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Decentralized Robust PI Controller Design For An Industrial Utility Boiler - An IMC Method

Batool Labibi, * **Horacio Jose Marquez and Tongwen Chen** **

* *Advanced Process Automation and Control (APAC) Research Group, K. N. Toosi University of Technology, Tehran, Iran.*
** *Electrical and Computer Engineering Department, University of Alberta, Edmonton, Canada.*

Abstract: This paper presents a scheme for designing a robust decentralized PI controller for an industrial utility boiler system. First, a new method for designing robust decentralized PI controllers for uncertain LTI MIMO systems is presented. Sufficient conditions for closed-loop stability and diagonal dominance of a multivariable system are given. For each isolated subsystem a first order approximation is obtained. Then, achieving robust stability and closed-loop diagonal dominance is formulated as local robust performance problems. It is shown by selecting time constants of the closed-loop isolated subsystems appropriately, these local robust performance problems are solved and the interactions between closed-loop stabilized subsystems are attenuated. The internal model control (IMC) method is used to design local PI controllers. The suggested design strategy is applicable to unstable systems as well. Thereafter, the nonlinear model of an industrial utility boiler is linearized about its operating points and the nonlinearity is modeled as uncertainty for a nominal LTI MIMO system. Using the new proposed method, a decentralized PI controller for the uncertain LTI nominal model is designed. The designed controller is applied to the real system. The simulation results show the effectiveness of the proposed methodology.

1. INTRODUCTION

PI controllers have shown to be robust and extremely beneficial in control of many important applications. Extensive research has been done over tuning PID controllers (Astrom et al. (1995)). A powerful and simple strategy for tuning PID controllers is the internal model control (IMC) method. This method directly shapes the closed-loop transfer functions; such as the sensitivity and complimentary sensitivity functions. The classical techniques of frequency domain design for single-input single-output systems can been generalized and applied to multivariable feedback systems by Nyquist like methods (Rosenbrock (1974)). If a good degree of diagonal dominance can be obtained, then decentralized control with Nyquist like methods can be very effective. Control of the interacting multivariable systems can be realized either by centralized MIMO controllers or by a set of SISO local controllers. The decentralized control is more desirable from the view point of implementation, requiring fewer parameters to tune and loop failure tolerance of the resulting control system. Therefore, in process control applications, more often than not, decentralized control is used.

The Syncrude Canada, Ltd. (SCL) integrated energy facility located in Mildred Lake, Alberta utilizes a complex header system for steam distribution. The normal plant operation requires tracking the steam demand while maintaining the steam pressure and the steam temperature of the 6.306-MPa header at their respective set points, despite variations of the steam load. Due to the physical characteristics, utility boilers are used to regulate the steam pressure (Tan et al. (2005)).

In this paper, a robust decentralize PI controller for the utility boiler systems in SCL is designed and applied to the real system. First, a method for robust decentralized PI design is proposed. Sufficient conditions for closed-loop stability and diagonal dominance under a decentralized control are achieved. If the isolated subsystems are of high order, first order models are first obtained. Then, the approximation error can be modeled as multiplicative uncertainty for each isolated subsystem. It will be shown that achieving diagonal dominance and robust stability can be guaranteed by solving certain local robust performance problems to be defined. By appropriately selecting the time constants of the closed-loop isolated subsystems, these local problems can be solved. Then the nonlinear model of the industrial utility boiler in SCL is linearized about its operating points and the nonlinearity is modeled as uncertainty for a nominal LTI system. Thereafter, based on the decentralized robust PI controller design method that we propose, a decentralized PI controller for the system is designed. The resulting controller is applied to the real system using SYNSIM to show the effectiveness of the proposed method. SYNSIM is a simulation package developed by Syncrude Canada with the purpose of simulating certain upset conditions and as a general tool for stability analysis.

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The rest of this paper is organized as follows: In section 2 the problem of finding suitable local dynamical controllers for subsystems of a linear large-scale system is presented. In section 3 sufficient conditions for closed-loop stability and diagonal dominance are given. These conditions are stated in sensitivity functions of closed-loop isolated subsystems. In section 4 the new method for decentralized PI controller design is given. Section 5 gives the method for robust decentralized PI controller design. In section 6 the nonlinear model of the drum boiler system in SCL is linearized about its operating points and the nonlinearity is modeled as uncertainty. By solving the appropriately defined local robust problems, a decentralized PI controller for the system is designed. Then, the designed controller is applied to the real system and the simulation results are given. Finally, concluding results are given in section 7.

2. PROBLEM FORMULATION

Consider an uncertain LTI system $\tilde{G}(s)$ with output multiplicative uncertainty as follows

$$\tilde{G}(s) = (I + \triangle(s)\bar{\bar{W}}_3(s))G(s), \qquad |\triangle(j\omega)| \leq 1 \qquad \forall\omega, \quad (1)$$

where $G(s)$ is the nominal plant, $\triangle(s)$ is any stable transfer function which at each frequency is less than or equal to one in magnitude and $\bar{\bar{W}}_3(s)$ is the weighting matrix which contains the frequency information for the uncertainties. Suppose the nominal system $G(s)$ has the following state-space equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \qquad (2)$$

where $x \in R^n$, $u \in R^m$, $y \in R^m$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{m\times n}$ and $D \in R^{m\times m}$. Assume $G(s)$ is composed of N linear time-invariant subsystems $G_i(s)$, described by

$$\begin{aligned} \dot{x_i}(t) &= A_{ii}x_i + B_{ii}u_i + \sum_{j=1_{j\neq i}}^{N} A_{ij}x_j + \sum_{j=1_{j\neq i}}^{N} B_{ij}u_j, \\ y_i(t) &= C_{ii}x_i + D_{ii}u_i + \sum_{j=1_{j\neq i}}^{N} C_{ij}x_j + \sum_{j=1_{j\neq i}}^{N} D_{ij}u_j, \end{aligned} \qquad (3)$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, $y_i \in R^{m_i}$, $A_{ii} \in R^{n_i\times n_i}$, $B_{ii} \in R^{n_i\times m_i}$, $C_{ii} \in R^{m_i\times n_i}$, $\sum_{i=1}^{N} n_i = n$ and $\sum_{i=1}^{N} m_i = m$. The terms $\sum_{j=1_{j\neq i}}^{N} A_{ij}x_j$, $\sum_{j=1_{j\neq i}}^{N} B_{ij}u_j$, $\sum_{j=1_{j\neq i}}^{N} C_{ij}x_j$, and $\sum_{j=1_{j\neq i}}^{N} D_{ij}u_j$ are due to interactions of the other subsystems. The objective is to design a local PI controller given by

$$K_i(s) = K_{ci}(\frac{1 + T_{Ii}s}{T_{Ii}s}), \qquad i = 1, ..., N, \qquad (4)$$

for each isolated subsystem $G_{ii}(s)$, described by

$$\begin{aligned} \dot{x}_i(t) &= A_{ii}x_i(t) + B_{ii}u_i(t), \\ y_i(t) &= C_{ii}x_i(t) + D_{ii}u_i(t), \end{aligned} \qquad (5)$$

such that the closed-loop subsystem is stabilized and at the same time effects of interactions of the other subsystems and uncertainties are minimized. By this, the decentralized controller

$$K(s) = \mathrm{diag}\{K_i(s)\}, \qquad (6)$$

stabilizes the overall uncertain system given in (1) if some sufficient conditions are satisfied.

3. CLOSED-LOOP STABILITY AND DIAGONAL DOMINANCE

In this section, sufficient conditions for closed-loop stability and diagonal dominance of the nominal system are obtained. To this end and in order to prove our theorems, the transformation proposed in (Labibi et al. (2006)) is used to transform the system given in (2) to an equivalent descriptor system representation. It should be noted that this representation is only for proving the related theorems and the control design will be done for conventional isolated subsystems. Since designing a dynamic controller for a system can be converted into designing a static controller for an augmented system, without loss of generality in this section we assume the designed controller is a static one.

Consider the system given by equations (2). In order to obtain an equivalent descriptor representation form, all of the inputs and outputs of the system are defined as state variables. Then the augmented system $\bar{G}(s)$ has the following equations

$$\begin{aligned} \bar{E}\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u(t), \\ y(t) &= \bar{C}\bar{x}(t) + \bar{D}u(t), \end{aligned} \qquad (7)$$

where

$$\bar{E} = \begin{bmatrix} 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & I & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & I & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \end{bmatrix}, \bar{x} = \begin{bmatrix} y_1 \\ x_1 \\ u_1 \\ \vdots \\ y_N \\ x_N \\ u_n^T \end{bmatrix},$$

$$\bar{A} = \begin{bmatrix} -I & C_{11} & D_{11} & \ddots & 0 & C_{1N} & D_{1N} \\ 0 & A_{11} & B_{11} & \ddots & 0 & A_{1N} & B_{1N} \\ 0 & 0 & -I & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & C_{N1} & D_{N1} & \ddots & -I & C_{NN} & D_{NN} \\ 0 & A_{N1} & B_{N1} & \ddots & 0 & A_{NN} & B_{NN} \\ 0 & 0 & 0 & \ddots & 0 & 0 & -I \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 & \ddots & 0 \\ 0 & \ddots & 0 \\ I & \ddots & 0 \\ \vdots & \dots & \vdots \\ 0 & \ddots & 0 \\ 0 & \ddots & 0 \\ 0 & \ddots & I \end{bmatrix},$$

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}, y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, \bar{C} = \begin{bmatrix} I & 0 & 0 & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & I & 0 & 0 \end{bmatrix},$$

and$\bar{D} = 0_{m\times m}$.

The transfer matrix of the closed-loop descriptor system in (7), $\bar{G}_{cl}(s)$ is given by

$$\bar{G}_{cl}(s) = \bar{C}(sE - \bar{A} + \bar{B}K\bar{C})^{-1}\bar{B}K + \bar{D} = \tag{8}$$
$$(I + C(sI - A)^{-1}BK + DK)^{-1}(C(sI - A)^{-1}B + D)K;$$

which is equal to $T(s)$, the transfer matrix of system (2) under the decentralized controller. Therefore control of system (7) results in controlling of system (2). Defining

$$\bar{A}_d = \text{diag}\{\bar{A}_{ii}\}, \tag{9}$$

where

$$\bar{A}_{ii} = \begin{bmatrix} -I & C_{ii} & D_{ii} \\ 0 & A_{ii} & B_{ii} \\ 0 & 0 & -I \end{bmatrix}, \tag{10}$$

it is easy to show that the transfer matrices of the closed-loop systems $(E, \bar{A}_d, \bar{B}, \bar{C}, \bar{D})$ and (A_d, B_d, C_d, D_d) under the decentralized controller $K = \text{diag}\{K_i\}$ are the same i.e.

$$T_d(s) = \bar{C}\bar{P}\bar{B}K = (I + C_d(sI - A_d)^{-1}B_dK + D_dK)^{-1}$$
$$(C_d(sI - A_d)^{-1}B_d + D_d)K, \tag{11}$$

where

$$\bar{P} = (sE - \bar{A}_d + \bar{B}K\bar{C})^{-1}. \tag{12}$$

Defining

$$\bar{H} = \bar{A} - \bar{A}_d, \tag{13}$$

the system $(E, \bar{A}_d, \bar{B}, \bar{C})$ is a block-diagonal system and the matrix $\bar{H}$ can be considered as uncertainty in the matrix $\bar{A}$.

3.1 Sufficient Conditions for Stability

The next theorem provides conditions for closed-loop stability.

Theorem 1 Suppose the decentralized controller K stabilizes the diagonal system (A_d, B_d, C_d, D_d). Then the closed-loop original system under the decentralized controller is stable if

$$\|\bar{P}\bar{H}\|_\infty < 1, \tag{14}$$

where $\bar{P}$ and $\bar{H}$ are given in equations (12) and (13), respectively and $\|.\|_\infty$ is the maximum singular value of (.).

Proof: The transfer matrix of the closed-loop system can be written as

$$T(s) = \bar{C}(I - \bar{P}\bar{H})^{-1}\bar{P}\bar{B}K. \tag{15}$$

Since $\bar{P}$ is stabilized by stabilizing the block-diagonal system (A_d, B_d, C_d, D_d), then the closed-loop system is stable if the transfer matrix $(I - \bar{P}\bar{H})^{-1}$ is stable. The transfer matrix $\bar{P}$ is stable and if the Nyquist plot of $det(I - \bar{P}\bar{H})$ does not encircle the origin, it means if the condition given in (14) is satisfied the closed-loop system is stable.

The matrix

$$\bar{P} = \text{diag}\{\bar{P}_i\} \tag{16}$$

with

$$\bar{P}_i = (sE_i - \bar{A}_{ii} + \bar{B}_iK_i\bar{C}_{ii})^{-1}, \tag{17}$$

$$\bar{E}_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \bar{B}_{ii} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \text{and} \bar{C}_{ii} = [\, I \;\; 0 \;\; 0 \,], \tag{18}$$

is a block-diagonal matrix. Then the following stability conditions

$$\|\bar{P}_i\|_\infty < \mu^{-1}(\bar{H}), \quad i = 1, ..., N, \tag{19}$$

where $\mu(.)$ is the maximum structured singular value of (.), give sufficient conditions for closed-loop stability at the subsystem level.

3.2 Sufficient Conditions for Diagonal Dominance

Theorem 2 The closed-loop nominal system given in (2) under the decentralized controller K is diagonal dominate, if

$$\|S_i\|_\infty < \frac{\alpha_i}{|\bar{w}_{1i}|}, \quad i = 1, ..., N, \tag{20}$$

where α_i is a positive scalar less than one and small enough, $\bar{w}_{1i}(s)$ is the weighting function that satisfies the following equation

$$|\bar{w}_{1i}(s)| > \sqrt{N}|(C_{ii}(sI - A_{ii})^{-1}H_{AB_i} + H_{CD_i}|, \tag{21}$$

S_i is the sensitivity function of the i-th closed-loop isolated subsystem,

$$H_{AB_i} = [\, AB_1 \;\; AB_2 \,], H_{CD_i} = [\, CD_1 \;\; CD_2 \,], \tag{22}$$

$$AB_1 = [\, A_{i1} \;\; B_{i1} \;\; \dots \;\; A_{ii-1} \;\; B_{ii-1} \;\; 0 \,],$$
$$AB_2 = [\, 0 \;\; A_{ii+1} \;\; B_{ii+1} \;\; \dots \;\; A_{iN} \;\; B_{iN} \,],$$
$$CD_1 = [\, C_{i1} \;\; D_{i1} \;\; \dots \;\; C_{ii-1} \;\; D_{ii-1} \;\; 0 \,],$$
$$CD_2 = [\, 0 \;\; C_{ii+1} \;\; D_{ii+1} \;\; \dots \;\; C_{iN} \;\; D_{iN} \,].$$

Proof: The closed-loop descriptor system under decentralized control has the following form

$$(\bar{C} - \bar{C}\bar{P}\bar{H})\bar{X} = \bar{C}\bar{P}\bar{B}KR. \tag{23}$$

If

$$\|\bar{C}\bar{P}\bar{H}\|_\infty < \alpha\sigma_{min}(\bar{C}), \tag{24}$$

where $\sigma_{min}(.)$ is the minimum singular value of (.) and α is a positive scalar less than one, then (Stewart (1973))

$$\bar{C} - \bar{C}\bar{P}\bar{H} \cong \bar{C}, \tag{25}$$

and $T(s)$, the transfer matrix of the closed-loop system is given by the following equation

$$T(s) \cong \tilde{C}\bar{P}\bar{B}K = T_d(s), \tag{26}$$

which is a block-diagonal transfer matrix. Based on the definition of $\bar{C}$ given in (7)

$$\sigma_{min}(\bar{C}) = 1, \tag{27}$$

which means that by minimizing $\|\bar{C}\bar{P}\bar{H}\|_\infty$ such that

$$\|\bar{C}\bar{P}\bar{H}\|_\infty < \alpha, \quad (28)$$

the closed-loop system is diagonal dominant. We know

$$\|\bar{C}\bar{P}\bar{H}\|_\infty \leq \sqrt{N}\|\bar{C}_{ii}\bar{P}_i\bar{H}_i\|_\infty, i = 1, ..., N, \quad (29)$$

(Skogestad et al.(2005)) with

$$\bar{H}_i = [\, H_{i1} \;\; H_{i2}\,], H_{i1} = \begin{bmatrix} 0 & C_{i1} & D_{i1} & \ldots & 0 & C_{ii-1} & D_{ii-1} & 0 & 0 \\ 0 & A_{i1} & B_{i1} & \ldots & 0 & A_{ii-1} & B_{ii-1} & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_{i2} = \begin{bmatrix} 0 & 0 & C_{ii+1} & D_{ii+1} & \ldots & 0 & C_{iN} & D_{iN} \\ 0 & 0 & A_{ii+1} & B_{ii+1} & \ldots & 0 & A_{iN} & B_{iN} \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \end{bmatrix}.$$

Denoting

$$S_i = (I + C_{ii}(sI - A_{ii})^{-1}B_iK_i + D_{ii}K_i)^{-1}, \quad (30)$$

as the sensitivity function of the i-th closed-loop isolated subsystem, it can be shown that

$$\bar{C}_{ii}\bar{P}_i\bar{H}_i = S_i(C_{ii}(sI - A_{ii})^{-1}H_{AB_i} + H_{CD_i}). \quad (31)$$

Therefore by defining $\bar{w}_{1i}$ as given in equation (21) by satisfying equations (20), the closed-loop system is diagonal dominant with the degree $\alpha = \max\{\alpha_i\}$. It follows that by designing local controllers such that

$$\|\bar{w}_{1i}S_i\|_\infty < \alpha_i, \quad i = 1, ..., N, \quad (32)$$

with α_i small enough, the closed-loop system is block-diagonal dominant. In fact the transfer matrix of the closed loop system is given as

$$\begin{aligned} T(s) &= \bar{C}\bar{P}_T\bar{B}K = \bar{C}\bar{P}\bar{H}(I - \bar{P}\bar{H})^{-1}\bar{P}\bar{B}K + \bar{C}\bar{P}\bar{B}K \\ &= \bar{C}\bar{P}\bar{H}\bar{P}_T\bar{B}K + \bar{C}\bar{P}\bar{B}K, \end{aligned} \quad (33)$$

with

$$\bar{P}_T = (sE - \bar{A} + \bar{B}K\bar{C})^{-1}. \quad (34)$$

Let $\bar{C}^+$ denotes the pseudoinverse of $\bar{C}$ because for the system given in (7) $\|\bar{C}^+\|_\infty = \frac{1}{\sigma_{min}(\bar{C})} = 1$, then by designing a decentralized controller such that $\|\bar{C}\bar{P}\bar{H}\|_\infty < \alpha$, we have

$$\begin{aligned} \|\bar{C}\bar{P}\bar{H}\bar{P}_T\bar{B}K\|_\infty &\leq \|\bar{C}\bar{P}\bar{H}\|_\infty\|\bar{C}^+\bar{C}\bar{P}_T\bar{B}K\|_\infty \\ &\leq \alpha\sigma_{min}(\bar{C})\|\bar{C}^+\|_\infty\|\bar{C}\bar{P}_T\bar{B}K\|_\infty\| = \alpha T(s). \end{aligned} \quad (35)$$

On the other hand from equation (33) we have

$$\bar{C}\bar{P}\bar{B}K = T(s) - \bar{C}\bar{P}\bar{H}\bar{P}_T\bar{B}K, \quad (36)$$

then

$$\begin{aligned} \|\bar{C}\bar{P}\bar{B}K\|_\infty &\geq \|T(s)|_\infty - \|\bar{C}\bar{P}\bar{H}\bar{P}_T\bar{B}K|_\infty \\ &\geq (1 - \alpha)\|T(s)\|_\infty. \end{aligned} \quad (37)$$

Considering equations (33), (35) and (37) we observe by selecting α small enough $\alpha < 0.5$, the norm of the block-diagonal transfer matrix $\bar{C}\bar{P}\bar{B}K$ is larger than the norm of the transfer matrix $\bar{C}\bar{P}\bar{H}\bar{P}_T\bar{B}K$. It means the block-diagonal transfer matrix $\bar{C}\bar{P}\bar{B}K$ highlights the transfer matrix of the overall system and it is a kind of closed-loop block-diagonal dominance. Then the stability and performance of the system which is diagonal dominant can be inferred directly from the stability and performance of the block-diagonal transfer matrix $T_d(s) = \bar{C}\bar{P}\bar{B}K$ (Rosenbrock (1974)).

4. DECENTRALIZED PI CONTROLLER DESIGN

In this section a new method for decentralized PI controller design is given. Before doing so, however we revisit the SISO PI design problem.

4.1 SISO PI Design using Internal Model Control Method

In designing a PI controller for a SISO system approximation of high order processes by first order models is a common practice. Once an approximated model is obtained a PI controller based on IMC method can be designed. Even though many industrial processes meet the assumptions sufficiently to be modeled with a first order model, there do exist many plants which can not be well approximated by the first order systems. In order to avoid instability due to approximation, in this paper the error of approximation is considered as multiplicative uncertainty for isolated subsystems. Then designing a local PI controller can be converted into solving an appropriately defined local robust performance problem. The strategy is described in the following subsection.

4.2 Decentralized PI Controller Design

In this subsection a new tuning criterion for MIMO PI controllers is proposed. The next theorem gives the methodology.

Theorem 3 Consider the i-th isolated subsystem which is approximated with a first order model and the approximation error is considered as multiplicative uncertainty weight $\bar{w}_{3i}(s)$. Then the closed-loop MIMO system can be stabilized if τ_{c_i} for the i-th isolated closed-loop subsystem is selected such that

$$\||\frac{1}{\alpha_i}\bar{w}_{1i}(s)\frac{\tau_{c_i}s}{\tau_{c_i}s + 1}| + |\bar{w}_{3i}(s)\frac{1}{\tau_{c_i}s + 1}|\|_\infty < 1. \quad (38)$$

Proof: By designing a local PI controller for the i-th isolated subsystem by an IMC based method, the i-th closed-loop subsystem has the following sensitivity and complementary sensitivity functions respectively (Skogestad et al. (2005))

$$S_i = \frac{\tau_{c_i}s}{\tau_{c_i}s + 1}, T_i = \frac{1}{\tau_{c_i}s + 1}. \quad (39)$$

In order to attenuate the interactions between subsystems by local controllers, the sensitivity functions of each isolated subsystem should satisfy condition (20). This condition can be considered as a nominal performance problem. If the isolated subsystem can not be approximated sufficiently well by a low order model, the modeling error may be considered as multiplicative uncertainty given by weighting function $\bar{w}_{3i}(s)$. The i-th approximated closed-loop isolated subsystem is stable if and only if (Skogestad et al. (2005))

$$\|T_i(s)\bar{w}_{3i}(s)\|_\infty < 1. \quad (40)$$

The i-th sensitivity and complementary sensitivity functions should satisfy the conditions given in (20) and (40) respectively to have diagonal dominance (nominal performance) for the overall system and robust stability for the subsystems. This is a robust performance problem for the isolated subsystems. Then by considering definitions given in (39) the closed-loop system is diagonal dominant if the local robust performance problems given in (38) are solved.

According to the theorem, by selecting appropriate values for τ_{c_i}'s and α_i to solve local problems (38) the closed-loop diagonal dominance is guaranteed.

5. ROBUST DECENTRALIZED PID CONTROL

Consider the uncertain system given in (1). The uncertainty is modeled as diagonal multiplicative output uncertainty (In this section without loss of generality we consider multiplicative output uncertainty. It is clear the same result can be obtained for multiplicative input uncertainty as well.). The closed-loop uncertain system is robust stable if and only if for multiplicative output uncertainty (Skogestad et al. (2005))

$$\|\bar{\bar{W}}_3 T\|_\infty < 1 \quad (41)$$

where $\bar{\bar{W}}_3(s) = diag\{\bar{\bar{w}}_{3i}(s)\}$ is a diagonal matrix representing multiplicative uncertainty of the system. If the closed-loop system is diagonal dominant with a good degree of diagonal dominance, then with a good approximation $T(s) \cong T_d(s)$ and the performance of the closed-loop system can be inferred from the block-diagonal part of the transfer matrix. We can show by solving the robust stability problem

$$\|\bar{\bar{W}}_3 T_d\|_\infty < 1, \quad (42)$$

or equivalently by solving local robust stability problems

$$\|\bar{\bar{w}}_{3i} T_i\|_\infty < 1, \quad (43)$$

the condition given in (41) can be satisfied.

Now, suppose the objective is to design a robust decentralized PI controller for an uncertain plant with multiplicative uncertainty to solve the robust stability problem given in equation (41). It was shown that for closed-loop diagonal dominance the local robust performance problems given in (38) and for treating uncertainty in the system the conditions given in (43) should be satisfied. Combining these conditions for designing a robust decentralized PI controller for a MIMO system, the problem can be converted into solving the following modified local robust performance problems

$$\| |w_{1i}(s) \frac{\tau_{c_i} s}{\tau_{c_i} s + 1}| + |\frac{w_{3i}(s)}{\tau_{c_i} s + 1}| \|_\infty < 1, \quad (44)$$

with

$$|\frac{\bar{w}_{1i}(s)}{\alpha_i}| \le |w_{1i}(s)|, \quad (45)$$

and

$$max\{|\bar{w}_{3i}(s)|, |\bar{\bar{w}}_{3i}(s)|\} \le |w_{3i}(s)|. \quad (46)$$

Remark 1: In the conditions given in (21) at high frequencies the left and right hand sides of the relation approach to one and $\|H_{CD_i}\|_\infty$ respectively. Then in order to satisfy these conditions $\|H_{CD_i}\|_\infty$ should be less than or equal to one. This is however not always the case. For solving this problem, it is possible to use similarity transformations. Since similarity transformations do not affect output feedback and the overall system is observable, it is possible by using the observability matrix of the overall system to find an appropriate transformation to transform the original system into the output-decentralized form, where the matrix C is block diagonal (Labibi et al. (2006)). Then $C_{ij} = 0, i \neq j$, and for strictly proper systems ($D = 0$) at high frequencies the right hand side of relation (21) approaches zero and this condition will always be satisfied. But for proper systems the proposed methodology is applicable only when $\|D - \text{diag}\{D_{ii}\}\|_\infty$ is less than one.

6. UTILITY BOILER

The utility boilers in Syncrude Canada are water tube drum boilers. Since the steam is used for generating electricity, the demand for the steam is variable. Thus the control objective of the system is to track the steam demand while maintaining the steam pressure and the steam temperature of the header at their respective set-points. In SCL the utility boilers are used to control the steam pressure and steam temperature. The main objective of this paper is to design a PI controller so that the utility boiler system keeps stability and reaches the desired performance. In the system the principal input variables are u_1 feedwater flow rate (kg/s); u_2 fuel flow rate (kg/s); u_3 attemperator spray flow rate (kg/s); and the principal output variables are y_1 drum level (m); y_2 drum pressure (kPa); y_3 steam temperature ^0C. For proper function of the boiler system, steam pressure of the 6.306-MPa header must be maintained despite variations in the amount of steam demanded by users. The amount of water in the steam drum must be maintained at the desired level to prevent overheating of drum components or flooding of steam lines. Indeed, the steam temperature must be maintained at the desired level to prevent overheating of the super heaters and to prevent wet steam entering turbines (Tan et al.(2002)).

6.1 The model

For the utility boilers in SCL a fairly accurate nonlinear model is identified (Labibi et al. (2007)). The derived model for the utility boiler is given as follows.

$$\begin{aligned}
\dot{x_1}(t) &= \frac{u_1 - 0.03\sqrt{x_2^2 - (6306)^2}}{155.1411}, \\
\dot{x}_2(t) &= (-1.8506 \times 10^{-7} x_2 - 0.0024)\sqrt{x_2^2 - (6306)^2} \\
&\quad - 0.0404 u_1 + 3.025 u_2, \\
\dot{x}_3(t) &= -0.0211\sqrt{x_2^2 - (6306)^2} + x_4 - 0.0010967 u_1 \\
&\quad + 0.0475 u_2 + 3.1846 u_3, \\
\dot{x}_4(t) &= 0.0015\sqrt{x_2^2 - (6306)^2} + x_5
\end{aligned}$$

$$
\begin{aligned}
&+0.001u_1 + 0.32u_2 - 2.9461u_3,\\
\dot{x}_5(t) &= -1.278 \times 10^{-3}\sqrt{x_2^2 - (6306)^2} - 0.00025831x_3\\
&- 0.029747x_4 - 0.8787621548x_5\\
&- 0.00082u_1 - 0.2652778u_2 + 2.491u_3,\\
y_1(t) &= 0.010157116x_1 + 1.8386 \times 10^{-4}\sqrt{x_2^2 - (6306)^2}\\
&- 0.001u_1 + 0.019814u_2 - 6.1982,\\
y_2(t) &= x_2,\\
y_3(t) &= x_3,\\
q_s(t) &= 0.03\sqrt{x_2^2 - (6306)^2}.
\end{aligned}
$$

The system works at three operating points, called low load, normal load and high load. In addition, the following limit constraints exist for the three control variables:

$$0 \le u_1 \le 120, \tag{47}$$

$$0 \le u_2 \le 7, \tag{48}$$

$$0 \le u_3 \le 10, \tag{49}$$

$$-0.017 \le \dot{u}_2 \le 0.017. \tag{50}$$

The nonlinear model is linearized about its operating points and the linear model at the normal load is considered as the nominal plant. The uncertainty in parameters of the state space matrices can be modeled as multiplicative uncertainty. By solving local robust problems given in (38) the decentralized controller is designed as follows

$$
K(s) = \begin{bmatrix} 212(1+\frac{1}{62.2424s}) & 0 & 0 \\ 0 & 0.01(1+\frac{1}{34.67s}) & 0 \\ 0 & 0 & -0.015(1+\frac{1}{4s}) \end{bmatrix}.
$$

The designed controller is applied to the real nonlinear system. In order to compensate the constraints given in (47-50) on control signals, as explained in (Tan et al.(2005)), these constraints can be ignored at the design stage. Then, the effects of the constraints are compensated after the controller design using anti-windup bump-less transfer (AWBT) techniques (Tan et al.(2005)). Applying the designed controller with AWBT compensation to the nonlinear system, figure 1 shows the responses of the closed-loop system in switching from normal load to high load. This figure shows good set point tracking of the closed-loop system. Figure 2 shows the related control signals and that the constraints given on control signals are satisfied.

7. CONCLUSION

In this paper a method for robust decentralized PI controller design for an industrial utility boiler is proposed. Sufficient conditions for robust stability and diagonal dominance of the overall closed-loop system are derived. These conditions are based on sensitivity functions of closed-loop subsystems and are formulated as local robust performance problems. It is shown by appropriately selecting the time constants of the closed-loop isolated subsystems, these sufficient conditions are satisfied. Then for the identified model of the utility boilers in SCL a decentralized PI controller is designed. Applying the designed controller to the real industrial utility boiler shows the effectiveness of the proposed method.

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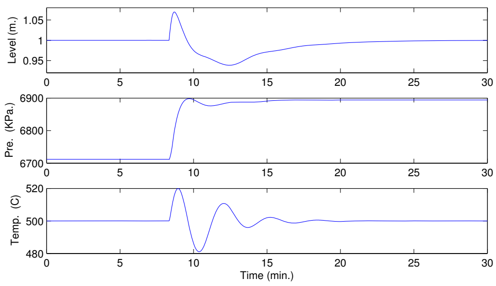

Fig. 1. Switching from normal load to high load (output signals)

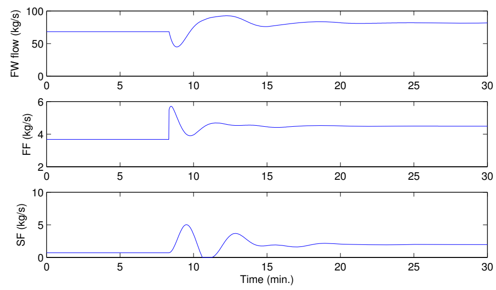

Fig. 2. Switching from normal load to high load (control signals)