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Transfer Functions for Natural Gas Pipeline Systems

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Abstract: Natural gas pipeline systems typically have very complex dynamic characteristics. However, extracting "basic" dynamic parameter values, like gains and time constants is sometimes necessary. Looking at the complex dynamical models of pipeline systems, this is not straightforward, and a method providing those linear model parameters is needed. The method described in the paper is based on discretization of a Partial Differential Equation model followed by linearization of the resulting Ordinary Differential Equation model of high order. Balanced truncation is applied on the linearized model, resulting in a radically reduced order linear state space model from which transfer functions and finally the gains and time constants are obtained.

Keywords: Natural gas pipelines; Pipeline dynamical models; Model reduction; Balanced truncation

1. INTRODUCTION

Modern control algorithms can use almost any linear or non-linear models, and classical linear SISO or MIMO transfer function models have lost their status of preferred model structures. However, process and control specialists find it often very convenient to have access to the parameters of linear transfer functions, namely, gains and time constants. These parameters have their own meanings, for example, one single number, the *dominating time constant*, is often interesting to know.

The models describing the dynamical behaviour of natural gas pipeline systems are basically non-linear partial differential equations (PDE). These models, or simplified models derived from these, do not directly reveal the values of the linear transfer function parameters. Some few attempts towards calculating the parameter values in simplified, limited cases have been done. (Lewandowski, 1995) linearised the PDE model of a pipeline segment and derived the solution for the gas pressure p as a function of the space co-ordinate along the pipeline, z, and time t: $p(z,t) = \exp(az)\sum_{k=1}^{\infty}\exp(-t/T_k)\sin(k\pi z)$, where the parameters T_k are the *mode time constants* of the pipeline segment. (Botros et. al., 1996) calculated the time constant for a pipeline in the case of blow-down of the gas at the other end of the pipeline. (Kra'lik et. al. 1984) presented a linearised staircase-type transfer function model for a pipeline segment divided into equal-length *nodes*. The pressure and flow propagation through each node is described by 4 linear transfer functions. The dynamic model parameter values for the whole segment can be calculated using this model, although the calculation procedure becomes complicated for a pipeline segment with many nodes.

Common to the three approaches above is, that they used simple pipeline segments only allowing no complexity typical to practical pipelines, such as multiple gas off-takes along the segment and pipe branches.

In this paper, we present a method for calculating gains and time constants of a pipeline system of any structure. Dead time does not need to be considered, since this depends on the speed of sound in the pressurised gas and is thus very small compared to the time constant values. The basic iso-thermal PDE model is first discretised with respect to the space variable and then linearised in the steady state operating point. A model reduction method is then applied to radically reduce the dimensions of the resulting linear state space model. The reduced, continuous time state space model is then converted to a transfer function, from which the parameters are obtained.

2. NATURAL GAS PIPELINE SYSTEM MODELS

2.1 The basic PDE model

A natural gas pipeline system consists of pipeline segments, compressor stations and gas offtakes. Figure 2 below (see section 4) illustrates a pipeline system with branches, offtakes and four compressors stations (CS). Only the pipeline segment from the supply point at the far left to CS1 is a simple segment. For a simple pipeline segment the following PDE model can be written (Osiadacz, 1996; Marque's and Morari, 1988)

$$\frac{\partial P}{\partial t} + \frac{b}{A}\frac{\partial q}{\partial z} = 0 \qquad (1)$$

$$\frac{\partial q}{\partial t} + A\frac{\partial P}{\partial z} + f\frac{b}{DA}\frac{q^2}{P} = 0 \qquad (2)$$

where P $\hat{=}$ P(z,t) is the gas pressure, q $\hat{=}$ q(z,t) is the mass flow of the gas, b=kRT where k is the (constant) gas compressibility, R is the universal gas constant and T is the (constant) gas temperature, A is the pipe cross-sectional area, D is the pipe diameter and f is the dimensionless Moody friction factor.

Each pipeline segment and branch require their own equations of type (1) and (2) together with suitable boundary conditions in order to form the complete model of the pipeline system.

If we discretise the PDE model with respect to the space coordinate using length elements Δz_i, i=1,…,N, where N is typically large, we obtain for each volume element, or node, "i" of the pipeline, using (1) and (2) directly:

$$\frac{dP_i}{dt} = \frac{b_i}{A_i \Delta z_i}(q_{i-1} - q_i) \tag{3}$$

$$\frac{dq_i}{dt} = \frac{A_i}{\Delta z_i}(P_i - P_{i+1}) - f_i \frac{b_i}{D_i A_i} \frac{q_i^2}{P_i} \tag{4}$$

In the sequel, we shall use the following parameter definitions: $\alpha_i = b_i / (A_i \Delta z_i)$, $\beta_i = A_i / \Delta z_i$, $\gamma_i = f_i b_i / (D_i A_i)$

Note, that for each individual node, individual cross-sectional area, diameter, friction and even b-parameter can be used, the latter allowing compressibility and temperature changes along the pipeline to be accounted for in an approximate and ad-hoc way. If a branch or an offtake is connected to node "k", the ODE's are written as follows (see figure 1):

$$\begin{aligned} \frac{dP_k}{dt} &= \alpha_k (q_{k-1} - q_k - q_0') \\ \frac{dq_k}{dt} &= \beta_k (P_k - P_{k+1}) - \gamma_k \frac{q_k^2}{P_k} \\ \frac{dq_0'}{dt} &= \beta_1' (P_k - P_1') - \gamma_1' \frac{(q_0')^2}{P_0'} \end{aligned} \tag{5}$$

If the flow q_0' represents a gas offtake, a differential equation for this flow is left away and q_0' is handled as a system input variable instead.

Equations (5) are valid for a branch into which gas flows from the main pipeline. Branches, which bring gas into the pipeline (into some specified node "k") can be modelled accordingly.

If we exclude the fact, that CS's may sometimes be operated so, that their operating constraints are encountered thus resulting in non-linear operation, we can for the purpose of this study model a CS at node "k" being a PI controller manipulating the mass flow q_{k-1} into that volume element:

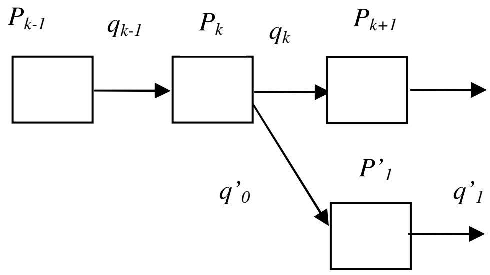

Figure 1. A pipeline branch leaving from node "k"

$$q_{k-1}(t) = K\left(P_{k,SET} - P_k(t)\right) + \frac{K}{T_i}\int_0^t \left(P_{k,SET} - P_k(\tau)\right) d\tau \tag{6}$$

where $P_{k,SET}$ is the desired pressure of node "k" (the discharge point of the CS), K is the controller gain and T_i is the controller integrator time constant. Taking the time derivative of (6) and applying (3) we obtain :

$$\frac{dq_{k-1}}{dt} = -K\beta_k (q_{k-1} - q_k) + \frac{K}{T_i}(P_{k,SET} - P_k) \tag{7}$$

which replaces the equation of type (4) for i=k-1

2.2 A linearised model

Linearisation of the node equations (3) and (4) leaves the state equation for pressure unchanged, except for redefining the variables as deviations from some steady state pressure and flow, and the flow rate equation becomes:

$$\frac{d\Delta q_i}{dt} = \beta_i (\Delta P_i - \Delta P_{i+1}) - 2\gamma_i \frac{\Delta q_i}{P_{i,SS}} + \gamma_i \frac{q_{i,SS}^2 \Delta P_i}{P_{i,SS}^2} \tag{8}$$

where $P_{i,SS}$ and $q_{i,SS}$ are the steady state values of pressure and mass flow, respectively, of node "i". The steady state values are obtained by setting all time derivatives in (3) and (4) to zero and solving the resulting system of 2N nonlinear algebraic equations. The resulting linear, continuous-time system is

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{9}$$

where the 2N-dimensional state vector **x** contains all node pressure and mass flow deviations from steady state and the m-dimensional input vector **u** contains CS discharge pressure set-point and offtake gas mass flow rate deviations. From a control system perspective, the CS pressure set-points are the true control inputs of the system whereas off-take flows are (known and possibly predicted, through consumption forecasts) disturbances. The output vector **y** may be any combination of pressures and flow rates, and the matrix **C** is a selection matrix consisting of only zeros and ones so that the row sums are equal to one and column sums are either zero or one. The transfer function is defined as:

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \tag{10}$$

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3. MODEL REDUCTION TECHNIQUES

The characteristic polynomial of (10) has 2N roots, which is typically a large number depending on the complexity of the pipeline system under consideration and on the accuracy of the space discretisation. Solving the roots of a high-degree characteristic polynomial may not be that difficult, but, as we will see below, transfer function zeros (or "numerator dynamics" of the transfer function) has significance in this case. Even for cases with moderate complexity, calculation of transfer function zeros may be difficult (Kwakernaak and Siwan, 1972). Up to recent times, calculation of transfer function zeros in electrical circuit analysis has been avoided because of implementation difficulties and numerical sensitivity (Ragavan et. al., 2005).

In order to be able to calculate the values of the transfer function parameters, we shall apply model reduction techniques prior to the conversion from state space to transfer function model.

Model reduction through *truncation* is based on calculating the N_r largest Hankel singular values of the original n-dimensional system, where N_r < n is the dimension of the reduced system (Antoulas, 2004). First, two Lyapunov type equations are solved to obtain the Grammians **P** and **Q**:

$$\begin{aligned} \mathbf{AP} + \mathbf{PA}^T + \mathbf{BB}^T &= \mathbf{0} \\ \mathbf{A}^T\mathbf{Q} + \mathbf{QA} + \mathbf{C}^T\mathbf{C} &= \mathbf{0} \end{aligned} \tag{11}$$

The Hankel singular vales are defined as the square roots of the eigenvalues of the matrix **PQ**, and the transformation matrix **T** is defined as $\mathbf{T} = [\mathbf{v}_1\ \mathbf{v}_2 \ldots \mathbf{v}_{2N}]$, where $\mathbf{v_1}$ is the eigenvector of **PQ** corresponding to the largest Hankel singular value, $\mathbf{v_2}$ is the eigenvector corresponding to the second largest and so on. A transformed system is defined as:

$$\widetilde{\mathbf{A}} = \mathbf{T}^{-1}\mathbf{AT}, \widetilde{\mathbf{B}} = \mathbf{T}^{-1}\mathbf{B}, \widetilde{\mathbf{C}} = \mathbf{CT} \tag{12}$$

The system matrices of the reduced system:

$$\begin{aligned} \frac{d\mathbf{x_r}(t)}{dt} &= \mathbf{A_r x_r}(t) + \mathbf{B_r u}(t) \\ \mathbf{y_r}(t) &= \mathbf{C_r x_r}(t) \end{aligned} \tag{13}$$

are obtained by selecting the first N_r matrix rows and/or columns:

$\mathbf{A_r} = \widetilde{\mathbf{A}}(1:N_r, 1:N_r), \mathbf{B_r} = \widetilde{\mathbf{B}}(1:N_r,:), \mathbf{C_r} = \widetilde{\mathbf{C}}(:,1:N_r)$.

In the case of *balanced truncation*, Cholesky decompositions for **P** and **Q** are calculated: $\mathbf{P} = \mathbf{UU}^T$, $\mathbf{Q} = \mathbf{LL}^T$ where **U** and **L** are upper and lower triangular matrices respectively.

Let $\mathbf{U}^T\mathbf{L} = \mathbf{Z\Sigma Y}^T$ be a singular value decomposition of $\mathbf{U}^T\mathbf{L}$. Define the matrices $\mathbf{W} = \mathbf{LY}_1\mathbf{\Sigma}_1^{-1/2}$ and $\mathbf{V} = \mathbf{UZ}_1\mathbf{\Sigma}_1^{-1/2}$, where subscripts "1" refer to the first N_r columns of **Y** and **Z** and the upper left N_r x N_r sub-matrix of **Σ**. The matrices for the reduced system (13) are calculated as follows: $\mathbf{A_r} = \mathbf{W}^T\mathbf{AV},\ \mathbf{B_r} = \mathbf{W}^T\mathbf{B}, \mathbf{C_r} = \mathbf{CV}$

4. EXPERIMENTAL RESULTS

A gas pipeline system with 4 CS:s and 7 pipeline segments is shown in figure 2. The gas offtakes are shown as small arrows together with the gas mass flow in kg/s. The gas supply point is at the far left through which the total gas flow of 170 kg/s flows in. Segment no. 3 between CS2 and CS4 has two branches (segments 5 and 7). The parameters of the pipeline system are shown in table 1.

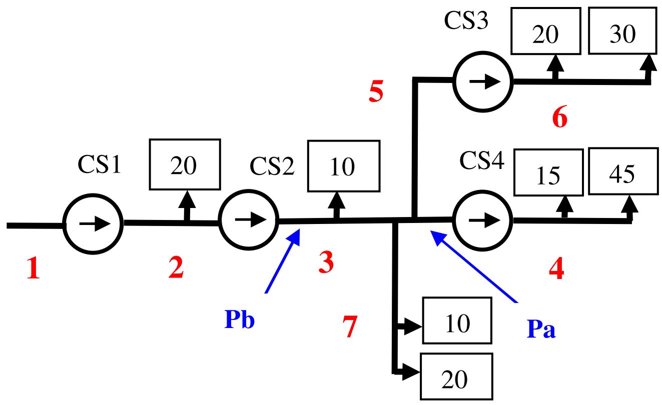

Figure 2 A natural gas pipeline system.

Segment	No. of nodes/ segment	node length, m	Pipe dia-meter,m
1	8	6000	1.2
2	10	5700	1.0
3	16	5000	1.0
4	10	6500	0.7
5	10	4000	0.75
6	10	4000	0.6
7	6	5000	0.5

Table 1. Parameters of the pipeline system of figure 2

The value of the friction parameter for all nodes is f=0.059. The steady state discharge pressures of all CS's is 52 bar. Assume, that we would like to know the linear transfer function parameters for the response from the discharge pressure of CS2 to a) the pressure far downstream, more specifically 13 nodes or 65 kilometres downstream of CS2 and b) the pressure near CS2 discharge point 5 nodes or 25 kilometres downstream of CS2. These are called P_a and P_b in figure 2, respectively.

The total number of nodes is 70 which means that the length of the state vector – recall one pressure and one flow variable for each node- is 140. The linearized system model is truncated to a state space model with 4 states (N_r =4) and the transfer function parameters are calculated using standard methods ("tf2zp" function of Matlab). Results for P_a and P_b are shown in table 2 below. Over-damped complex pair zeros and poles with small contribution to the overall response occur, which is typical in this kind of cases. These are left away from the tables below. Note, that for P_a there is a pole-zero cancellation situation, The general form of the transfer functions are:

$$G(s) = \frac{K(T_a s+1)(T_b s+1)...}{(T_1 s+1)(T_2 s+1)(T_3 s+1)...} \tag{14}$$

Gain, K (dimensionless, bar/bar)	1.44
Numerator time constants (minutes) T_a, T_b, …	116.9
Denominator time constants (minutes) T_1,T_2,….	117.7, 190.6

Table 2 Transfer function parameters, CS2 discharge pressure to P_a

Gain, K (dimensionless, bar/bar)	1.16
Numerator time constants (minutes) T_a, T_b, …	83.5
Denominator time constants (minutes) T_1,T_2,….	11.2, 183.6

Table 3 Transfer function parameters, CS2 discharge pressure to P_b

The typical feature of increasing gain with increasing downstream distance from the CS discharge node is seen. Also, a (relevant) numerator time constant appears when the distance is small as for P_b.

The simulated step responses for P_a and P_b to a 1 bar step in CS2 discharge pressure using the non-linear, large dimensional ODE model are shown in figure 3 below.

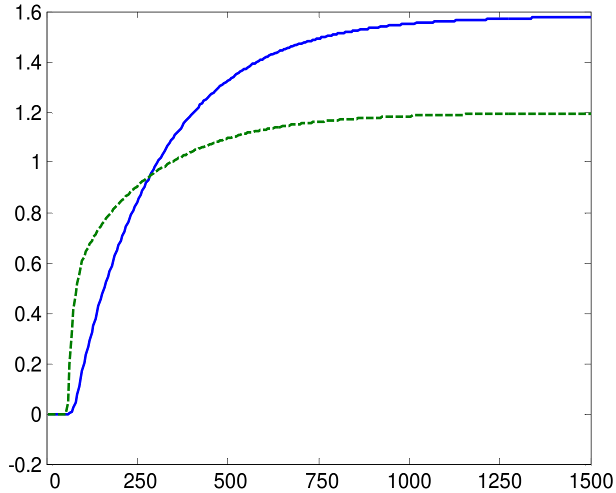

Figure 3. P_a (solid line) and P_b (dashed line) step responses (deviations from steady state values, bar) to a one bar step change in CS2 discharge pressure. Time axis tick is one minute.

The step responses of P_a for the ODE model and the reduced linear model are shown in figure 4 below. The full-dimensional linearised and reduced linear model step responses are practically speaking equal, the maximum error between the step response being 0.04 % The difference between the two curves in the figure comes from the linearisation error and can be made smaller by selecting smaller node lengths.

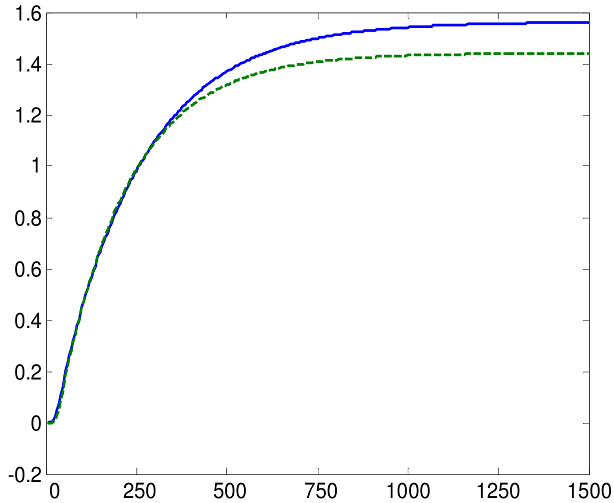

Figure 4 P_a step response, non-linear model (solid line) and linear reduced model (dashed line). Time axis tick is one minute.

Next, let us calculate the parameters of the transfer function from the gas flow of the last offtake in segment 4, figure 2 (45 kg/s steady state flow) , to the pressure at a point in the middle of segment 4 (about 33 km downstream of CS4) as shown in table 4.

Gain, K (bar/(kg/s))	-0.33
Numerator time constants (minutes) T_a, T_b, …	3.1
Denominator time constants (minutes) T_1,T_2,….	6.58 , 76.6

Table 4. Parameters for transfer function for offtake flow to pressure in segment no. 4

The step responses of the pressure using the original ODE model and the reduced linear model are shown in figure 5. Some gain error is seen similarly as in the previous case (see figure 4).

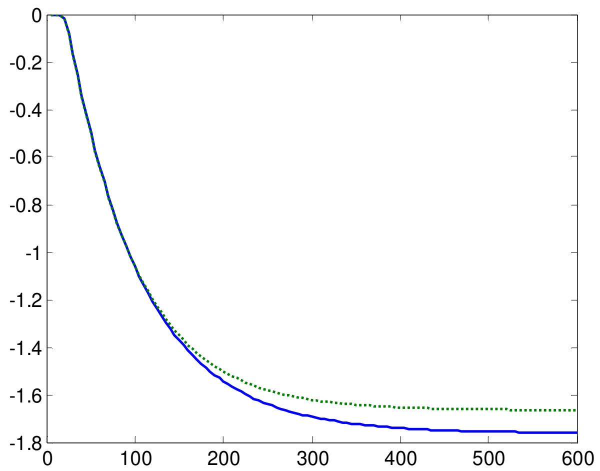

Figure 5. Pressure change in bar as a response to a 5 kg/s step change in the last offtake flow of segment 4. Time axis tick is one minute. Solid line = original ODE model, dotted line = reduced, linear model.

From an operations or control point of view, it may be interesting to know what kind of dynamics can be found for the CS4 discharge pressure to the pressure in the middle of segment 4. The transfer function parameters are shown in table 5. The principal dynamics (time constants) are the same as in table 4, which they should. In terms of numerator time constants, the control response is only slightly faster than the disturbance response (T_a 11.3 versus 3.1 minutes). This either requires aggressive, high-gain feedback control of the pressure at the specified point in the pipeline or feed-forward control utilising measured information of the gas offtake mass flow rate, or a combination of these.

Gain, K (dimensionless, bar/bar)	1.21
Numerator time constants (minutes) T_a, T_b, …	11.3
Denominator time constants (minutes) T_1,T_2,….	6.58, 76.6

Table 5. Parameters for transfer function for CS4 discharge pressure to pressure in the middle of segment no. 4

5. DISCUSSION

Values of transfer function parameters of gas pipelines can be obtained by various identification methods directly from true pipeline operational data or from data provided by a dynamic pipeline system simulator. Especially with a true pipeline process, executing the required process tests, repeated frequently in case of gas consumption changes or if there are other variations in the operating conditions in the pipeline, is tedious.

Both steady state and dynamic pipeline simulators typically use input files to describe the details: pipeline segment lengths, diameters and friction parameters as well as the pipeline system geometry. This data may be easily automatically converted to the parameters needed by the ODE model described above. If nodes in the simulator are too big, they may, as part of the conversion procedure, be split into smaller values of Δz. To combine calculation of transfer function parameters and simulators is an advantage in cases, where one wants to look at the pipeline dynamics (in terms of gains and time constants) in the design phase of the pipeline before it is even built.

In the study above, a full-scale nonlinear model was first linearised and then reduced. Reverse methods, to first reduce the nonlinear model to a smaller dimensional nonlinear model and the linearize, exist, such as Proper Orthogonal Decomposition (POD), (Hahn and Edgar, 2002). Intuitively, it seems like it is better to first linearise, because linear transfer functions are what we need in the end. However, to apply POD on this case is still a possible subject for further research.

The model (1) and (2) is restricted in the sense that it assumes isothermal conditions, low gas speed and horizontal pipeline segments (Osiadacz, 1996). Another subject for further research would be to challenge the model reduction procedure by including in the PDE model non-isothermal conditions and allowing high gas speed and inclined pipeline segments.

CS's isolate pipeline segments from each other, under the condition that they operate within operational constraints. If the suction pressure of a CS fluctuates (because of upstream CS discharge pressure variations or offtake flow changes), it has no effect on the gas flow through the compressor station nor the CS discharge pressure. The opposite is not true, i.e. a CS discharge pressure or offtake flow change propagates upstream through CS's all the way to "the beginning" of a pipeline system. This property may be utilised to decrease the extent of the original ODE model in cases, where downstream responses are analysed. For example, if responses downstream to CS2 are under consideration, pipeline segments 1,2, 4 and 6 can be left away from the model.

REFERENCES

Antoulas, A. C., (2004), Lectures on the approximation of linear dynamical systems, *Advances in Design and Control*, SIAM, Philadelphia, 2004

Botros, K. K., W.M. Jungowski and M.H. Weiss (1989), Models and Methods of Simulating Gas Pipeline Blowdown, *The Canadian Journal of Chemical Engineering*, Vol. **67**, August 1989, pp. 529-539

Hahn J and T. F. Edgar, Balancing Approach to Minimal Realisation and Model Reduction of Stable Nonlinear Systems, *Ind. Eng. Chem. Res*. 2002, Vol. **41**, pp. 2204-2212

Kra'lik J., P. Stiegler, Z. Vostry' and J. Za'vorka (1984), Modeling the Dynamics of Flow in Gas Pipelines, *IEEE Trans. on Systems, Man and Cybernetics*, Vol **SMC-14**, No. 4, July/August 1984, pp. 586-596

Kwakernaak H and R Siwan (1972), *Linear Optimal Control Systems*, 608 pages, Wiley Interscience

Lewandowski, A. (1995), New numerical methods for transient modelling of gas pipeline networks, *Pipeline Simulation Interest Group, Annual Conference* October 18-20, 1995. Albuquerque, New Mexico, USA, www.psig.org/papers

Marque´s D.and M. Morari (1988), On-line Optimisation of Gas Pipeline Networks, *Automatica*, Vol. **24**, No. 4, pp. 455-469

Osiadacz ,A. J. (1996), Different Transient Models-Limitations, Advantages and Disadvantages, *Pipeline Simulation Interest Group*, Annual Conference October 23-25,1996, San Francisco, USA, www.psig.org/papers

Ragavan, K. and L. Satish (2005), An Efficient Method to Compute Transfer Function of a Transformer from its Equivalent Circuit. *IEEE Transactions on Power Delivery*, Volume **20**. No. 2, April 2005, pp. 780-788