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Integrated Design of Robust Controller and Fault Estimator for Linear Parameter Varying Systems

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Abstract: An integrated design of robust controller and fault estimator for linear parameter varying systems is presented in this paper. Based on gain-scheduled H_∞ design strategy and scaled bounded real lemma, a linear parameter varying controller is developed, which can generate both control signals and fault estimates. To demonstrate the effectiveness of the proposed method, an uncertain system with actuator faults is studied.

1. INTRODUCTION

The issue of Fault Detection and Isolation (FDI) has been an active research area in the last two decades, a useful survey on FDI can be found in the book of Chen and Patton (1999). Early papers on FDI suffered from problems due to modelling uncertainties. To achieve robustness in the presence of disturbances and uncertainty, optimization-based FDI schemes have been proposed where an appropriately selected performance index is chosen to enhance sensitivity to the faults and simultaneously attenuate disturbances. One of the popular methods is the so-called robust H_∞ FDI (Liu & Frank, 1999, Stoustrup & Niemann 2002, Zhong *et al*., 2003, Casavola *et al*., 2005).

In the literature dealing with FDI, the filters for FDI have in general been considered as a separate design problem from the design of feedback controllers. However, some attention has been paid to the integrated design of the controller and the FDI filter (Jacobson & Nett, 1991, Stoustrup *et al*., 1997, Marcos & Balas, 2005, Castro *et al*., 2006). Stoustrup *et al*. (1997) have shown that the optimal integrated design is equal to the optimal separate design of the controller and detection filter if there is no model uncertainty. When there is uncertainty the design is coupled and then an integrated approach would present a more advantageous framework for the trade-off between performance and robustness. Indeed the integrated approach is very important for the design and development of fault-tolerant control schemes

Recently, FDI for linear parameter varying (LPV) systems has attracted many investigators (Bokor & Balas, 2004, Casavola *et al*., 2008). LPV systems are linear time-varying plants whose state-space matrices are fixed functions of some vector of varying parameters. An LPV system can be reduced to a linear time-varying (LTV) system for a given parameter trajectory and it can also be transformed into a linear time-invariant (LTI) system on a constant trajectory. From a practical point of view, a large class of nonlinear systems can be reduced to LPV systems by using the linearization along trajectories of the parameters. LPV methods have been successfully used in control design to provide guaranteed stability and performance (Apkarian *et al*., 1995), its extension to the FDI problem has not been studied thoroughly.

In this paper, we study the integrated design of a robust controller and fault estimator for a class of LPV plants which depend *affinely* on a vector of time-varying parameters. The resulting LPV controller has the same parameter dependence as the plants, and can generate both the control action and the fault estimates.

2. PROBLEM FORMULATION

The setup considered is illustrated in Fig. 1.

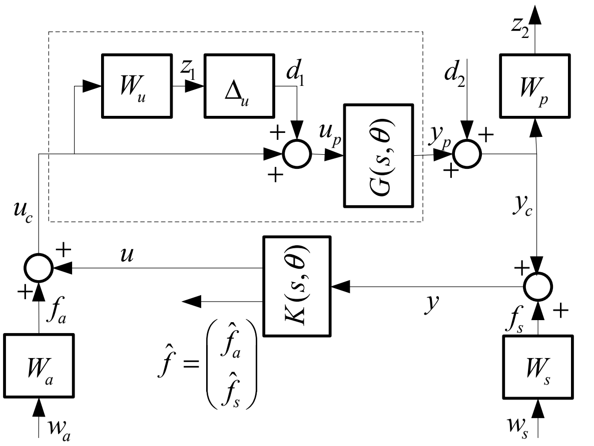

Fig. 1. Control system with faults

Where $G(s,\theta)$ is the LPV plant, θ is a vector of varying parameters measured in real time during system operation, Δ_u is the uncertainty block which reflects the modelling uncertainty or unmodelled dynamics, and satisfies $\|\Delta\|_\infty \le 1/\gamma$, W_p and W_u are weighting functions for performance and robustness, respectively. W_a and W_s are the fault models of the actuator faults signals f_a and sensor faults signals f_s, respectively. $\hat{f}$ is the estimation of the actuator and sensor faults, and the exogenous signal d_2 is uncertain disturbance.

The objective of this paper is to design a LPV controller $K(s,\theta)$ for Fig. 1 such that the system has satisfactory control performance and fault estimation performance.

Define a fault estimation error z_3 as:

$$z_3 = \begin{pmatrix} f_a \\ f_s \end{pmatrix} - \begin{pmatrix} \hat{f}_a \\ \hat{f}_s \end{pmatrix} := f - \hat{f} \tag{1}$$

Then Fig. 1 can be rearranged as Fig. 2 which is the standard structure in H_∞ theory. The transfer function from d_2 to z_2 defines the performance of the closed-loop control system and the transfer function from (w_a, w_s) to z_3 defines the performance of the fault estimator. Now the integrated design problem can be expressed as:

Problem 1: Find an internally stabilizing controller $K(s,\theta)$ such that the closed-loop system is internally stable and the H_∞ norm of the operator mapping $\begin{pmatrix} d_2^T & w_a^T & w_s^t \end{pmatrix}^T$ into $\begin{pmatrix} z_2^T & z_3^T \end{pmatrix}^T$ is bounded by γ for all $\|\Delta_u\|_\infty \le 1/\gamma$.

By inserting fictitious perturbation blocks, Problem 1 can be further reduced to the robust stability problem as shown in Fig. 3 where Δ_p ($\|\Delta_p\|_\infty \le 1/\gamma$) and Δ_f ($\|\Delta_f\|_\infty \le 1/\gamma$) blocks represent performances of the closed-loop system and fault estimation, respectively.

The equivalent uncertainty block for the robust stability problem in Fig. 3 has the following structure:

$$\begin{aligned} \Delta := \{&\mathrm{diag}\left(\Delta_u, \Delta_p, \Delta_f\right): \\ &\Delta_u \in \mathbf{C}^{q_1\times q_1}, \Delta_p \in \mathbf{C}^{q_2\times q_2}, \Delta_f \in \mathbf{C}^{q_3\times q_3}\} \end{aligned} \tag{2}$$

This structured uncertainty problem can be solved by existing methods, such as μ-synthesis. In this paper we use the scaled H_∞ control theory. The set of scaling matrices associated with the structure Δ is:

$$\begin{aligned} L_\Delta := \{&\mathrm{diag}(\sigma_1 I_{q_1}, \sigma_2 I_{q_2}, \sigma_3 I_{q_3}): \\ &\sigma_i \in \mathbf{R}, \sigma_i > 0, \forall i = 1,2,3\} \end{aligned} \tag{3}$$

According to the small gain theorem, the sufficient condition for robust stability of the system in Fig.3 can be expressed as Problem 2.

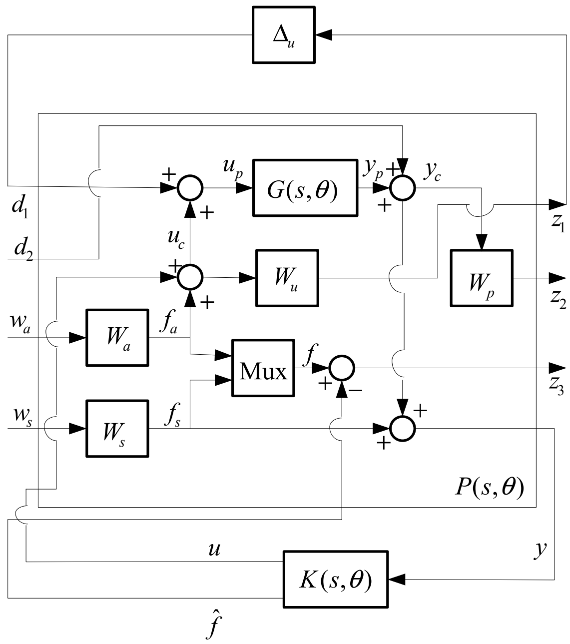

Fig. 2. Standard structure for robust control and fault estimation

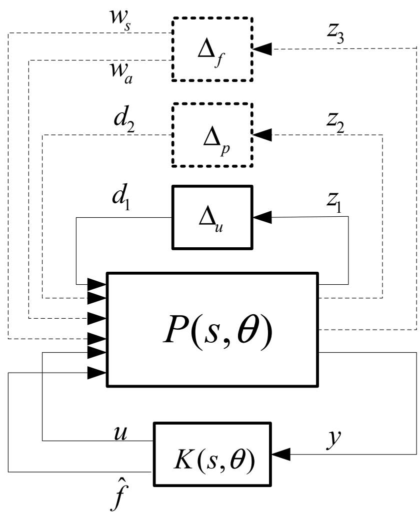

Fig. 3. Equivalent structure for robust stability

Problem 2: The system in Fig.3 is internally stable for all $\|\Delta_u\|_\infty \le 1/\gamma$, $\|\Delta_p\|_\infty \le 1/\gamma$, $\|\Delta_f\|_\infty \le 1/\gamma$, if there exists a scaling matrix $L \in L_\Delta$ and a controller $K(s,\theta)$ such that the nominal system is internally stable and the closed-loop transfer function $F_l(P(s,\theta), K(s,\theta))$ form (d_1, d_2, w_a, w_s) to (z_1, z_2, z_3) satisfies

$$\left\| L^{\frac{1}{2}} F_l(P(s,\theta), K(s,\theta)) L^{-\frac{1}{2}} \right\|_\infty < \gamma \tag{4}$$

Remark 1: If $K(s,\theta)$ satisfies Problem 2 then it also satisfies Problem 1. In the remaining of this paper, we will focus on the solution to Problem 2.

3. DESIGN METHOD

This section will discuss the solution to Problem 2. Suppose the LPV plant $G(s,\theta)$ has the following state-space representation

$$G(s,\theta)=\left[\begin{array}{c|c} A_g(\theta) & B_g \\ \hline C_g & 0 \end{array}\right] \tag{5}$$

Here we assume that the matrices $\boldsymbol{B}_g$ and $\boldsymbol{C}_g$ are parameter independent. This assumption can be alleviated by pre- and/or post-filtering of the control inputs and/or the measured outputs.

More assumptions on the LPV plant $\boldsymbol{G}(s,\boldsymbol{\theta})$

(A1) The vector of varying parameters $\boldsymbol{\theta}(t)$ varies in a polytope $\boldsymbol{\Theta}$ of vertices $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_r$, i.e.:

$$\begin{aligned}\theta(t)\in\Theta &:= Co\{\theta_1,\theta_2,\cdots,\theta_r\} \\ &= \left\{\sum_{i=1}^{r}\alpha_i\theta_i : \alpha_i\ge 0, \sum_{i=1}^{r}\alpha_i = 1\right\}\end{aligned} \tag{6}$$

(A2) The matrix $\boldsymbol{A}_g(\boldsymbol{\theta})$ depends affinely on $\boldsymbol{\theta}(t)$.

(A3) The pairs ($\boldsymbol{A}_g(\boldsymbol{\theta})$, $\boldsymbol{B}_g$) and ($\boldsymbol{A}_g(\boldsymbol{\theta})$, $\boldsymbol{C}_g$) are quadratically stabilizable and quadratically detectable over $\boldsymbol{\Theta}$, respectively.

From (A1) and (A2), it is clear that the LPV system (5) is a polytopic system, i.e.

$$\begin{bmatrix} A_g(\theta) & B_g \\ C_g & 0 \end{bmatrix} \in Co\left\{\begin{bmatrix} A_g(\theta_i) & B_g \\ C_g & 0 \end{bmatrix}, i=1,\cdots,r\right\} \tag{7}$$

The state-space representations of the weightings are as follows:

$$W_u=\left[\begin{array}{c|c} A_u & B_u \\ \hline C_u & D_u \end{array}\right] \qquad W_p=\left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & D_p \end{array}\right]$$

$$W_a=\left[\begin{array}{c|c} A_a & B_a \\ \hline C_a & D_a \end{array}\right] \qquad W_s=\left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array}\right]$$

The state-space realization of the generalized plant $P(s,\theta)$ is:

$$P(s,\theta)=\left[\begin{array}{c|cc} A(\theta) & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{8}$$

Where:

$$A(\theta)=\begin{bmatrix} A_g(\theta) & 0 & 0 & B_gC_a & 0 \\ 0 & A_u & 0 & B_uC_a & 0 \\ B_pC_g & 0 & A_p & 0 & 0 \\ 0 & 0 & 0 & A_a & 0 \\ 0 & 0 & 0 & 0 & A_s \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} B_1 & B_2 \end{bmatrix}=\left[\begin{array}{cccc|ccc} B_g & 0 & B_gD_a & 0 & B_g & 0 & 0 \\ 0 & 0 & B_uD_a & 0 & B_u & 0 & 0 \\ 0 & B_p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_s & 0 & 0 & 0 \end{array}\right] \tag{10}$$

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}=\left[\begin{array}{ccccc} 0 & C_u & 0 & D_uC_a & 0 \\ D_pC_g & 0 & C_p & 0 & 0 \\ 0 & 0 & 0 & C_a & 0 \\ 0 & 0 & 0 & 0 & C_s \\ \hline C_g & 0 & 0 & 0 & C_s \end{array}\right] \tag{11}$$

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}=\left[\begin{array}{cccc|ccc} 0 & 0 & D_uD_a & 0 & D_u & 0 & 0 \\ 0 & D_p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & D_a & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & D_s & 0 & 0 & -I \\ \hline 0 & I & 0 & D_s & 0 & 0 & 0 \end{array}\right] \tag{12}$$

The dimensions of the generalized plant (8) are given by

$$A(\theta)\in\mathbf{R}^{n\times n},\ D_{11}\in\mathbf{R}^{p_1\times p_1},\ D_{22}\in\mathbf{R}^{p_2\times m_2} \tag{13}$$

The LPV controller $K(s,\theta)$ to be designed in this paper has the same parameter dependence as the plant, i.e.

$$\begin{aligned}\mathcal{K}(\theta) &:= \begin{bmatrix} A_K(\theta) & B_K(\theta) \\ C_K(\theta) & D_K(\theta) \end{bmatrix} \\ &\in Co\left\{\mathcal{K}_i=\begin{bmatrix} A_K(\theta_i) & B_K(\theta_i) \\ C_K(\theta_i) & D_K(\theta_i) \end{bmatrix}, i=1,\cdots,r\right\}\end{aligned} \tag{14}$$

As shown in Fig. 3, by closing the loop with the k^{th}-order controller: $K(s,\theta)$, we have the following closed-loop system:

$$\begin{aligned}\dot{x}_{cl} &= A_{cl}(\theta)x_{cl}+B_{cl}(\theta)w \\ z &= C_{cl}(\theta)x_{cl}+D_{cl}(\theta)w\end{aligned} \tag{15}$$

Where:

$$w=\left(d_1^T\ d_2^T\ w_a^T\ w_s^T\right)^T$$

$$z=\left(z_1^T\ z_2^T\ z_3^T\right)^T$$

$$A_{cl}(\theta)=A_0(\theta)+\mathcal{B}\mathcal{K}(\theta)\mathcal{C}$$

$$B_{cl}(\theta) = B_0 + \mathcal{B}\mathcal{K}(\theta)\mathcal{D}_{21}$$
$$C_{cl}(\theta) = C_0 + \mathcal{D}_{12}\mathcal{K}(\theta)\mathcal{C}$$
$$D_{cl}(\theta) = D_{11} + \mathcal{D}_{12}\mathcal{K}(\theta)\mathcal{D}_{21} \tag{16}$$

and with

$$A_0(\theta) = \begin{pmatrix} A(\theta) & 0_{n\times k} \\ 0_{k\times n} & 0_{k\times k} \end{pmatrix},\ B_0 = \begin{pmatrix} B_1 \\ 0_{k\times p_1} \end{pmatrix}$$
$$C_0 = \begin{pmatrix} C_1 & 0_{p_1\times k} \end{pmatrix}$$
$$\mathcal{B} = \begin{pmatrix} 0_{n\times k} & B_2 \\ I_{k\times k} & 0_{k\times m_2} \end{pmatrix},\ \mathcal{C} = \begin{pmatrix} 0_{k\times n} & I_{k\times k} \\ C_2 & 0_{p_2\times k} \end{pmatrix}$$
$$\mathcal{D}_{12} = \begin{pmatrix} 0_{p_1\times k} & D_{12} \end{pmatrix},\ \mathcal{D}_{21} = \begin{pmatrix} 0_{k\times p_1} \\ D_{21} \end{pmatrix} \tag{17}$$

Since the state-space matrices of the closed-loop system (15) depend affinely on $\boldsymbol{\theta}(t)$, the following statements are equivalent:

(i) $A_{cl}(\theta)$ is stable and there exists $L \in L_\Delta$ such that $\| L^{1/2}(C_{cl}(\theta)(sI - A_{cl}(\theta))^{-1}B_{cl}(\theta) + D_{cl}(\theta))L^{-1/2} \|_\infty < \gamma$ for all possible parameter trajectories $\boldsymbol{\theta}(\boldsymbol{t})$ in the polytope $\boldsymbol{\Theta}$.

(ii) There exist positive definite solutions X and $L \in L_\Delta$ to the matrix inequalities:

$$\begin{pmatrix} A_{cl}^T(\theta_i)X + XA_{cl}(\theta_i) & XB_{cl}(\theta_i) & C_{cl}^T(\theta_i) \\ B_{cl}^T(\theta_i)X & -\gamma L & D_{cl}^T(\theta_i) \\ C_{cl}(\theta_i) & D_{cl}(\theta_i) & -\gamma L^{-1} \end{pmatrix} < 0 \tag{18}$$
$$i = 1, 2, \cdots, r$$

Remark 2: The statement (i) is the sufficient condition for robust stability of the system in Fig. 3 (see Problem 2). The equivalence between statement (i) and (ii) is an extension of the scaled bounded real lemma for linear time-invariant systems (Apkarian & Gahinet, 1995) to LPV systems. Based on this extension, and through some manipulations, we have the following theorem which gives a solution to the Problem 1.

Theorem 1

Consider the LPV plant (5) with assumptions (A1)-(A3). Let $\mathcal{N}_R$ and $\mathcal{N}_s$ denote bases of the null spaces of $(B_2^T, D_{12}^T, 0)$ and $(C_2, D_{21}, 0)$, respectively. The Problem 1 is solvable if and only if there exist two symmetric matrices (R, S) in $\mathrm{R}^{n\times n}$ and (L, J) in L_Δ such that

$$\mathcal{N}_R^T \begin{pmatrix} A_iR + RA_i^T & RC_1^T & B_1 \\ C_1R & -\gamma J & D_{11} \\ B_1^T & D_{11}^T & -\gamma L \end{pmatrix} \mathcal{N}_R < 0, i = 1, \cdots, r \tag{19}$$

$$\mathcal{N}_s^T \begin{pmatrix} A_i^TS + SA_i & SB_1 & C_1^T \\ B_1^TS & -\gamma L & D_{11}^T \\ C_1 & D_{11} & -\gamma J \end{pmatrix} \mathcal{N}_s < 0, i = 1, \ldots, r \tag{20}$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0 \tag{21}$$

$$LJ = I \tag{22}$$

where A_i denotes the value of $A(\theta)$ at the vertices $\theta = \theta_i$ of the parameter polytope $\boldsymbol{\Theta}$.

Constraint (22) is non-convex, the global optimization algorithm based on convex area search (Yamada *et al.*, 1995) is employed in this paper. Once the solution (R, S, L, J) of the matrix inequalities (19)-(22) has been given, the state-space data $\mathcal{K}(\theta)$ of the LPV controller $K(s, \theta)$ can be computed as follows (Apkarian & Gahinet, 1995, Apkarian *et al.*, 1995):

Algorithm 1

1). Compute the full column rank matrices $M, N \in \mathrm{R}^{n\times k}$ such that

$$MN^T = I - RS \tag{23}$$

2). Compute X as the unique solution of the linear matrix equation

$$X\begin{pmatrix} I & R \\ 0 & M^T \end{pmatrix} = \begin{pmatrix} S & I \\ N^T & 0 \end{pmatrix} \tag{24}$$

3). Compute $\mathcal{K}_i$ by solving the linear matrix inequality (18).

4). Compute the state-space data $\mathcal{K}(\theta)$ of the controller $K(s, \theta)$.

$$\mathcal{K}(\theta) = \sum_{i=1}^{r} \alpha_i \mathcal{K}_i \tag{25}$$

where α_i is any solution of the following convex decomposition problem:

$$\theta = \sum_{i=1}^{r} \alpha_i \theta_i \tag{26}$$

4. EXAMPLE

In this section we present an example illustrating the integrated design method. The plant considered has the following state-space description:

$$\dot{x}_p = \begin{pmatrix} 0 & 1 \\ -p_1 & -p_2 \end{pmatrix} x_p + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_p$$
$$y_p = \begin{pmatrix} 10 & 1 \end{pmatrix} x_p \tag{27}$$

where x_p, u_p and y_p are the vectors of states, input and output of the plant, respectively. p_1 and p_2 are time-varying parameters which can be measured in real time, $p_1 \in [6,14]$, $p_2 \in [9,13]$. The plant is subject to multiplicative uncertainties and actuator faults, the interconnections of the generalized plant are shown in Fig. 1 and Fig. 2. The weighting functions are selected as:

$$W_u = \frac{s+10}{s+1000} \tag{28}$$

$$W_p = \frac{10s+1000}{100s+1} \tag{29}$$

$$W_a = \frac{10}{s+10} \tag{30}$$

The purpose is to design a combined controller and estimator such that the system has satisfactory control performance and fault estimation performance.

Applying Theorem 1 to this problem and solving the matrix inequalities (19)-(22) yields a performance level of γ = 0.71433. Then according to the steps in Algorithm 1, a polytopic LPV controller can be constructed.

Fig. 4 shows a simulation with an actuator fault. The disturbance d_2 is Gaussian white noise with standard deviation of 0.2. The parameter trajectories of p_1 and p_2 are

$$p_1(t) = 6 + 2t,\ p_2(t) = 9 + t,\ 0 \le t \le 4$$

Simulation results show that the designed controller can identify the actuator fault effectively, the disturbance has little effect on the fault estimation, and the actuator fault does not have disastrous effects on the system.

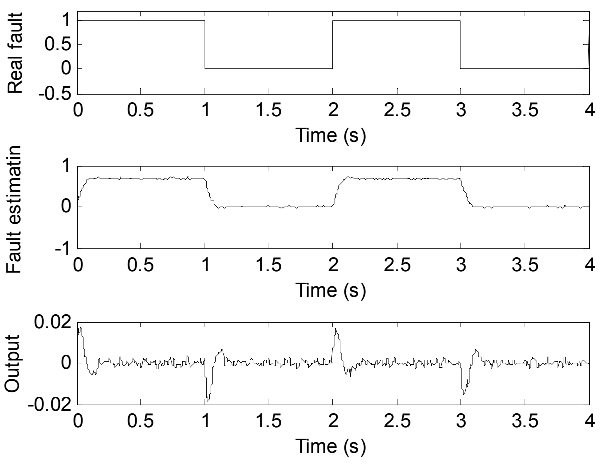

Fig. 4. Simulation for integrated design

5. CONCLUSIONS

A new framework for the simultaneous design of a robust controller and fault estimator for LPV systems has been outlined in this paper based on scaled H_∞ theory and gain-scheduled techniques. The work forms a part of an on-going study by the authors on the development of new methods for Fault Tolerant Control. Research is already underway on application of these methods to practical laboratory and engineering systems.

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