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MODELLING OF BUSINESS SYSTEM DEVELOPMENT ON THE BASES OF PETRI NETS AND GRAPHS OF INCREMENTS

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Abstract: We suggest a formal dynamic model designed to represent the processes of business system development. The model consists of three working units: the objectives unit, the evaluation unit, and the operations unit. The theoretical bases of the suggested triad model are Petri nets. In this paper, we articulate the functioning of the triad model, as well as the conditions that allow us to establish a balance in the transition process. We have developed an algorithm of preliminary analysis for a system's performance based on the triad model.

1. INTRODUCTION

The survival and success of a business under conditions of economic and political instability and fierce competition depend on the ability of management to anticipate and respond to possible internal and external economic changes. Valuable assistance in addressing this problem can be provided by complex formal models if those models appropriately reflect the reality and are clear to analysts and experts. This work discusses a formal dynamic model of business systems development that can be applied in the context of the known "Balanced Scorecard" (BS) model [Kaplan, Norton, 1996].

The triad model consists of three interacting units:

1. The unit of objectives, which describes the hierarchy of the business system objectives, such as profitability or market share.
2. The evaluation unit, which reflects the state of the system for the objectives achieved by using performance indicators.
3. The unit of operations, which reflects activities performed in a defined order during the process of the system development.

According to the BS model, performance indicators and objectives are divided into four categories: financial, operational, customer-related, and employee-related. The system objectives can be short-term, medium-term and long-term depending on the time horizon and objectives themselves.

To model the dynamics of objectives and performance indicators, we use a graph of increments. To model the process operations (activities), we use a graph of operations represented by Petri nets that are loaded by the logical functions [Yuditskiy, Magergood, 1987].

There are internal and external interactions in the triad model. The former reflect the mutual influence of objectives or performance indicators (factors). The latter realizes the next principle of the triad model's functioning: at certain moments of a perturbation, defined by experts as the increments of some, and possibly all, external factors, the unit of operations generates and transmits these increments to the unit of objectives and to the evaluation unit. The increments, which can be positive or negative, initiate a transition process in the evaluation unit. In this paper we investigate a transition process conditions under which the components of the vector of increments decrease in absolute value; some become smaller than threshold values specified by the experts. These threshold values will provide a reason to equate each component of the vector of increments to zero, corresponding to the achievement of a balance in the evaluation unit for the current perturbation. We assume that the next perturbation can occur only after the unit of increments reaches a balance. In our model we assume that the factors, as well as their interrelations and initial values, have been given by experts.

The triad model is designed to emulate experiments that describe the dynamics of the objectives, the performance indicators, and the operations sequence achieved by the business system.

Most of the existing literature considers business system development either in a stationary environment or in isolation from internal and/or external economic factors. Our approach is based on the fuzzy cognitive cards methods of modelling of weakly structured situations ([Kosko 1986], [Roberts 1976], [Kosko 1993], [Kononov et al., 1999], [Liu, Zhang 2003], [Kuznetsov et al., 2006]) that has become an effective instrument for the analysis and control of social economic systems functioning. The detailed survey of the literature related to our research is given in [Yuditskiy et al., 2008].

The remainder of this paper is organized as follows. Section 2 presents the architecture of the triad model. Section 3 provides the definition of the graph of increments and describes its functioning rules. Section 4 presents information about the unit of operations. Section 5 describes the algorithm of the graph of operations functioning over time. Section 6 summarizes our findings.

2. THE TRIAD MODEL ARCHITECTURE

The interactions mentioned above between the three units of the triad model, as well as their variables, are given in Fig. 1. The curly brackets denote a set of similar variables; the circle inside of the unit of operations denotes the converter that associates operations (p_i) with the initial increments of the business objectives (Δc_i) and the increments of the performance indicators (Δd_i). Variables v_i, c_i, d_i, p_i can be arguments of the indicator logical functions (f_i) defined in Section 4.

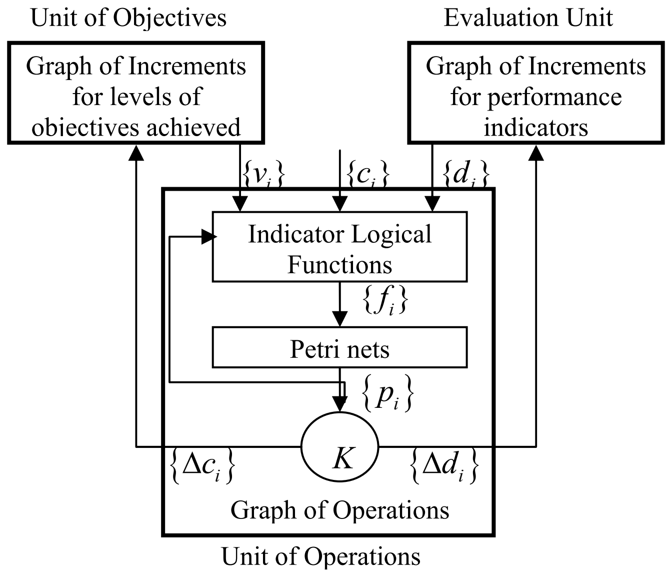


Fig. 1. The architecture of the triad model: v_i are external influences; c_i are objectives; d_i are performance indicators; f_i are indicator logical functions; p_i are operations.

3. GRAPH OF INCREMENTS

The graph of increments is defined as a 5-tuple, $GP = \langle A, Q, D, W, \alpha \rangle$, where $A = \{\Delta a_i\}$, $i = 1,\ldots,n,$ is a finite set of the places corresponding to the increments; $Q = \{q_i\}$, $i = 1,\ldots,n,$ is a finite set of the transitions between the places such that $|A| = |Q|$; $D \subset (A \times Q) \bigcup (Q \times A)$ is a set of directed arcs connecting the places and the transitions with the number of output arcs from the place i to be equal to one denoted as $a_i^* = 1$ for $\forall i$; $W = \{w_j\}, j = 1,\ldots,b,$ is a set of weights for arcs leading from the transitions into the places. If an arc connects places a_g and $a_i, g \neq i$ through a transition, then the weight of such an arc is $w(g,i) = sign(g,i) * z(g,i), sign(g,i) \in \{+,-\}$, where the plus sign corresponds to the increment of Δa_i; $z(g,i) \in \,]0,1[$ is an attenuation coefficient in the transfer of the increment; $\alpha : Q \times A \to W$ is a weight function for the arcs that lead into places. If Δa_g increases, then the minus sign describes the decrement of Δa_i. An example of the graph of increments is given in Fig. 2, where places and transitions are depicted by circles and bars, respectively.

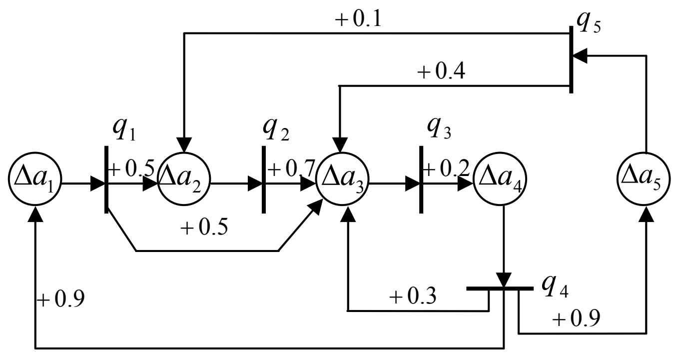


Fig. 2. An example of the graph of increments.

Consider the functioning of the graph of increments over time. The graph of increments is balanced at time τ if $a_i(\tau) = 0$, $i = 1,\ldots,n$. If at some time $\tau' > \tau$, the vector of increments $\Delta a(\tau) = (\Delta a_1(\tau),\ldots,\Delta a_n(\tau))$ changes to $\Delta a(\tau') = (\Delta a_1(\tau'),\ldots,\Delta a_n(\tau'))$, for which the condition

$$\exists i_1,\ldots,i_k \in \{1,\ldots,n\}, \Delta a_{i_r}(\tau') \neq 0, r \in \{1,\ldots,k\} \quad (1)$$

is true, then we can describe the graph's functioning as two successive steps performed simultaneously for all its places in times τ'', τ''':

$$\Delta a_{i_r}(\tau'') = 0, \quad (2)$$

$$\Delta a_{j_s}(\tau''') = \Delta a_{i_r}(\tau') * w(i_r, j_s), \quad (3)$$

where $\tau''' \geq \tau'' \geq \tau'$, and $\Delta a_{j_s}, s \in \{1,\ldots,\ell\}$ are direct successors of $\Delta a_{j_r}, r \in \{1,\ldots,k\}$. In other words, the non-zero value of the input places are emptied, and a previous value of the input places multiplied by the weight of the arc is brought as a new value for the output places.

If there are several arcs entering the output place, then its new value is a sum of all corresponding values. For example, the graph in Fig.2, $\Delta a(\tau'') = (0,1,1.7,0,0)$, where $\Delta a_2(\tau'') = 2 * 0.5 = 1, \Delta a_3(\tau'') = 2 * 0.5 + 1 * 0.7 = 1.7.$

During the simulation of the business system by using the triad model (Fig. 1), we define the modelling interval

$[\tau_0, \tau_m]$ on the axis of time, which is marked with points $\tau_0 < \tau_1 < \cdots < \tau_{m-1} < \tau_m$, where τ_0, τ_m correspond to the beginning and end of the simulation. In the moments $\tau_j, j = 1, \ldots, m$, the graph of operations transfers the perturbation to the graph of increments, which launches a transition process described by equations (2) and (3). The transition process is described as a sequence of vectors attributed to moments $\tau_{j_1} < \tau_{j_2} < \cdots < \tau_{j_k}$ inside the time interval $[\tau_j, \tau_{j+1}]$.

Now we need to establish conditions for the structure of the graph of increments or conditions for the initial perturbation $\Delta a(\tau_{j_1})$, under which the transition process of the graph of increments reaches a balance.

By topology, i.e. structure, the graph of increments can be classified into simple or complex. In the simple graph of increments there is a single arc going out from the transition, and, hence, a single arc entering each place, that is $q_i^* = {}^*\Delta a_i = 1, i = 1, \ldots, n.$

In the complex graph of increments, there are transitions and places that have more than one arc, that is

$$\exists i, j \in \{1, \ldots, n\}, (q_i^* > 1) \wedge ({}^*\Delta a_j > 1). \qquad (4)$$

An example of the complex graph of increments is given in Fig. 2.

It is not difficult to show that starting from any initial perturbation $\Delta a(\tau_j)$, the transition process in a simple graph of increments will reach a balance. Indeed, each non-zero component $\Delta a_i(\tau_{j_1})$ of initial perturbation goes through a sequence of algebraic manipulations during the transition process such that the absolute value of each component of the successive vector $\Delta a_i(\tau_{j_1})$ is less than the previous value. Since we multiply the non-zero components $\Delta a_i(\tau_{j_1})$ by arc weights, which are smaller than 1 in absolute value, all components of $\Delta a(\tau_{j_k})$ will be equal to zero at time τ_{j_k}, that is, a transition process initiated at time τ_{j_1} reaches a balance.

For the complex graph of increments, there is a problem in determining whether there exists an initial perturbation $\Delta a(\tau_{j_1})$, such that for a given number of steps, k, the balance is reached in a graph of increments. This problem could be solved by, first, introducing new variables $x_i, i = 1, \ldots, h, h \le n$, that correspond to the non-zero components of the vector $\Delta a(\tau_{j_1})$. Next, we need to construct a sequence of vectors $\Delta a(\tau_{j_1}), \ldots, \Delta a(\tau_{j_k})$, with each vector component being a linear function of x_i. Finally, we could solve the system of linear inequalities for components of the vector $\Delta a(\tau_{j_k})$. For the complex graph of increments in Fig. 2, the sequence of vectors with linear components and number of steps of the transition process $k = 4$ is represented as a graph in Fig. 3. Arcs of the graph are labelled by a set of transitions that synchronously fire and initiate transformation of vector of increments during the transition process.

Setting the threshold values for the factors $\Delta a_i(\tau_{j_k})$ to $\delta_i, i = 1, \ldots n,$ we come to the following system of linear inequalities for the moment τ_{j_4}:

$$\begin{aligned}
&0.063x_1 < \delta_1, \\
&0.054x_1 + 0.076x_2 < \delta_2, \\
&0.083x_1 + 0.123x_2 < \delta_3, \\
&0.006x_1 + 0.008x_2 < \delta_4,
\end{aligned}$$

that can be solved by any suitable method.

	$\Delta a_1(\tau)$	$\Delta a_2(\tau)$	$\Delta a_3(\tau)$	$\Delta a_4(\tau)$	
τ_j	x_1	x_2	0	0	0
τ_{j1}	0	$0.5x_1$	$0.5x_1 + 0.7x_2$	0	0
τ_{j2}	0	0	$0.35x_1$	$0.1x_1 + 0.14x_2$	0
τ_{j3}	$0.09x_1 + 0.126x_2$	0	$0.03x_1 + 0.042x_2$	$0.07x_1$	
τ_{j4}	$0.063x_1$	$0.054x_1 + 0.076x_2$	$0.08x_1 + 0.12x_2$	$0.006x_1 + 0.008x_2$	$0.063x_1$

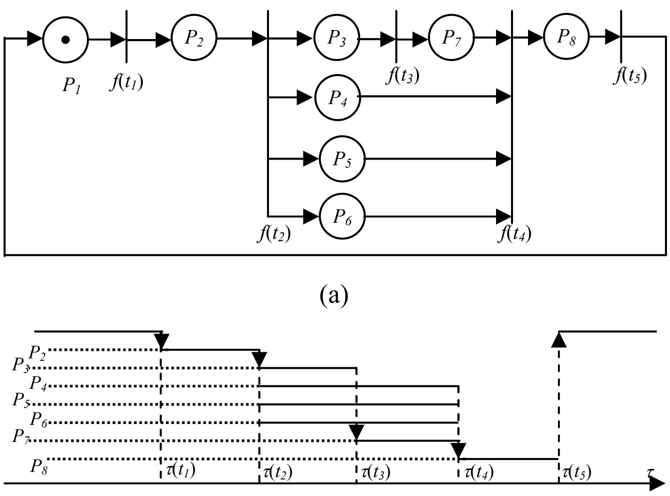
Fig.3. Example of transition process in the graph of increments.

If this system of linear inequalities does not have a solution, or there are solutions that are not acceptable by the experts, we could increase the number of steps of the transition process, k, adjust the threshold values, change the graph's structure and/or weights of the arcs.

4. GRAPH OF OPERATIONS

Graph of operations is defined as a 3-tuple $GO = \langle N, F, \beta \rangle$, where $N = \langle P, T, E, M_0 \rangle$ is a Petri net; $P = \{p_i\}, i = 1, \ldots, b$ is a finite set of places corresponding to the process operations; $T = \{t_i\}, i = 1, \ldots, \ell$ is a finite set of transitions; $E \subseteq (P \times T) \cup (T \times P)$ is a set of arcs; M_0 is an initial marking of a Petri net [Peterson, 1981]. Set $F = \{f_i\}, i = 1, \ldots, r$, is a set of indicator logical functions consisting of indicators of type $(x\#\lambda)$ with the logical connections $s \in \{\wedge, \vee, \neg, \forall, \exists\}$, where x is a numerical variable, $x \in \{p_i, c_i, v_i, d_i\}$ (see Fig. 1), λ is a constant, $\#$ is a comparison sign, $\# \in \{=, \neq, >, \geq, <, \leq\}$ [Vladislavlev et al., 2005]; $\beta : T \rightarrow F$ is a function that maps transitions into the indicator logical functions.

An example of a graph of operations is given in Fig. 4(a), and the time diagram of its states as a result of a simulation is given in Fig. 4(b). Here, the horizontal axis for this time diagram is a time interval for implementation of operations.



(a)

(b)

Fig.4. Example of the implementation of the graph of operations with τ being simulation time.

An example of the indicator logical function is next

$$f(t_1) = (v_1 > \lambda_1) \wedge ((d_1 \geq \lambda_2) \vee (d_3 < \lambda_4)). \quad (5)$$

The dynamics of the graph of operations are determined by firing enabled transitions t_i, with emptying out input positions and the indicator logical function value being true, i.e. $f(t_i) = 1$.

5. THE ALGORITHM OF SIMULATION FOR THE BUSINESS SYSTEM DYNAMIC

The result of the business system simulation is the time diagram of the graph of operations and the time-based pictures of changes in the levels of objectives achieved and levels of the performance indicators of the system, received through the graphs of increments. As mentioned above, the moments $\tau_j, j = 1, \ldots, m,$ of the time interval $[\tau_0, \tau_m]$ of the system's simulation correspond to the firing of enabled transitions of the graph of operations. For each such transition, the experts set the initial perturbations that launch the transition process in the graph of increments. The business system's simulation in the interval $[\tau_0, \tau_m]$ proceeds as follows:

1. A vector of initial factors $a(\tau_0)$ is set at time τ_0.
2. A sequence of factors increments $a(\tau_{1,1}), \ldots, a(\tau_{1,k_1})$ is formed at time τ_1.
3. For each factor a_i, the maximum and minimum increments in this sequence are determined as follows:

$$\Delta a_i^{\max}(\tau_1) = \max_{\tau_{1,1},\ldots,\tau_{1,k_1}} \{\Delta a_i(\tau_{1,1}), \ldots, \Delta a_i(\tau_{1,k_1})\},$$
$$\Delta a_i^{\min}(\tau_1) = \min_{\tau_{1,1},\ldots,\tau_{1,k_1}} \{\Delta a_i(\tau_{1,1}), \ldots, \Delta a_i(\tau_{1,k_1})\}. \quad (6)$$

4. The maximum and minimum values of $a_i(\tau_1)$ at time τ_1 are found as follows:

$$a_i^{\max}(\tau_1) = a_i(\tau_0) + \Delta a_i^{\max}(\tau_1),$$
$$a_i^{\min}(\tau_1) = a_i(\tau_0) + \Delta a_i^{\min}(\tau_1). \quad (7)$$

5. The vectors of maximum and minimum values of the factors at the time τ_1 are formed as follows:

$$a^{\max}(\tau_1) = (a_1^{\max}(\tau_1), \ldots, a_n^{\max}(\tau_1)),$$
$$a^{\min}(\tau_1) = (a_1^{\min}(\tau_1), \ldots, a_n^{\min}(\tau_1)). \quad (8)$$

6. Steps 2-5 are repeated for the moment τ_2, with the exception that in (7) we use $a_i^{\max}(\tau_1), a_i^{\min}(\tau_1)$, respectively, as initial values instead of $a_i(\tau_0)$. We continue to implement these steps up to the moment τ_m.

The algorithm gives values of factors in an interval representation that is suitable for prediction of the business system dynamic in an uncertain future.

6. CONCLUSIONS

There are three important features of the suggested approach:

- *Complexity*: The triad model consists of three units (objectives unit, evaluation unit, operations unit) and considers their interactions through a set of variables.
- *Formalism*: A new tool of the graph of increments is introduced, structurally similar to Petri nets, but with different principles of functioning that provide additional opportunities.
- *Interval values of the variables*: This approach is based on the preliminary analysis of business system

behaviour, which defines the ranges of possible values of the variables. Specific values are chosen during the subsequent detailed analysis.

Our main contribution is in providing a general framework for investigation of the conditions for successful functioning of business systems. The suggested triad model incorporates the ability to describe internal business processes and events, as well as external events, which reflect the influence of socio-economic and political factors. Future work will include an illustration of such modelling approaches with real-life examples.

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