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Discrete-time Intermittent Iterative Learning Controller with Independent Data Dropouts ⋆

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Abstract: In networked control systems (NCS), it is considered essential to design a robust controller such that the networked-system is stable against data dropouts during the network transfer. It has been shown that there is a critical data dropout rate over which the networked-system could be unstable; hence the desired task cannot be achieved. This paper shows that a desired task or trajectories can be still achieved even though there are feedback signal dropouts if the desired task is repetitive, as in the iterative learning control case. Specifically this paper shows how to design stochastic iterative learning control systems such that the networked-system with a repetitive task is robust stable against measurement and process noises and independent, intermittent output channel dropouts.

1. INTRODUCTION

In the field of networked control systems (NCS), a major research interest is to understand how to compensate for signal delay and data dropout effects during network data transfer, so as to enable stable tele-operation and remote control (see Tipsuwan and Chow (2001); Walsh et al. (1999)). Recently, data dropout problems during network transmission have attracted a lot of research attention (see Wang et al. (2003); Smith and Seiler (2003)), though the problem was also addressed in the late 1980's (see Vangal (1989); Lynch et al. (1990)). From existing intermittent estimation theory (see Ling and Lemmon (2004, 2002); Zhang et al. (2001)), we can see that there is a critical data dropout rate above which an NCS is not stable. Recently we showed that if intermittent Kalman filtering is used, but enhanced by a learning control scheme, a repetitive or iterative NCS system could be made robustly stable, as long as there is not complete data dropout during the network transfer. Hence, the critical data dropout rate condition can be relaxed (see Ahn et al. (2007, 2006)). However, our earlier results were restricted to the case when the output channels have dependency (i.e., they are either all delivered together or all are dropped). In this paper we improve the theory given in Section 8.3 of Ahn et al. (2007) and in Ahn et al. (2006) by considering independent data loss between measurement output channels at every sampling instant. This is practically important and represents the situation, for example, when the data from different sensors in a process is transmitted over different network connections (e.g., wirelessly) or in separate packets.

Iterative learning control (ILC) is a control method, which tries to achieve perfect trajectory tracking when the system operates repetitively (see Arimoto et al. (1984); Moore et al. (1992)). In fact learning control is a kind of multi-pass system (see Edwards (1974); Chow and Fang (1998); Owens et al. (2000)), whereby the ILC systems try to achieve the desired trajectories in the time domain, but the convergence is guaranteed along the iteration domain. Various robustness issues such as model uncertainty, Cheah (2001), nonlinear robustness, Xu et al. (2000), parameter interval uncertainty, Ahn et al. (2007), the initial reset problem, Hillenbrand and Pandit (2000), stochastic noise, Saab (2001), disturbance rejection, Norrlöf and Gunnarsson (2001), data delays, Sun and Wang (2001), etc., along the iteration axis, have been investigated. However the data dropout problem in the context of ILC has not been completely studied beyond our initial results in Ahn et al. (2007) and Ahn et al. (2006). This paper investigates the implementation of learning control in a network control system setting, specifically focusing on compensation for intermittent data dropout when the desired task or desired trajectories are repetitive. It will be shown that the critical data dropout rate condition can be removed if the system is enhanced by learning control.

To motivate the current research and to highlight the contribution over our previous work, let us consider a

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situation of data dropouts. Data packets are transferred as a data stream. Since data is transferred at every sampling instant, there is a gap between a data packet for one sample and the other data packet for another sample. Also, in each packet, data is transferred in a stream. In Ahn et al. (2007) and Ahn et al. (2006), it was supposed that a data packet is either completely missed during the network transfer or delivered successfully. However, it is more natural to assume that some data bytes in individual data packet are lost; but still remaining data bytes are safely delivered. Thus, it is natural to say that part of output data $Y = [y^1, y^2, \ldots, y^n]$ is missed; but part of them is transferred safely, which means data bytes are independently intermittent each other. The key objective of this paper thus is to design a learning controller for such a case with measurement and process random noises.

The paper is organized as follows. In Section 2, we provide a detailed problem set-up and in Section 3, we derive an iteration-varying learning gain matrix and analyze the convergence of the stochastic learning control system with intermittent data dropouts. The results are verified through numerical simulations in Section 4. Conclusions will be given in Section 5.

2. PROBLEM SET-UP FOR ILC WITH INDEPENDENT DATA DROPOUTS

This section presents the problem set-up for intermittent data dropout networked-systems enhanced by stochastic learning control scheme. Let us consider the following discrete-time, 2-dimensional system:

$$x_k(t+1) = Ax_k(t) + Bu_k(t) + w_k(t) \quad (1)$$

$$y_k(t) = Cx_k(t) + v_k(t) \quad (2)$$

where $w_k(t)$ and $v_k(t)$ are process and measurement noises respectively, and $A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times m}$, and $C \in \mathbf{R}^{l\times n}$. The system operates repeatedly in the iteration domain and at the k-th iteration, the system is described by the discrete state equations (1) and (2). As explained in Ahn et al. (2006), the intermittent ILC problem is to design a robust controller when the output measurement is missed with some stochastic characteristics. In the typical discrete-time learning controller, the control input $u_{k+1}(t)$ is calculated by $u_{k+1}(t) = u_k(t) + K_k(t)e_k(t+1)$, where $e_k(t+1) = y_d(t+1) - y_k(t+1)$, $K_k(t)$ is the learning gain matrix, y_d is the desired output, and y_k is the actual measured output. In this update, it is supposed that the stored control input $u_{k+1}(t)$ or the output measurement y_k can be missed during the network transfer. In the intermittent ILC problem it is further supposed that the remote plant can detect when the current control signal u_{k+1} is missed; and the remote controller can also recognize whether the output measurement is dropped or not. Thus, the remote plant uses the past control signal $u_k(t)$ when the current control signal $u_{k+1}(t)$ is missed and the remote controller updates the control signal by $u_{k+1}(t) = u_k(t)$ when the output is missed. Therefore, in terms of the remote plant, the control signal is updated either by $u_{k+1}(t) = u_k(t) + K_k(t)e_k(t+1)$ or by $u_{k+1}(t) = u_k(t)$, which can be represented by

$$u_{k+1}(t) = u_k(t) + K_k(t)\eta e_k(t+1) \quad (3)$$

where $\eta \in \{0, 1\}$ (see Ahn et al. (2007, 2006)). If $\eta = 0$, then there could be control signal dropout and/or the measurement dropout. However, if $\eta = 1$, there is no data dropout during the network transfer. Thus, in Ahn et al. (2007) and Ahn et al. (2006), the data dropout indicator η is a binary random parameter. However, it is possible that a part of the data packet can be missed during the network transfer. In other words, in the multi-input multi-output 2 dimensional system modeled by (1) and (2), only a part of the output channels of $y_k = [y_k^1, \ldots, y_k^l]^T$ [1] can be dropped. Similarly in the control signal vector $u_k = [u_k^1, \ldots, u_k^m]^T$, a part of signal channels can be dropped. Thus, it is more general to model the data dropout by

$$u_{k+1}(t) = u_k(t) + K_k(t)\Sigma e_k(t+1) \quad (4)$$

where Σ is a diagonal matrix indicating the data dropouts such as:

$$\Sigma = \mathrm{diag}[\eta_i] := \begin{pmatrix} \eta_1 & 0 & \cdots & 0 \\ 0 & \eta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \eta_l \end{pmatrix} \quad (5)$$

where each η_i is a binary random variable such as $\eta_i \in \{0, 1\}$, $i = 1, \ldots, l$ and $E[\eta_i \eta_j] = 0$ if $i \neq j$. It is assumed that $E[\eta_i] := \overline{\eta_i}$ is known, which implies that the expectation of Σ is given as $\overline{\Sigma} = \mathrm{diag}[\overline{\eta_i}]$. Introducing $\eta_i = \overline{\eta_i} + \widetilde{\eta_i}$, it is clear that $\widetilde{\eta_i}$ is a zero-mean stochastic sequence. Here for our main result, we introduce $\widetilde{\Sigma} = \mathrm{diag}[\widetilde{\eta_i}]$, which means $\Sigma = \overline{\Sigma} + \widetilde{\Sigma}$. Also, since the variance of $\widetilde{\eta_i}$ is $(1 - \overline{\eta_i})\overline{\eta_i}$, the variance of $\widetilde{\Sigma}$ is given as $\mathrm{diag}[(1 - \overline{\eta_i})\overline{\eta_i}]$. For convenience, we denote $\widetilde{\Sigma}_{\sigma^2} := \mathrm{diag}[(1-\overline{\eta_i})\overline{\eta_i}]$, and denote $u_d(t), x_d(t)$, and $y_d(t)$ as the desired input, state, and output signals, respectively. Also we introduce $\delta u_{k+1}(t) = u_d(t) - u_{k+1}(t)$ and $\delta x_k(t) = x_d(t) - x_k(t)$. Then, following the same procedure as given in Saab (2001) and Ahn et al. (2006), we obtain the auxiliary system given in (6). Using $\Sigma = \overline{\Sigma} + \widetilde{\Sigma}$, we change (6) to (7). Here, for a brevity of presentation, we define $X^+ := \begin{bmatrix} \delta u_{k+1}(t) \\ \delta x_k(t+1) \end{bmatrix}$, $X := \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix}$ and $W := \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix}$, and assume no correlation between X and W. In the following section, we will calculate $P^+ := E[X^+(X^+)^T]$ and analyze the convergence of the system.

3. INTERMITTENT ILC: DESIGN AND ANALYSIS

3.1 Optimal Learning Gain Matrix

First we need to calculate the variance of the third term of the right-hand side of (7) as shown in (8) (shown on the next page). In (8), Θ_1 is calculated as:

$$\begin{aligned} \Theta_1 = {} & K_k\widetilde{\Sigma}CB\delta u_k \delta u_k^T (CB)^T \widetilde{\Sigma}^T K_k^T \\ & + K_k\widetilde{\Sigma}CA\delta x_k \delta u_k^T (CB)^T \widetilde{\Sigma}^T K_k^T \\ & + K_k\widetilde{\Sigma}CB\delta u_k \delta x_k^T (CA)^T \widetilde{\Sigma}^T K_k^T \\ & + K_k\widetilde{\Sigma}CA\delta x_k \delta x_k^T (CA)^T \widetilde{\Sigma}^T K_k^T. \end{aligned} \quad (9)$$

[1] In the output y_k, y_k^1 is the first channel; y_k^2 is the second channel; $\ldots$, y_k^l is the l-th channel.

$$\begin{bmatrix} \delta u_{k+1}(t) \\ \delta x_k(t+1) \end{bmatrix} = \begin{bmatrix} I - K_k(t)\Sigma CB & -K_k(t)\Sigma CA \\ B & A \end{bmatrix} \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix} + \begin{bmatrix} K_k(t)\Sigma C & K_k(t) \\ -I & 0 \end{bmatrix} \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix} \tag{6}$$

$$\begin{aligned} \begin{bmatrix} \delta u_{k+1}(t) \\ \delta x_k(t+1) \end{bmatrix} &= \begin{bmatrix} I - K_k(t)\overline{\Sigma} CB & -K_k(t)\overline{\Sigma} CA \\ B & A \end{bmatrix} \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix} + \begin{bmatrix} K_k(t)\overline{\Sigma} C & K_k(t) \\ -I & 0 \end{bmatrix} \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix} \\ &+ \begin{bmatrix} -K_k(t)\widetilde{\Sigma} CB & -K_k(t)\widetilde{\Sigma} CA \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta u_k(t) \\ \delta x_k(t) \end{bmatrix} + \begin{bmatrix} K_k(t)\widetilde{\Sigma} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_k(t) \\ v_k(t+1) \end{bmatrix}. \end{aligned} \tag{7}$$

$$E\left[\begin{bmatrix} -K_k(t)\widetilde{\Sigma} CB & -K_k(t)\widetilde{\Sigma} CA \\ 0 & 0 \end{bmatrix} XX^T \begin{bmatrix} -K_k(t)\widetilde{\Sigma} CB & -K_k(t)\widetilde{\Sigma} CA \\ 0 & 0 \end{bmatrix}^T \right] = E\left[\begin{pmatrix} \Theta_1 & 0 \\ 0 & 0 \end{pmatrix} \right] \tag{8}$$

$$E[K_k\widetilde{\Sigma} CB\delta u_k \delta u_k^T (CB)^T \widetilde{\Sigma}^T K_k^T] = K_k \widetilde{\Sigma}_{\sigma^2} \mathrm{Diag} E\left[CB\delta u_k \delta u_k^T (CB)^T\right] K_k^T = K_k \widetilde{\Sigma}_{\sigma^2} \mathrm{Diag}[CBP_{11}(CB)^T] K_k^T \tag{10}$$

Since elements of $\widetilde{\Sigma}$ are independent each other, the equation given in (10) is true (shown on the next page). In (10), Diag[·] is an operator that selects diagonal terms of matrix [·] and $P_{11} = E[\delta u_k \delta u_k^T]$. Therefore, we obtain

$$\begin{aligned} E[\Theta_1] = & K_k\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CBP_{11}(CB)^T]K_k^T \\ & + K_k\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CBP_{12}(CA)^T]K_k^T \\ & + K_k\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CAP_{21}(CB)^T]K_k^T \\ & + K_k\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CAP_{22}(CA)^T]K_k^T \\ = & K_k\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CBP_{11}(CB)^T + CBP_{12}(CA)^T \\ & + CAP_{21}(CB)^T + CAP_{22}(CA)^T]K_k^T \end{aligned} \tag{11}$$

where $P_{12} = E[\delta u_k \delta x_k^T]$, $P_{21} = E[\delta x_k \delta u_k^T]$, and $P_{22} = E[\delta x_k \delta x_k^T]$. Similarly we calculate the variance of the fourth term of the right-hand side of (7) such as:

$$\begin{aligned} & E\left[\begin{bmatrix} K_k(t)\widetilde{\Sigma} C & 0 \\ 0 & 0 \end{bmatrix} WW^T \begin{bmatrix} K_k(t)\widetilde{\Sigma} C & 0 \\ 0 & 0 \end{bmatrix}^T \right] \\ & = \begin{pmatrix} K_k\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CQ_{11}C^T]K_k^T & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{12}$$

where $Q_{11} = E[w_k w_k^T]$. Next using (11) and (12), we calculate P^+ such as

$$\begin{aligned} P^+ = & \Phi P \Phi^T + \Psi Q \Psi^T \\ & + \begin{pmatrix} E[\Theta_1] + K_k\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CQ_{11}C^T]K_k^T & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{13}$$

where

$$\Phi := \begin{bmatrix} I - K_k(t)\overline{\Sigma} CB & -K_k(t)\overline{\Sigma} CA \\ B & A \end{bmatrix}$$

$$\Psi := \begin{bmatrix} K_k(t)\overline{\Sigma} C & K_k(t) \\ -I & 0 \end{bmatrix}$$

$$P^+ := E[X^+ X^{+T}];\ P := E[XX^T];\ Q := E[WW^T].$$

Now, in order to find an optimal learning gain matrix $K_k(t)$, we use the trace of P^+. In what follows, for simplicity, we omit subscripts the k and $\tilde{\eta}$, and the time index t. Now, we are able to compute the traces of both sides of (13) as (14) (shown on the next page). In (14), we partitioned P and Q according to:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix};\ Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

Next, to simplify the expression, we use the following substitutions

$$V_1 := (CB,\ CA),\ V_2 := (B,\ A),\ V_3 = (I,\ 0)$$

Then, (14) is expressed as:

$$\begin{aligned} \mathrm{trace}(P^+) = & \mathrm{trace}[K\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[V_1PV_1^T]K^T \\ & + K\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CQ_{11}C^T]K^T \\ & + K\overline{\Sigma}V_1PV_1^T\overline{\Sigma}^TK^T + V_2PV_2^T \\ & - V_3PV_1^T\overline{\Sigma}^TK^T - K\overline{\Sigma}V_1PV_3^T \\ & + K(\overline{\Sigma}CQ_{11} + Q_{21})(\overline{\Sigma}C)^TK^T \\ & + K(\overline{\Sigma}CQ_{12} + Q_{22})K^T + Q_{11}] \end{aligned} \tag{15}$$

Therefore, we have

$$\begin{aligned} \frac{\partial \mathrm{trace}(P^+)}{\partial K} = & 2K\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[V_1PV_1^T] \\ & + 2K\widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[CQ_{11}C^T] \\ & + 2K\overline{\Sigma}V_1PV_1^T\overline{\Sigma}^T - 2V_3PV_1^T\overline{\Sigma}^T \\ & + 2K(\overline{\Sigma}CQ_{11} + Q_{21})(\overline{\Sigma}C)^T \\ & + 2K(\overline{\Sigma}CQ_{12} + Q_{22}) \end{aligned} \tag{16}$$

Here, assuming no correlation between $w_k(t)$ and $v_k(t)$, we consider $Q_{12} = Q_{21} = 0$. Now, defining

$$\begin{aligned} \Pi := & \widetilde{\Sigma}_{\sigma^2}\mathrm{Diag}[V_1PV_1^T + CQ_{11}C^T] \\ & + \overline{\Sigma}V_1PV_1^T\overline{\Sigma}^T + \overline{\Sigma}CQ_{11}\overline{\Sigma}C^T + Q_{22}, \end{aligned} \tag{17}$$

when $\frac{\partial \mathrm{trace}(P^+)}{\partial K}$ is set equal to zero, we finally calculate the optimal learning gain as follows:

$$K_k(t) = V_3PV_1^T\overline{\Sigma}^T\Pi^{-1}. \tag{18}$$

3.2 Analysis of convergence

In this subsection, we analyze the convergence of the intermittent ILC system updated by the learning gain given by (18). For this purpose, we need to find a recursive update formula for the covariance matrix $P = E[XX^T]$. Using the same method given in Saab (2001), it can be shown that $P_{12,k}(t) = P_{21,k}(t) = 0$. For the convergence analysis of the diagonal term $P_{11,k}(t)$, we derive the following formula which is similar to Eq. (16) of Ahn et al. (2006):

$$\begin{aligned}\text{trace}(P^+) = \text{trace}\Big[&K_k\widetilde{\Sigma}_{\sigma^2}\text{Diag}[CBP_{11}(CB)^T + CBP_{12}(CA)^T + CAP_{21}(CB)^T + CAP_{22}(CA)^T]K_k^T \\ &+K_k\widetilde{\Sigma}_{\sigma^2}\text{Diag}[CQ_{11}C^T]K_k^T + [(I - K\overline{\Sigma}CB)P_{11} - K\overline{\Sigma}CAP_{21}][I - K\overline{\Sigma}CB]^T \\ &+[(I - K\overline{\Sigma}CB)P_{12} - K\overline{\Sigma}CAP_{22}][-K\overline{\Sigma}CA]^T + (BP_{11} + AP_{21})B^T + (BP_{12} + AP_{22})A^T \\ &+(K\overline{\Sigma}CQ_{11} + KQ_{21})(K\overline{\Sigma}C)^T + (K\overline{\Sigma}CQ_{12} + KQ_{22})K^T + Q\Big] \end{aligned} \quad (14)$$

$$\begin{aligned}&E[\delta u_{k+1}(t)\delta u_{k+1}(t)^T] \\ &= K\widetilde{\Sigma}_{\sigma^2}\text{Diag}[V_1PV_1^T]K^T + K\widetilde{\Sigma}_{\sigma^2}\text{Diag}[CQ_{11}C^T]K^T \\ &+K\overline{\Sigma}V_1PV_1^T\overline{\Sigma}^TK^T + V_3PV_3^T - V_3PV_1^T\overline{\Sigma}^TK^T \\ &-K\overline{\Sigma}V_1PV_3^T + K\overline{\Sigma}CQ_{11}C^T\overline{\Sigma}^TK^T + KQ_{22}K^T. \end{aligned} \quad (19)$$

We can now change (19) as follows:

$$\begin{aligned}&E[\delta u_{k+1}(t)\delta u_{k+1}(t)^T] \\ &= K\Pi K^T + V_3PV_3^T - V_3PV_1^T\overline{\Sigma}^TK^T - K\overline{\Sigma}V_1PV_3^T \\ &= V_3PV_1^T\overline{\Sigma}^T\Pi^{-1}\Pi[V_3PV_1^T\overline{\Sigma}^T\Pi^{-1}]^T + V_3PV_3^T \\ &\quad - V_3PV_1^T\overline{\Sigma}^T[V_3PV_1^T\overline{\Sigma}^T\Pi^{-1}]^T \\ &\quad - [V_3PV_1^T\overline{\Sigma}^T\Pi^{-1}]\overline{\Sigma}V_1PV_3^T \\ &= V_3PV_3^T - [V_3PV_1^T\overline{\Sigma}^T\Pi^{-1}]\overline{\Sigma}V_1PV_3^T \end{aligned} \quad (20)$$

Then, inserting $V_1 = (CB, CA)$ and $V_3 = (I, 0)$ into the above equation yields

$$\begin{aligned}P_{11,k+1} &:= E[\delta u_{k+1}(t)\delta u_{k+1}(t)^T] \\ &= (I - K_k(t)\overline{\Sigma}CB)P_{11,k}. \end{aligned} \quad (21)$$

In the sequel, we will show that the spectral radius of $I - K_k(t)\overline{\Sigma}CB$ is less than 1 (i.e., $\rho(I - K_k(t)\overline{\Sigma}CB) < 1$). The result is summarized in the following theorem.

Theorem 1. If $\overline{\Sigma}CB$ is full rank and P_{11} is positive definite, then $\rho(I - K_k(t)\overline{\Sigma}CB) < 1$.

Proof. Due to page limitations, we omit a detailed proof.

Theorem 1 shows that $P_{11,k} \to 0$ as $k \to \infty$. Next we consider the convergence of $P_{22,k}(t)$. For this we have the following theorem.

Theorem 2. If there is no initial resetting error at every iteration, then

$$P_{22,k} \to \sum_{i=0}^{t-1} A^{t-1-i}Q_{11}\sum_{i=0}^{t-1}(A^T)^{t-1-i}. \quad (22)$$

Proof. The proof is direct by following the proof of Theorem 3.2 of Ahn et al. (2006).

Remark 3. In Theorem 2, it is shown that $P_{22,k}$ converges to a fixed value as the number of iterations increases. It is observed that the final converged value of P_{22} depends on A and Q_{11}. If there is no noise, then $P_{22} \to 0$.

Remark 4. Similar to Saab (2001) and Ahn et al. (2006), we can develop an algorithm for updating $K_k(t)$ and for propagating the ILC system on the iteration domain. For this purpose, we use the following propagation, which is derived from (13):

$$P_{22,k}(t+1) = BP_{11,k}(t)B^T + AP_{22,k}(t)A^T + Q_{11}. \quad (23)$$

Assuming that $P_{11,k}(t)$, when $k = 0$, is available and $P_{22,k}(t)$, when $t = 0$, is also available, we generate the following process:

- Use (18) for updating $K_k(t)$.
- From (23), when $k = 0$, we calculate $P_{22,k}(t+1)$.
- Calculate $u_{k+1}(t)$ using (4).
- Use (21) to update $P_{11,k}(t)$.
- Repeat whole process (i.e., $k = k + 1$).

4. SIMULATION ILLUSTRATIONS

For illustration and verification, we use the following two input, two output linear time invariant discrete system:

$$\begin{aligned}x_k(t+1) &= \begin{pmatrix} 0.5 & -0.025 & 0.015 \\ 0.03 & -0.5 & -0.5 \\ -0.75 & 0.025 & -0.025 \end{pmatrix} x_k(t) \\ &+ \begin{pmatrix} 0.1 & 0.5 \\ -0.1 & 0.5 \\ 0.0 & 1.0 \end{pmatrix} u_k(t) + w_k(t) \end{aligned} \quad (24)$$

$$y_k(t) = \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.1 \end{pmatrix} x_k(t) + v_k(t) \quad (25)$$

where $E[w_kw_k^T] = \text{diag}[10^{-4}]$ and $E[v_kv_k^T] = \text{diag}[10^{-4}]$. The number of samples in each iteration is fixed at 50 (i.e., $t = 0, 1, 2, \dots, 49$) and the desired output trajectories are given as $y_{d1}(t) = \sin(2\pi t/50)$ and $y_{d2}(t) = 1.5\sin(\pi t/50)$. For simulation purposes, we choose $P_{11,k=0} = \text{diag}[10^4]$ and $P_{22,k}(t = 0) = \text{diag}[10^{-2}]$. To see various intermittent rates, we tested five different cases; Case-1: $\overline{\eta_1} = 0.9$ and $\overline{\eta_2} = 0.9$; Case-2: $\overline{\eta_1} = 0.7$ and $\overline{\eta_2} = 0.7$; Case-3: $\overline{\eta_1} = 0.5$ and $\overline{\eta_2} = 0.5$; Case-4: $\overline{\eta_1} = 0.3$ and $\overline{\eta_2} = 0.3$; Case-5: $\overline{\eta_1} = 0.1$ and $\overline{\eta_2} = 0.1$. Note that the intermittent random variables η_1 and η_2 are independent.

For comparison of the proposed intermittent stochastic learning controller with a fixed learning gain strategy, we selected a simple Arimoto-type gain matrix given by $K_k = \begin{pmatrix} 2.1818 & -1.8182 \\ 0.3636 & 0.3636 \end{pmatrix}$ which makes $\|I - K_kCB\| = 0.6$. The selected K_k was made from $\alpha(CB)^{-1}$ where α was tuned to be 0.4. Fig. 1 is the transient response of the ILC system with the fixed Arimoto-type gain matrix. The top plot is the norm of the error $E_k(1) = Y_d(1) - Y_k(1)$ vs. iteration number and the bottom plot is the norm of error $E_k(2) = Y_d(2) - Y_k(2)$ [2] vs. iteration number. Fig. 1 shows that the desired trajectories have been achieved perfectly as iterations increase by the fixed learning gain matrix without stochastic noises and intermittent signal dropouts. Fig. 2 to Fig. 6 illustrate the transient responses with stochastic noises and intermittent signal dropouts.

[2] $E_k(1)$, $Y_d(1)$, $Y_k(1)$, $E_k(2)$, $Y_d(2)$, and $Y_k(2)$ are length N column vectors defined based on $[Y_d(1), Y_d(2)] = [y_d(1), y_d(2), \dots, y_d(N)]^T$ where N is the length of discrete time points at every iteration.

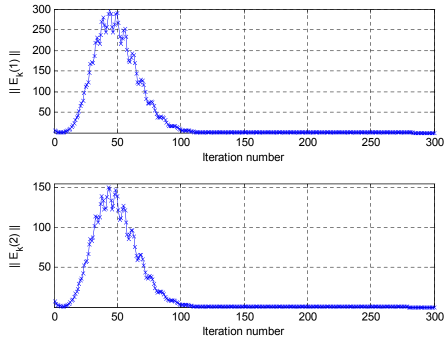

Fig. 1. The error of achieved trajectories when the fixed gain K_k was used for the nominal system which has no stochastic noises and intermittent signal dropouts. Top: Norm of error $E_k(1) = Y_d(1) - Y_k(1)$ vs. iteration number. Bottom: Norm of error $E_k(2) = Y_d(2) - Y_k(2)$ vs. iteration number.

Fig. 2 is for Case-1; Fig. 3 is for Case-2; Fig. 4 is for Case-3; Fig. 5 is for Case-4; and Fig. 6 is for Case-5. In the individual figures, the left plots are results from the proposed learning controller and the right plots are results from the Arimoto-type learning gain matrix. From the comparison between the left plots and the right plots, we observe that the desired trajectories are satisfactorily achieved in the left plots even though there exist steady-state errors as iterations increase, whereas the desired trajectories are not achieved in the right plots. Also as shown from the comparison between Case-1, Case-2, Case-3, Case-4, and Case-5, there tend to be more and higher overshoots during the transient response and convergence speed gets slower as intermittent rate increases. It remains a topic of future research to explore the steady-state error shown in the simulations. This is related to the base-line error of ILC discussed by the authors in previous work.

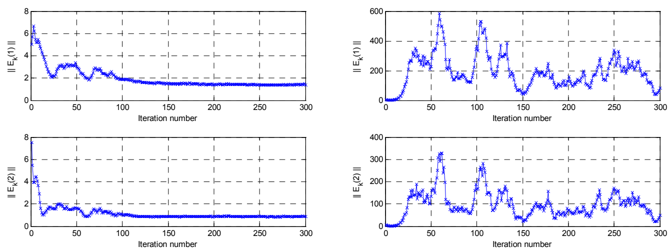

Fig. 2. Case-1: $\overline{\eta_1} = 0.9$ and $\overline{\eta_2} = 0.9$. Left-top: Norm of error $E_k(1) = Y_d(1) - Y_k(1)$ vs. iteration number. Left-bottom: Norm of error $E_k(2) = Y_d(2) - Y_k(2)$ vs. iteration number. Right-top: Error of $Y_k(1)$ from the fixed gain matrix. Right-bottom: Error of $Y_k(2)$ from the fixed gain matrix.

5. CONCLUDING REMARKS

This paper presented a stochastic iterative learning controller for the compensation of data dropouts in networked control systems. Specifically, we presented a design method to find an iteration-varying learning gain matrix, which was developed considering independent, intermittent data dropouts. The main theoretical result is that if Kalman filtering is enhanced by the learning control scheme, the system with repetitive desired trajectories is robustly stable as long as there is not complete data dropout. Thus the major contribution of this paper over the existing intermittent estimation theories, thus relaxing the classical result for non-repetitive systems that there is a critical data dropout rate above which the system is not stable can In our future efforts, we will consider model uncertainty as well as data dropouts. It will be also important to consider data delay in addition to the data dropouts during the network transfer of data between the remote plant and the remote server.

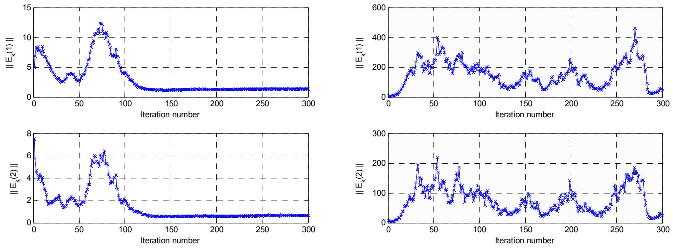

Fig. 3. Case-2: $\overline{\eta_1} = 0.7$ and $\overline{\eta_2} = 0.7$. Left-top: Norm of error $E_k(1) = Y_d(1) - Y_k(1)$ vs. iteration number. Left-bottom: Norm of error $E_k(2) = Y_d(2) - Y_k(2)$ vs. iteration number. Right-top: Error of $Y_k(1)$ from the fixed gain matrix. Right-bottom: Error of $Y_k(2)$ from the fixed gain matrix.

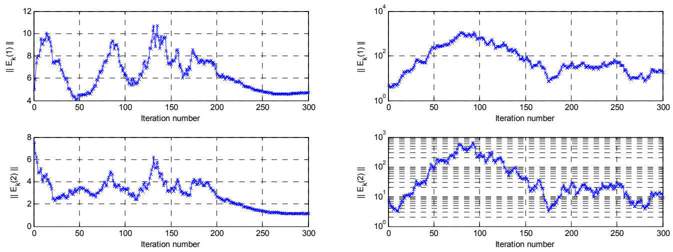

Fig. 4. Case-3: $\overline{\eta_1} = 0.5$ and $\overline{\eta_2} = 0.5$. Left-top: Norm of error $E_k(1) = Y_d(1) - Y_k(1)$ vs. iteration number. Left-bottom: Norm of error $E_k(2) = Y_d(2) - Y_k(2)$ vs. iteration number. Right-top: Error of $Y_k(1)$ from the fixed gain matrix. Right-bottom: Error of $Y_k(2)$ from the fixed gain matrix.

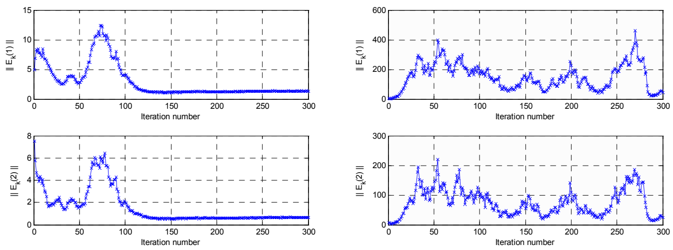

Fig. 5. Case-4: $\overline{\eta_1} = 0.3$ and $\overline{\eta_2} = 0.3$. Left-top: Norm of error $E_k(1) = Y_d(1) - Y_k(1)$ vs. iteration number. Left-bottom: Norm of error $E_k(2) = Y_d(2) - Y_k(2)$ vs. iteration number. Right-top: Error of $Y_k(1)$ from the fixed gain matrix. Right-bottom: Error of $Y_k(2)$ from the fixed gain matrix.

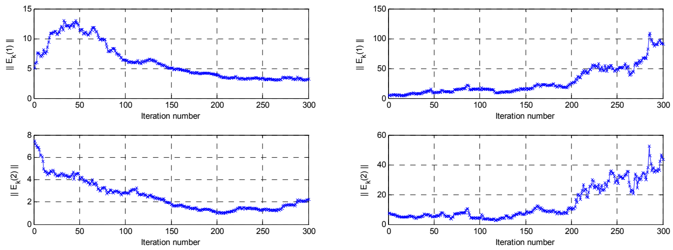

Fig. 6. Case-5: $\overline{\eta_1} = 0.1$ and $\overline{\eta_2} = 0.1$. Left-top: Norm of error $E_k(1) = Y_d(1) - Y_k(1)$ vs. iteration number. Left-bottom: Norm of error $E_k(2) = Y_d(2) - Y_k(2)$ vs. iteration number. Right-top: Error of $Y_k(1)$ from the fixed gain matrix. Right-bottom: Error of $Y_k(2)$ from the fixed gain matrix.

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