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A Hybrid Meta-heuristic Method for Multimodal Logistics Network Design over Planning Horizon

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Abstract: Logistics optimization has been acknowledged increasingly as a key issue of supply chain management to improve the business efficiency under global competition and diversified customer demands. Concerning a multimodal logistics optimization problem under multi-periods, in this paper, we have extended a method termed hybrid tabu search that was developed previously by the authors. The attempt aims at deploying a strategic planning more concretely so that it can link to an operational decision making. It is a two-level iterative method composed of tabu search to solve the location problem in the upper level while graph algorithm to solve the transformed minimum cost flow problem for the route selection in the lower level. Through numerical experiments, we have verified the effectiveness of the proposed method in comparison with the commercial software.

Keywords: Logistics network optimization; Hybrid tabu search; Multimodal transport; Multi-term planning.

1. INTRODUCTION

Logistic optimization has been acknowledged increasingly as a key issue of supply chain management to improve the business efficiency under global competition and diversified customer demands. Though many studies have been made in the area of operations research associated with combinatorial optimization (for example, Campbell, 1994), we need to make different efforts to cope with complex and complicated real world problems. From such aspects, we concerned various logistic optimization problems for strategic or static decision making so far. (Shimizu and Wada, 2004; Wada, Shimizu and Yoo, 2005; Shimizu, Matsuda and Wada, 2006).

During the planning horizons, however, there usually occur various deviations assumed constant in the strategic or static model. Taking into accounts such a dynamic circumstance, we can make a more reliable and operational decision making. In this study, therefore, we have extended our previous approach termed hybrid tabu search so as to incorporate a production planning such as an inventory management and a multimodal transport into the logistics network design optimization.

After presenting a general formulation and its solution method, the validity of the proposed method is shown through numerical experiment in comparison with the results obtained from the commercial software.

2. PROBLEM FORMULATION

2.1 Preliminary Statements

Many studies in the area of operations research make point to develop new algorithms and compete their abilities through simple benchmarking, and/or to reveal theoretical truth about how fast, how exactly and how large problem to be solvable. However, easy applications following these outcomes often cause a dramatic increase in problem size in real world problems, and accordingly such a difficulty that makes almost impossible to solve the resulting problem by any currently available software.

Under such understanding, to cope with the specific problem in complex and complicated real world situation, we concerned various logistics optimization problems for strategic or static decision making. They are interested in certain conditions like realistic discount of transportation cost, flexibility against demand deviations, multi-commodity delivery and so on. Moreover, we recently focused on the role of inventory management of warehouse or distribution centre (DC) through introducing a dynamic nature into the network model (Yamazaki, Wada, Fujikura and Shimizu, 2007).

The hybrid tabu search used for those studies decomposes the original problem into upper-level and lower-level sub-problems, and applies a suitable method for each sub-problem. The upper level sub-problem decides the locations of DC by the sophisticated tabu search.

Tabu search (TS; Glover, 1989, 1990) is a metaheuristic algorithm on a basis of local search technique. TS repeats the local search iteratively to move from a current solution x to a possible and best solution x' in the neighbor of $x, N(x)$. To avoid the cycling of the solution, TS uses a short-term memory structure termed tabu list that prohibits transition to any solutions for a while even if this will improve the current solution. The basic iteration process of TS is outlined for minimization problem as follows:

Step 1: Generate an initial solution x and let $x^* := x$, where x^* denotes the current best solution. Set $k = 0$ and let the tabu list $T(k)$ be empty.
Step 2: If $N(x) - T(k)$ becomes empty, stop. Otherwise, set $k := k + 1$ and select x' such that $x' = \text{argmin } f(x)$ for $\forall\ x\ \in N(x) - T(k)$.
Step 3: If x' outperforms the current solution x^*, *i.e.*, $f(x') \leq f(x^*)$, let $x^* := x'$.
Step 4: If a chosen number of iterations has elapsed either in total or since x^* was last improved, stop. Otherwise, update $T(k)$, and go back to Step 2.

On the other hand, the lower level sub-problem needs to decide the network routes under the prescribed upper level decision. It refers to a linear program possible to transform into a minimum cost flow (MCF) problem. In practice, this transformation is carried out by adding virtual nodes and edges to the physical configuration as illustrated in Fig.1. Then the resulting MCF problem can be solved by the graph algorithm, for which especially fast solution algorithm such as CS2 (Goldberg, 1997) is known.

Now by returning to the upper-level, neighbor locations are to be evaluated following the algorithm of the sophisticated tabu search whose local search operation is summarized in Table 1. Thereat, to enhance the efficiency of algorithm, selection probability of each operation is decided based on the following ideas. It is likely that the search types like "Add" and/or "Subtract" might be used rather often at the earlier stage of search while "Swap" is more suitable at the final stage where the number of open DCs is almost optimal.

Letting it be a basis to decide the probability in the table, we further present an idea to do it more effectively using the long-term memory or a history of so far search processes. That is, the probability of each operation will be increased by a certain rate if it has brought about the update of solution. In contrast, these values are reset when the tentative solution is not updated by the consecutive duration prescribed a priori and/or a feasible solution has not been obtained.

These procedures will be repeated until a certain convergence criterion has been satisfied. Figure 2 illustrates schematically this solution procedure.

2.2 Multimodal Model over Planning Horizon

We have extended the foregoing static or single-term development to cope with the multi-term problem in a practical manner. By making use of the available stock of DC to the descendent periods (inventory control), we revealed such formulation could bring about significant effects on the strategic decision making. In fact, our multi-term model can derive the result as shown in Fig.3 where the stocks at the DC are utilized to meet the customer demands beyond the upper bound of production ability.

Table 1. Employed neighborhood search operations

Search type	Selection probability	Neighborhood operation
Add	p_{add}	Let closed DC open.
Subtract	p_{subtract}	Let open DC close.
Swap	p_{swap}	Let closed DC open and open DC close.

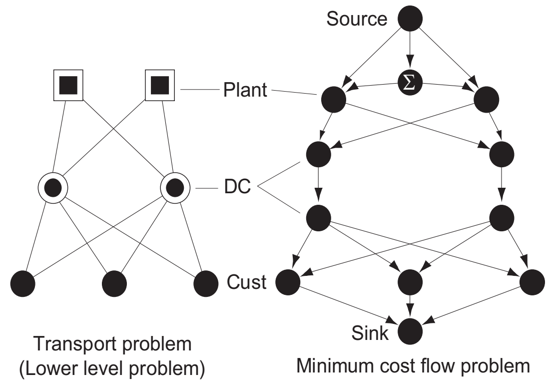

Fig. 1. Transformation of network to MCF graph

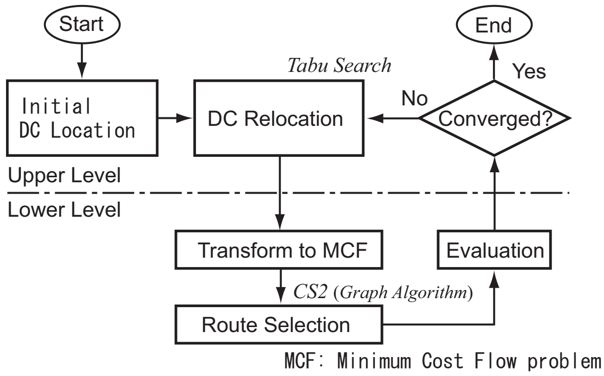

Fig. 2. Schematic procedure of hybrid tabu search

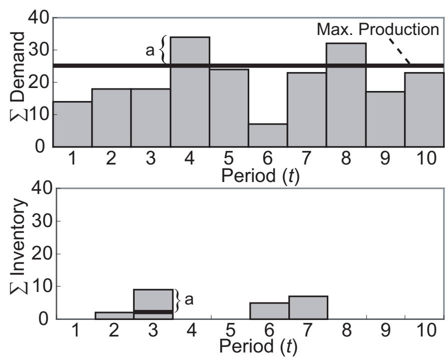

Fig. 3. Role of inventory in multi-term planning

However, in such a dynamic circumstance, we can make a more reliable and comprehensive decision by taking into accounts the multimodal transport that is commonly known as a transport operation carried out using different modes of transport, *e.g.*, truck, train, ship, *etc.*

This is because we can make proper use of transportation vehicles depending on the situation, say, either fast (short lead time) but expensive and small one or slow (long lead time) but cheap and large one as illustrated in Fig.4. However, introduction of such interests into the model will expands drastically the difficulty to solve the resulting problem. To cope with the problem, we provide two approaches mentioned below.

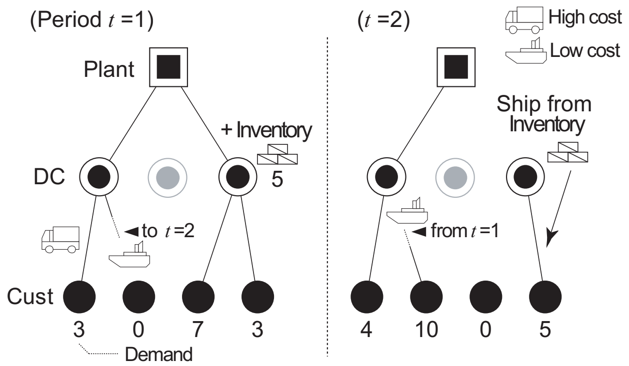

Fig. 4. Scheme of multimodal logistics network model

First, we formulate mathematically the problem under consideration as follows.

$$(p.1)\ \min_{x,f,s}\quad f(\boldsymbol{x},\boldsymbol{f},\boldsymbol{s}) = \sum_{t\in T}\sum_{t_a\in T}\sum_{i\in I}\sum_{j\in J}(E_{ij}^{tt_a}+C_i^t+H_j^t)f_{ij}^{tt_a} + \sum_{t\in T}\sum_{t_a\in T}\sum_{j\in J}\sum_{j'\in J}(E_{jj'}^{tt_a}+H_{j'}^t)f_{jj'}^{tt_a} + \sum_{t\in T}\sum_{t_a\in T}\sum_{j\in J}\sum_{k\in K}E_{jk}^{tt_a}\cdot f_{jk}^{tt_a} + \sum_{t\in T}\sum_{t_a\in T}\sum_{I\in I}\sum_{k\in K}(E_{ik}^{tt_a}+C_i^t)f_{ik}^{tt_a} + \sum_{t\in T}\sum_{j\in J}K_j^t\cdot s_j^t + \sum_{j\in J}F_j\cdot x_j \quad (1)$$

subject to

$$\sum_{t_s\in T}\sum_{j\in J}f_{jk}^{t_st} + \sum_{t_s\in T}\sum_{i\in I}f_{ik}^{t_st} = D_k^t, \quad \forall\, k\in K,\quad \forall\, t\in T \quad (2)$$

$$\sum_{t_s\in T}\sum_{i\in I}f_{ij}^{t_st} + \sum_{t_s\in T}\sum_{j'\in J}f_{j'j}^{t_st} + s_j^t \le U_j^t\cdot x_j, \quad \forall\, j\in J,\quad \forall\, t\in T \quad (3)$$

$$\sum_{t_a\in T}\sum_{j\in J}f_{ij}^{tt_a} + \sum_{t_a\in T}\sum_{k\in K}f_{ik}^{tt_a} \le \bar{P}_i^t, \quad \forall\, i\in I,\quad \forall\, t\in T \quad (4)$$

$$\sum_{t_a\in T}\sum_{j\in J}f_{ij}^{tt_a} + \sum_{t_a\in T}\sum_{k\in K}f_{ik}^{tt_a} \ge \underline{P}_i^t, \quad \forall\, i\in I,\quad \forall\, t\in T \quad (5)$$

$$\sum_{t_s\in T}\sum_{i\in I}f_{ij}^{t_st} + \sum_{t_s\in T}\sum_{j'\in J}f_{j'j}^{t_st} + s_j^t = \sum_{t_a\in T}\sum_{j'\in J}f_{jj'}^{tt_a} + \sum_{t_a\in T}\sum_{k\in K}f_{jk}^{tt_a} + s_j^{t+1}, \quad \forall\, j\in J,\quad \forall\, t\in T \quad (6)$$

$$\boldsymbol{f},\boldsymbol{s}\ge \boldsymbol{0}\quad \boldsymbol{f},\boldsymbol{s}\in \mathrm{R} \quad (7)$$

$$\boldsymbol{x}\in\{0,1\} \quad (8)$$

Below, we summarize the notations used to describe this problem.

Variable

- x_j : it takes one if DC is open at place j, otherwise 0
- $f_{uv}^{t_st_a}$: quantity that leaves facility u in period t_s and arrive at facility v in period t_a
- s_j^t : stock remaining in the next period at DC j in period t

Index set

- I : Index set indicating plant
- J : Index set indicating DC
- K : Index set indicating customer
- T : Index set indicating planning horizon

Parameter

- $\underline{P}_i^t$: lower bound for production at plant i in period t
- $\bar{P}_i^t$: upper bound for production at plant i in period t
- C_i^t : unit production cost at plant i in period t
- F_j : fixed-charge for opting DC j
- U_j^t : maximum capacity of DC j in period t
- H_j^t : unit operational cost of DC j in period t
- K_j^t : unit inventory cost of DC j in period t
- D_k^t : demand of customer k in period t
- $E_{ij}^{t_st_a}$: unit transport charge for the transportation that leaves plant i in period t_s and arrive at DC j in period t_a
- $E_{jj'}^{t_st_a}$: unit transport charge for the transportation that leaves DC j in period t_s and arrive at DC j' in period t_a
- $E_{jk}^{t_st_a}$: unit transport charge for the transportation that leaves DC j in period t_s and arrive at customer k in period t_a
- $E_{ik}^{t_st_a}$: unit transport charge for the transportation that leaves plant i in period t_s and arrive at customer k in period t_a

Noticing these, let us explain the above optimization problem. The first, forth, sixth and seventh terms of the objective function (1) correspond to the total transportation costs between plant and DC, DC and DC, DC and customer, and plant and customer, respectively. The second and eighth terms are the total production costs at plant, and the third and fifth terms denote the total costs spent for the operations between plant and DC, and DC and DC, respectively. Moreover, the ninth term represents the total holding cost at DC while the tenth term total fixed-charge for opening DC.

On the other hand, the first constraint (2) requires to meet the demand of every customer every period. The capacity constraint at each DC is given by (3) every period. Moreover, (4) and (5) are respectively the upper and lower bounds on the production ability of each plant every period. Finally, (6) describes the material balance at each DC every period. Additionally, non-negative conditions on the material flows and binary condition on the open/close selection are given by (7) and (8), respectively.

In this model, the transportation modes are distinguished by the difference of transportation lead time. It is described by the superscripts as $t_s t_a$, which means that the product leaves in period t_s and arrive in period t_a. Then we assumed the different lead time implies the different transportation mode, and the larger it is, the cheaper the transportation cost is, and vice versa.

In the numerical solution, we often impose certain additional conditions that might enhance the solution speed by limiting the search space properly. The followings are the augmented constraints and variables for this purpose (tight bound constraints).

$$\sum_{j \in J} s_j^1 + \sum_{j \in J} s_j^T = 0 \tag{9}$$

$$f_{ij}^{t_s t_a} \leq M \cdot A_{ij}^{t_s t_a}, \quad \forall\, i \in I,\ j \in J,\ \forall\, t_s \in T,\ \forall\, t_a \in T \tag{10}$$

$$f_{jj'}^{t_s t_a} \leq M \cdot A_{jj'}^{t_s t_a}, \quad \forall\, j \in J,\ j' \in J,\ \forall\, t_s \in T,\ \forall\, t_a \in T \tag{11}$$

$$f_{jk}^{t_s t_a} \leq M \cdot A_{jk}^{t_s t_a}, \quad \forall\, j \in J,\ k \in K,\ \forall\, t_s \in T,\ \forall\, t_a \in T \tag{12}$$

$$f_{ik}^{t_s t_a} \leq M \cdot A_{ik}^{t_s t_a}, \quad \forall\, i \in I,\ k \in K,\ \forall\, t_s \in T,\ \forall\, t_a \in T \tag{13}$$

In the above, (9) restricts the initial and final stocks to be zero. Equations (10) thru (13) exclude the infeasible transportations explicitly where $A_{ij}^{t_s t_a}$ denotes the variable it takes one if there exists such a transportation that leaves place i in period t_s and arrives at place j in period t_a, and otherwise zero. Moreover, M is a very large number. By solving thus formulated problem with the tight bounds, we can reduce the solution time as shown in Fig.5. Relying on this fact, in the following numerical experiments, we will adopt this formulation when we solve the problems using the commercial software.

3. THE HYBRID TABU SEARCH FOR MULTIMODAL TRANSPORT

Adding some ideas to the original method, we show it possible to apply effectively the hybrid tabu search for the present case as outlined below. Under multi-term condition, the lower level sub-problem of the hybrid tabu search needs to decide the network routes for every period. It refers to a linear program whose coefficient matrix becomes almost block diagonal per each period.

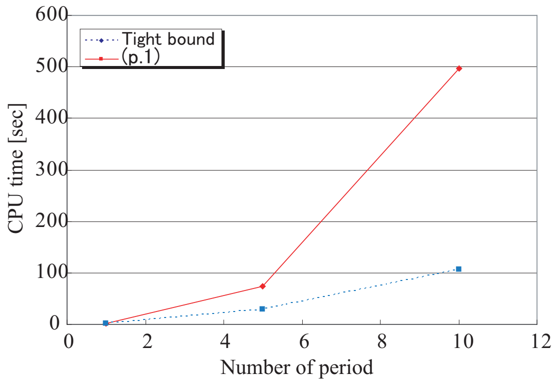

Fig. 5. Effect of tight bound formulation

Noticing such a special topological structure and adding further a dealing for the multimodal transport, we have invented a systematic procedure to transform the bulk original problem into the compact minimum cost flow problem as follows.

Step 1: For every period, place the nodes that stand for plant, DC (doubled), and customer. Next, virtual nodes termed source, Σ, and sink are prepared. Then connect the nodes between source and Σ (ID 1), source and plant (ID 2), Σ and plant (ID 3), duplicated DC nodes (ID 5), and customer and sink (ID 9). This results in the graph as depicted in Fig.6 (a).

Step 2: Letting z be the total amount of customer demand over planning horizon, $z = \sum_{t \in T} \sum_{k \in K} D_k^t$, flow this amount in the source, and flow out from the sink.

Step 3: To constrain the amounts of flow, set the capacities on the edges identified by ID 1, 2, 3, 5 and 9 as shown in "capacity column" in Table 2, respectively. Apparently, there never induce any costs on edge ID 1 and 9 for the connections.

Step 4: To allow the stock at DC, add the edges from down-DC node to up-DC node in the next period as shown in Fig.6 (b). See the "ID 10" row of the table for the labeling.

Step 5: According to the transportation mode, connect the edges between plant and DC (ID 4), DC and DC (ID 6), DC and customer (ID 7) and plant and customer (ID 8) every period (See Fig.6 (c)).

Step 6: Finally, place the appropriate label on each edge (See Fig.6 (d)).

From all of these, we have the final graph shown in Fig.6 (d) that makes it possible to still apply the graph algorithm. Consequently, we can solve the extensively expanded problem extremely fast compared with the linear programs.

4. NUMERICAL EXPERIMENT

4.1 Preliminary Experiments

In a long term planning, instead of the dynamic model, a strategic or conceptual decision is often made based on the averaged values that will fluctuate in reality over the planning horizon such as demand. This is equivalent to say that we attempt to obtain the result from the static or single-term problem for the time being. To verify the

Table 2. Labeling on the edge

Edge ID	Cost	Capacity	Description
#1	0	$\sum_{t\in T}\sum_{i\in I}\underline{P}_i^t$	source–Σ
#2	C_i^t	$\overline{P}_i^t - \underline{P}_i^t$	source-plant i (period t)
#3	C_i^t	$\underline{P}_i^t$	Σ– plant i (period t)
#4	$E_{ij}^{tt_a}$	∞	plant i–DC j (leave in period t and arrive in t_a)
#5	H_j^t	U_j^t	between doubled nodes representing DC (period t)
#6	$E_{jj'}^{tt_a}$	∞	DC j–DC j' (leave in period t and arrive in t_a)
#7	$E_{jk}^{tt_a}$	∞	DC j–customer k (leave in period t and arrive in t_a)
#8	$E_{ik}^{tt_a}$	∞	plant i–customer k (leave in period t and arrive in t_a)
#9	0	D_k^t	customer k–sink (period t)
#10	K_j^t	∞	stock at DC j (period t)

advantage of considering the dynamic model that enables us to make use of the stock of inventory, we compared the results between the (averaged) single-term model and the multi-term model using small size benchmark problems.

In Table 3, we summarize the results taken place under the conditions of demand deviations and single modal transport. Thereat, we know that the dynamic model can derive decisions with less total costs (the value of average model is represented as the rate to the value of the multi-period model to be one hundred). Particularly, it is remarkable that there appeared the case where the average model could not obtain the feasible solution against the demand deviations while the multi-period model could cope with every situation by virtue of the inventory control.

Table 3. Effect of dynamic model

Properties of problem				Total cost	
Plant	DC	Cust	Period	Multi-period model	Average model
1	10	20	5	100.0	107.9
1	15	25	10	100.0	N/A
1	20	30	20	100.0	138.2

Next, to examine a rising tendency of computational load with the increase in numbers of modal transport along the problem size (number of planning horizon), we carried out another numerical experiment. Figure 7 shows a comparison of CPU time required to solve the problem till 25 periods using commercial software known as CPLEX 9.0 between uni- and two-modal transport when $|I| = 2, |J| = 25$ and $|K| = 30$. We know the computation load grows rapidly when the two-modal model is considered even for those small size problems. It implies we need to resolve the difficulties associated with the dimensionality for real world applications.

4.2 Evaluation of the Proposed Algorithm

From so far discussions, it is interesting to examine the effectiveness of the proposed method in terms of problem size. In Table 4, we summarize the computation environment for the present numerical experiments.

Table 4. Computation environment for numerical experiments

Method	CPU type	Memory	OS
CPLEX	Pentium4 3.0 GHz	512 MB	Windows XP
This work	Pentium4 3.0 GHz	512 MB	Debian 3.0

Figure 8 shows the CPU times compared along with the number of planning horizon between the proposed method and the CPLEX 9.0 (The result of the proposed method is averaged over five trials, and the problem size is set as same as the foregoing one in Sec. 4.1).

Thereat, we can observe the increase is moderate (almost linear) for the proposed method while it is exponential for the CPLEX. In addition, we confirm the approximation rate or the accuracy of the objective function value of the proposed method is high as long as we can compare the results with each other (1, 5, 10, 15, 20 and 25 periods). As shown in Table 5, for every problem, the proposed method can derive the same results as those by CPLEX with very short CPU times. After all, in terms of these numerical experiments, we can ascertain the proposed method is promising for real world applications.

Table 5. Comparison of performance with commercial software

Period	CPLEX	Hybrid tabu	
	CPU time [s]	CPU time [s]	Approx. rate*
1	1.08	0.11	1.0000
5	28.84	1.17	1.0000
10	106.67	3.02	1.0000
15	315.72	5.31	1.0000
20	605.88	7.59	1.0000
25	1141.73	10.26	1.0000

* Objective value of Hybrid tabu/CPLEX

5. CONCLUSIONS

This paper concerned a multimodal and multi-period logistics optimization problem. To cope with such problem, we have extended a method termed hybrid tabu search developed previously. In practice, we have invented a systematic procedure to transform the mathematical model into the graph model and complete it to the multi-term and multimodal model finally.

Numerical experiments revealed that the inventory control and multimodal transport could bring about an economical effect and robustness against demand deviations during the planning horizon. The validity of the proposed method as a solution method was also shown through comparison with the results obtained from the commercial software.

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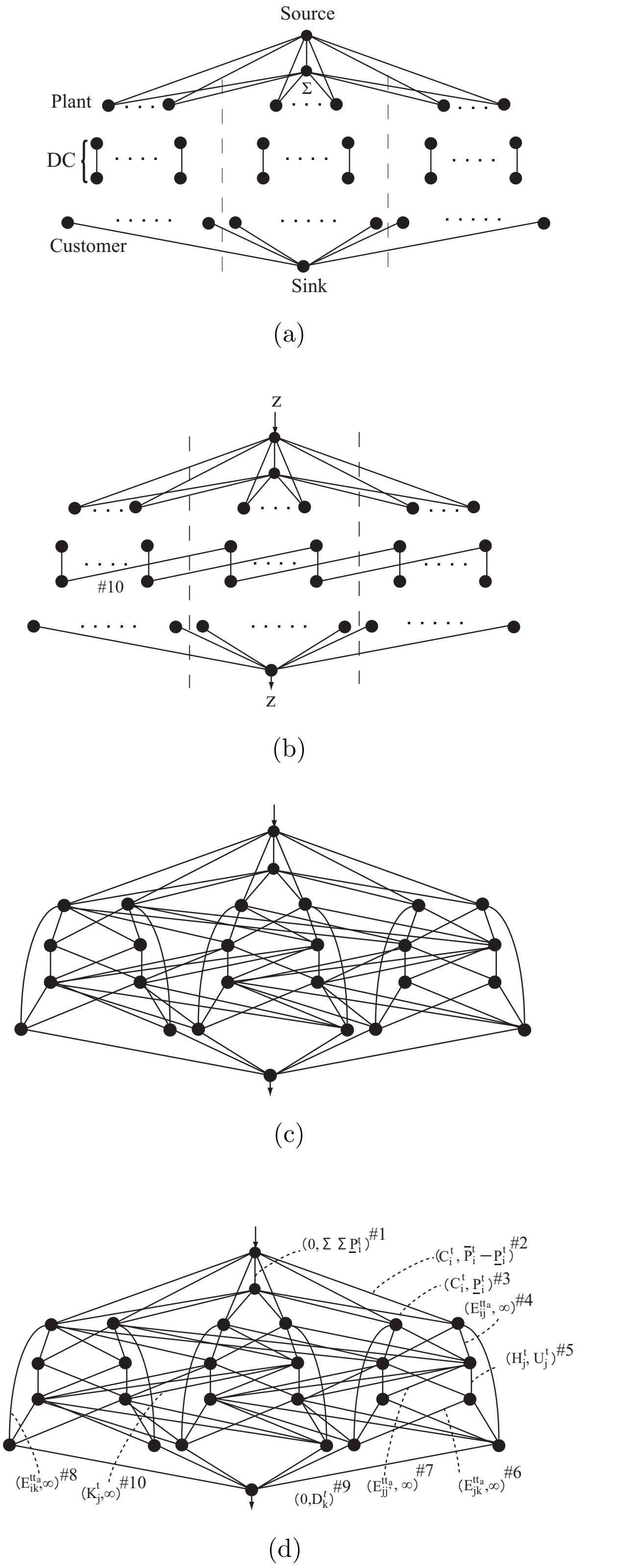

Fig. 6. Transformation procedure to aggregate MCF graph for three periods problem: (a) Initial, (b), (c) Intermediate, and (d) Final stages

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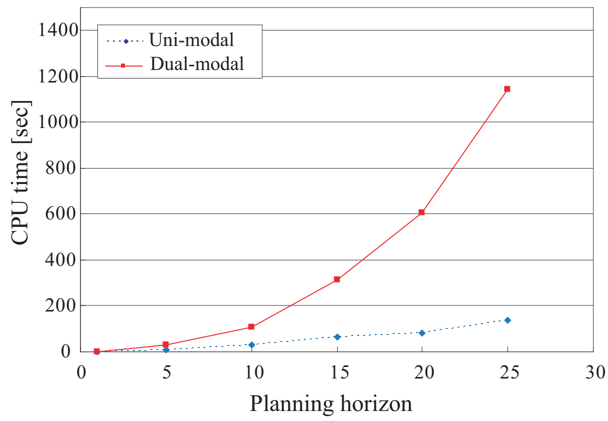

Fig. 7. Comparison of computational loads regarding multimodal transport

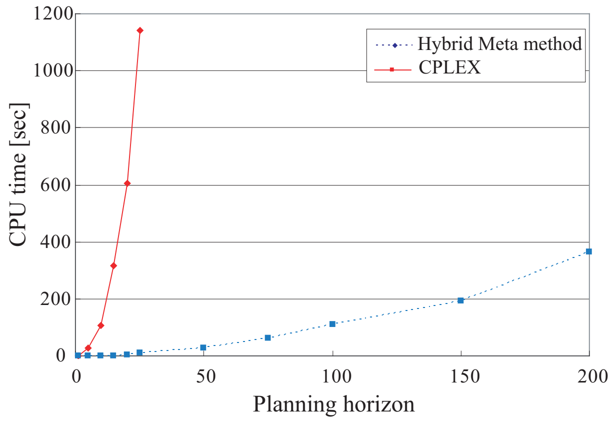

Fig. 8. Comparison of computational loads with commercial software

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