hat{q}) \end{aligned} \tag{7}$$

where the output injection gain $L_2 = QC_2^T/\mu, \mu > 0$ and

$$(I + A_2)Q + Q(A_2 + I)^T - QC_2^T C_2 Q/\mu + \pi B_2 B_2^T = 0, \tag{8}$$

with $\pi > 0$ sufficiently large.

Lemma 1 (Doyle and Stein, 1979) : Since $(A_2 + I, B_2, C_2)$ is minimum-phase, the solution Q of the observer Riccati equation (8) satisfies $\lim_{\pi\to\infty} \frac{Q}{\pi} = 0$.

Based on Lemma 2, one can now show that the proposed LTR observer achieves small estimation errors of q by using a sufficiently large design parameter π in the observer Riccati equation (8).

Theorem 2 : The LTR observer (7) and (8) achieves a small estimation error $\tilde{q} = q - \hat{q}$ in the sense that $\lim_{t\to\infty} \|\tilde{q}(t)\| \leq \epsilon_1\|x\| + \epsilon_2|u| + \epsilon_3$ where $\lim_{\pi\to\infty} \epsilon_i = 0$, for $i = 1, 2, 3$.

Proof: Define the state estimation error $\tilde{q} = q - \hat{q}$; it satisfies, via (5) and (7),

$$\dot{\tilde{q}} = (A_2 - L_2C_2)\tilde{q} + B_2\Delta p. \tag{9}$$

Set $V = \tilde{q}^T Q^{-1}\tilde{q}$ for the error dynamics (9). Its time derivative along the trajectoy of (9) is

$$\dot{V} \leq -2V - \frac{1}{\mu}\|C_2\tilde{q}\|^2 - \pi\|B_2^T Q^{-1}\tilde{q}\|^2 + 2\|\Delta p\|\|B_2^T Q^{-1}\tilde{q}\|.$$

The maximum of the last two terms in the above equation occurs when $\|B_2^T Q^{-1}\tilde{q}\| = \|\Delta p\|/\pi$, with the maximum value being $\|\Delta p\|^2/\pi$. Hence,

$$\dot{V} \leq -2V - \frac{1}{\mu}\|C_2\tilde{q}\|^2 + \frac{\|\Delta p\|^2}{\pi} \leq -V - (V - \frac{\|\Delta p\|^2}{\pi})$$

From the last equation, $\dot{V} < -V$ as long as $V > \|\Delta p\|^2/\pi$; therefore eventually one has $\lim_{t\to\infty} V(t) \leq \|\Delta p\|^2/\pi$. Using $V(t) \geq \underline{\sigma}(Q^{-1})\|\tilde{q}(t)\|^2 = 1/\bar{\sigma}(Q)\ \|\tilde{q}(t)\|^2$, one derives

$$\lim_{t\to\infty} \|\tilde{q}(t)\| \leq \sqrt{\frac{\bar{\sigma}(Q)}{\pi}}\ \|\Delta p\| \leq \epsilon_1\|x\| + \epsilon_2|u| + \epsilon_3, \quad (10)$$

where the last inequality is obtained by noting that Δp defined in (6) is a linear combination of potentially unbounded x, u, and bounded d and $\dot{d}$. Finally, by quoting Lemma 2, one concludes that $\lim_{\pi\to\infty} \epsilon_i = 0$ for all $i = 1, 2, 3$. End of proof.

Since $|\tilde{y}^{(r)}| \leq \|\tilde{q}\|$, it follows from Theorem 3 that the estimate $\hat{y}^{(r)}$ of $y^{(r)}$ obtained from the proposed LTR observer (7) achieves a small estimation error, whose magnitude can be controlled by the design parameter π in the observer Riccati equation (8). Following the notations introduced at the end of Section 1, one can say that

$$\tilde{y}^{(r)}(t) = o(\|x\|) + o(|u|) + o(1). \quad (11)$$

Recall that for the dynamic sliding mode control, the new sliding variable s is defined as in (2), where the first term $y^{(r)}$ can now be approximately estimated by the robust LTR observer (7). Hence, an estimate of s is obtained as

$$\begin{aligned}\hat{s} &= \hat{y}^{(r)} + \lambda_{r-1}y^{(r-1)} + \cdots + \lambda_1\dot{y} + \lambda_0 y, \quad (12)\\ &= \hat{y}^{(r)} + \lambda_{r-1}CA^{r-1}x + \cdots + \lambda_1 CAx + \lambda_0 Cx\end{aligned}$$

with $\hat{y}^{(r)}$ the second element of $\hat{q}$ in the observer (7).

3. CONTROL DESIGN IN DSMC

In the previous section, one has introduced the design of sliding variable s in dynamic sliding mode control, and presented a solution to its evaluation problem via the LTR observer. Having obtained an estimate $\hat{s}$ in (12) for the sliding variable s, one can now present the control design for the control variable $w = \dot{u}$ in Figure 1, which aims to drive s to almost zero,

$$u = \int w dt = \int [\phi(\hat{s}) + v(\hat{s})]dt, \quad (13)$$

where $\phi(\hat{s})$ is a nominal control for the situation when there is no system uncertainty,

$$\begin{aligned}\phi(\hat{s}) = \frac{1}{CA^{r-1}B}\{&-CA^{r+1}x - CA^r Bu - \lambda_{r-1}\hat{y}^{(r)}\\ &- \cdots - \lambda_1 CA^2 x - \lambda_0 CAx - \sigma\hat{s}\} \quad (14)\end{aligned}$$

and $v(\hat{s})$ a switching control to suppress the adverse effects of system uncertainty,

$$\begin{aligned}v(\hat{s}) &= -\frac{\delta(x,u)}{CA^{r-1}B}sgn(\hat{s}), \quad (15)\\ \delta(x,u) &= \rho_1\|x\| + \rho_2|u| + \rho_3,\end{aligned}$$

in which $\rho_i(> 0)$'s are chosen large enough so that $\delta(x, u)$ is an upper bound of $|\Delta p|$ in (6).

To facilitate the stability analysis, introduce a state transformation,

$$x = T\begin{bmatrix} z \\ \eta \end{bmatrix}, \quad T \in R^{n\times n}. \quad (16)$$

The first group of new state $z \in R^r$ is called the *external state* (Marquez, 2003), consisting of output derivatives,

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_r \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(r-1)} \end{bmatrix} = \begin{bmatrix} Cx \\ CAx \\ \vdots \\ CA^{r-1}x \end{bmatrix} \in R^r, \quad (17)$$

where r is the relative degree of the system (1). The second group of new state $\eta \in R^{n-r}$ is the *internal state* or *zero dynamic state* (Marquez, 2003) satisfying

$$\dot{\eta} = Q\eta + Pz, \quad (18)$$

for some matrices Q, P, where Q is a square matrix whose eigenvalues are open-loop zeros of the system (A, B, C) (Marquez, 2003). Since by design, (A, B, C) has only stable zeros, Q is known to be stable. Note that the state transformation (16) is introduced here only for the purpose of stability analysis; it is not required in the computation of the proposed control law.

The stabilizing property of the proposed control (13) is proved by the theorem below.

Theorem 3 : The proposed dynamic sliding mode control (13) *practically* stabilizes the system (1) with bounded control u, in the sense that the system state is asymptotically driven into a residual set around the origin, with the size of residual set approaching zero as the design parameter π in observer Riccati equation (8) approaches infinity.

Proof: In this proof, one denotes $\tilde{s} = s - \hat{s}$, where s and $\hat{s}$ are as given in (2) and (12). Choose $V_2 = \frac{1}{2}s^2$ and check its derivative under the proposed control w in (13),

$$\begin{aligned}\dot{V}_2 &= s[CA^{r-1}Bv + \Delta p - \sigma\hat{s} + \lambda_{r-1}\tilde{y}^{(r)}] = -\sigma s^2\\ &+ s(\sigma + \lambda_{r-1})\tilde{y}^{(r)} + s[-\delta(x,u)sgn(\hat{s}) + \Delta p] \quad (19)\end{aligned}$$

where one has used $\hat{s} = s - \tilde{s}$ and $\tilde{s} = \tilde{y}^{(r)}$ to derive the second equality. There may be two possible cases for the square brackets in the above equation.

Case 1. $|s| \leq |\tilde{s}|$: In this case, it follows from $\tilde{s} = \tilde{y}^{(r)}$ and (11) that

$$|s| \leq \epsilon_1\|x\| + \epsilon_2|u| + \epsilon_3. \quad (20)$$

Case 2. $|s| > |\tilde{s}|$: In this case, $sgn(\hat{s}) = sgn(s - \tilde{s}) = sgn(s)$. Equation (19) then becomes

$$\begin{aligned}\dot{V}_2 &\leq -\sigma s^2 + s(\sigma + \lambda_{r-1})\tilde{y}^{(r)} - |s|(\delta(x,u) - |\Delta p|)\\ &\leq -\sigma s^2 + s(\sigma + \lambda_{r-1})\tilde{y}^{(r)}\\ &\leq -\sigma|s|[|s| - (1 + \frac{\lambda}{\sigma})(\epsilon_1\|x\| + \epsilon_2|u| + \epsilon_3)]\end{aligned}$$

where the second inequality results from $\delta(x,u) > |\Delta p|$, and the last inequality results from (11). From the last inequality, one concludes that after some finite time,

$$|s| \leq (1 + \frac{\lambda}{\sigma})(\epsilon_1 \|x\| + \epsilon_2 |u| + \epsilon_3). \quad (21)$$

Judging from the conclusion (20) in Case 1 and the conclusion (21) in Case 2, one concludes that

$$s = o(\|x\|) + o(|u|) + o(1). \quad (22)$$

From (2), one can write $y = 1/D(s)s$, where $D(s) = s^r + \lambda_{r-1}s^{r-1} + \cdots + \lambda_0$ is by design a stable polynomial. Following this, the external state z in (17) is related to s by $z^T = \left[\frac{1}{D(s)}, \frac{s}{D(s)}, \cdots, \frac{s^{r-1}}{D(s)}\right] s$. Since all transfer functions in this equation are proper and stable, it follows from (22) that

$$\|z\|_t = o(\|x\|_t) + o(|u|_t) + o(1). \quad (23)$$

Since Q in (18) is a stable matrix, quoting (23) yields

$$\|\eta\|_t = o(\|x\|_t) + o(|u|_t) + o(1). \quad (24)$$

Finally, it follows from (23), (24) and the state transformation (16) that the system state $\|x\|_t = o(\|x\|_t) + o(|u|_t) + o(1)$. This equation can be re-arranged as

$$\|x\|_t = o(|u|_t) + o(1). \quad (25)$$

Since $y = 1/D(s)s$, as stated earlier, one has $y^{(r)} = s^r/D(s)s$, where the transfer function $s^r/D(s)$ is proper and stable. It then follows from (22) that $y^{(r)} = o(\|x\|_t) + o(|u|_t) + o(1)$. Substituting this result into (3) shows that $u(t) = (y^{(r)} - CA^r x - CA^{r-1}B\Delta ax - CA^{r-1}Bd)/(CA^{r-1}B) = O(\|x\|_t) + O(1) + o(|u|_t)$. This leads to $|u|_t = O(\|x\|_t) + O(1) + o(|u|_t)$. Re-arranging the equation gives

$$|u|_t = O(\|x\|_t) + O(1). \quad (26)$$

Substituting (26) into (25) yields $\|x\|_t = o(\|x\|_t) + o(1)$, implying that

$$\|x\|_t = o(1). \quad (27)$$

This suggests that after some finite time the proposed control achieves $|x(t)| \leq |x|_t \leq \epsilon$ with $\lim_{\pi \to \infty} \epsilon = 0$. In other words, $x(t)$ will eventually be driven into a small residual set around the state space origin, with the size of residual set approaching zero as the design parameter π in the observer Riccati equation (8) approaches infinity. Finally, substituting (27) into (26), one concludes that the control input $|u(t)| \leq |u|_t = O(1)$ is uniformly bounded. The boundedness of $w = \dot{u}$ follows from a careful examination of (13)-(15). End of proof.

Remark : According to the statement of Theorem 3, the final control accuracy is controlled by the LTR observer parameter π in (8), while the chattering reduction of the control signal u is achieved by low pass filtering. Hence, the mechanism of chattering reduction and that of accuracy control are decoupled in the dynamic sliding mode control design.

Notice that the conventional boundary layer control is susceptible to measurement noise when the noise is of a level larger than the boundary layer width. When this happens, the measurement noise will be reflected and amplified in the control signal. In contrast, in the dynamic sliding mode control, the low-pass filter $(1/s)$ in Figure 1 can to some extent filter the noise contained in the signal w; making the dynamic sliding mode control better immune to measurement noise. To compare how these two different controls perform in noisy environments, two simulation examples will be presented below.

Example 1 : Boundary layer control. Consider a relative-degree-three system (1) with

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1.6 \\ 1.6 \end{bmatrix}, C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$x^T(0) = [1, 1, 1]$, and a disturbance uncertainty $e = cos(t)$. One tests the conventional boundary layer control with a small boundary layer width 0.05. When the state measurement is noise-free, the control signal will be smooth because of the use of a boundary layer. But when the state measurement is contaminated with a uniform noise with zero-mean and standard deviation 0.001, the results are quite different as shown in Figure 2. The upper plot of Figure 2 shows the system state (with noise removed), and the lower plot shows the control input. It is seen that although the system state is successfully regulated to almost zero in the face of unknown disturbance, the control signal has severe chattering due to the measurement noise.

Example 2 : Dynamic sliding mode control. The same system as in Example 1 is tested again for the proposed dynamic sliding mode control (13). The sliding variable in (2) are chosen with $\lambda_2 = 30, \lambda_1 = 300, \lambda_0 = 1000$. Other design parameters are $\delta(x,u) = 35$ in (15), $\sigma = 3$ in (14), $\mu = 1$, $\pi = 10^8$ in (8). The upper plot of Figure 3 shows the time history of system state, which achieves almost the same performance as that with the boundary layer control in Example 1. However, note from the lower plot of Figure 3 that the dynamic sliding mode design has successfully removed the control chattering even in this noisy environment.

ACKNOWLEDGEMENTS

This work was supported by the National Science Council of the Republic of China under the grant number NSC-94-2213-E-002-080.

REFERENCES

Bartolini, G. (1989). Chattering phenomena in discontinuous control systems. *INT. J. SYSTEMS SCI.*, **20**, 2471-2481.

Bartolini, G., and P. Pydynowski (1996). An improved, chattering free, V.S.C scheme for uncertain dynamical systems. *IEEE Trans. Autom. Control*, **41**, 1220-1226.

Bartolini, G., A. Ferrara, and E. Usai (1998). Chattering avoidance by second-order sliding mode control. *IEEE Trans. Autom. Control*, **43**, 241-246.

Burton, J. A. and A. S. I. Zinober (1986). Continuous approximation of variable structure control. *Int. J. System Science*, **17**, 875-885.

Doyle, J. C. and G. Stein (1979). Robustness with observers. *IEEE Trans. Automat. Contr.*, **24**, 607-611.

Hung, J. Y., W. B. Gao, and J. C. Hung (1993). Variable structure control: a survey. *IEEE Trans. Ind. Electron.*, **40**, 2-22.

Levant, A. (1998). Robust exact differentiation via sliding mode technique. *Automatica*, **34**, 379-384.

Marquez, H. J. (2003). *Nonlinear Control Systems Analysis and Design*, John Wiley & Sons, Hoboken, New Jersey.

Narendra, K. S. and A. M. Annaswamy (1989). *Stable Adaptive Systems*, Prentice-Hall, Englewood Cliffs, New Jersey.

Sira-Ramirez, H. (1992). On the sliding mode control of nonlinear systems. *Systems & Control Letters*, **19**, 303-312.

Sira-Ramirez, H. (1992). Dynamical sliding mode control strategies in the regulation of nonlinear chemical processes. *Int. J. Control*, **56**, 1-21.

Sira-Ramirez, H. (1993). On the dynamical sliding mode control of nonlinear systems. *Int. J. Control*, **57**, 1039-1061.

Slotine, J. J. E. and S. S. Sastry (1983). Tracking control of nonlinear systems using sliding surfaces with application to robot manipulator. *Int. J. Contr.*, **38**, 931-938.

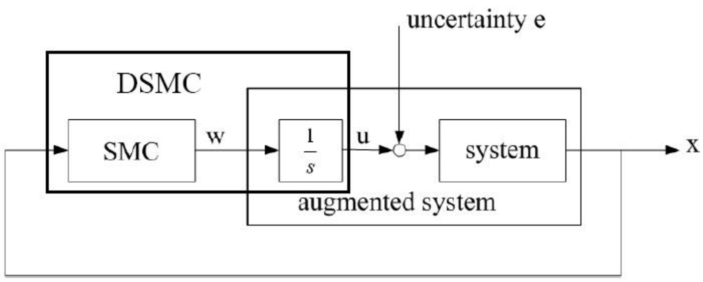

Fig. 1. Dynamic sliding mode control(DSMC)

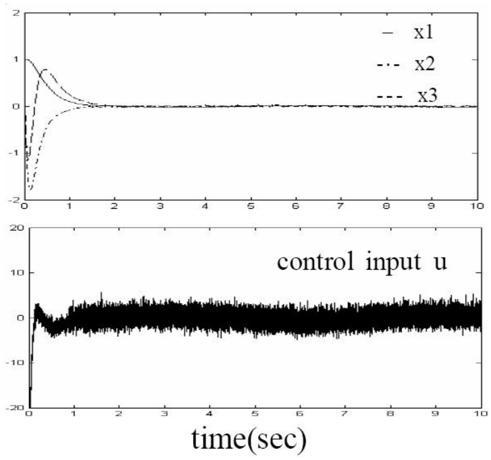

Fig. 2. Performance of boundary layer control

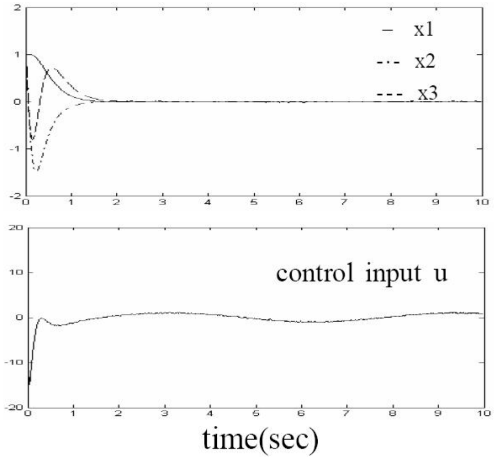

Fig. 3. Performance of dynamic sliding mode control