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Stability Guaranteed Predictive Control of Constrained Continuous-time PWL Systems

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Abstract: In this work, the stability of constrained continuous-time piecewise linear (PWL) systems in closed-loop is investigated based on model predictive control (MPC) and bounded control (BC). Firstly, bounded control framework is developed to stabilize the class of systems. Then, to reconcile the stability and optimality properties, a control strategy mixing model predictive control with bounded control is proposed further. For each subsystem, the switching idea is employed between model predictive controller and bounded controller for a set of all initial conditions within the stability region of the bounded controller. Switching laws of controllers in each subsystem are derived to safeguard against any possible instability and infeasibility under MPC. The switching constraints of regions between different subsystems are considered to ensure that the Lyapunov function for each subsystem is non-increasing wherever the mode is reactivated, thereby guaranteeing global closed-loop stability. The proposed method avoids computing terminal invariant set for guaranteeing stability and reduces on-line complexity of computing stabilizing controller for constrained continuous-time PWL systems. Finally, the implementation of the proposed method is illustrated with an example.

1. INTRODUCTION

Technological innovation pushes towards the consideration of systems of a mixed continuous and discrete nature, which are called hybrid systems. A general model of hybrid systems usually leads to a high level of complexity with respect to analysis and controller design techniques. However, piecewise affine (PWA) systems have become popular due to their accessible mathematical description and their ability to model a broad class of (hybrid) systems. The modelling power of PWA systems has already been shown in several applications, such as switched power converters (Leenaerts, 1996) and optimal control of DC-DC converters.

The control of PWA systems has attracted a great deal of attention in recent years, motivated by the fundamentally hybrid nature. Several control strategies have been proposed for PWA systems (Bemporad *et al*., 1999; Kerrigan and Mayne, 2002; Lazar *et al*., 2004). Optimal control of PWA systems was developed by solving mixed-integer optimization problems on-line or a number of multi-parametric programs off-line. But this control approach does not deal with constrained PWA systems well. Therefore, it is necessary to design a control algorithm that can incorporate constraints of PWA systems in the control design. As a result, MPC has been extended to PWA systems (Lazar *et al*., 2004).

Piecewise linear (PWL) systems are a particular class of PWA systems. How to guarantee closed-loop stability of MPC for PWL systems is a difficult problem. A common idea of existing works for guaranteeing stability of discrete-time PWL systems based on MPC is an extension of the terminal cost and constraint set approach in linear or nonlinear MPC (Lazar *et al*., 2004). In this work, the discrete-time PWL system is considered as a whole, where predicted state at different instant may correspond to different subsystem and region. In order to guarantee stability, the terminal state has to be constrained to an invariant set containing the origin. However, there are some problems existed: 1) the terminal invariant sets for discrete-time PWL systems are difficult to develop; 2) the difficulty of identifying the set of initial conditions starting from where feasibility and closed-loop stability are guaranteed is pronounced; 3) the computation burden of optimization problem and terminal stability constraints is heavy. Moreover, the existing methods are not suitable for continuous-time PWL systems. Due to continuity, the evolution of predictive state can not be estimated which region it belongs to in some prediction horizon, and it is difficult to design controller of continuous-time PWL systems as a whole. Therefore the development of novel stable control scheme for constrained continuous-tine of PWL systems is necessary as well as natural.

In this paper, instead of considering the whole PWL systems, we propose a mixed control strategy with switching constraints to guarantee closed-loop stability for each subsystem when it is activated. The switching laws of controllers between MPC and bounded control are designed to guarantee the Lyapunov function of the subsystem non-increasing. The switching constraints laws of regions between the various PWL subsystems are presented to track the evolution of the state corresponding to different region and guarantee asymptotic stability of the global closed-loop PWL systems. In this method, a set of initial conditions that is explicitly characterized by the bounded controller in

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individual subsystem is computed, which can avoid computing the terminal invariant set. In each set, we merge MPC with the bounded control in a way that allows both approaches to complement the stability and optimality properties of each other. Therefore, we only compute quadratic programming problems for each subsystem instead of solving MIQP problems for the whole PWL systems as in (Lazar *et al*., 2004). The rest of the paper is organized as follows. In Section 2, we present preliminaries and pose the problem we consider. In Section 3, the bounded controller of PWL systems is addressed. We then proceed in Section 4 to formulate a novel mixed controller scheme with a theorem to guarantee asymptotical stability. Finally, in Section 5, the implementation of the algorithm is demonstrated using a chemical process example.

2. PRELIMINARIES

2.1 Problem Formulation

Consider the class of continuous-time PWL systems described by the following form:

$$\dot{x}(t) = A_i x(t) + B_i u(t) , \qquad x(t) \in \mathcal{X}_i , \tag{1}$$

where, $x(t) \in X \subseteq \mathrm{R}^n$ is the continuous-time state vector, $u(t) \in U \subseteq \mathrm{R}^m$ is the manipulated inputs, U is in the input constraints defined as $U := \left\{ u \in \mathrm{R}^m \middle| \|u\| \le u_{\max} \right\}$. $A_i \in \mathrm{R}^{n \times n}$, $B_i \in \mathrm{R}^{n \times m}$, $f_i \in \mathrm{R}^n$, $i \in S$ with $S := \{1, 2, \cdots s\}$ and s denoting the number of modes. A partition of $\{ X_i | i \in S \}$ is assumed to be a closed convex polyhedron, and H_{ij} is the boundary between X_i and X_j . Here we consider $\cup_{i \in S} \mathrm{X}_i = X$, and $0 \in H_{ij} = \mathrm{X}_\mathrm{i} \cap \mathrm{X}_\mathrm{j}$.

For continuous-time PWL systems, it is difficult to describe evolution of the predictive state corresponding to different region in some prediction horizon and solve the optimization problem, which leads to inefficiency in the approaches used in discrete-time case (Lazar *et al*., 2004). Therefore, how to develop novel control framework to stabilize continuous-time PWL systems is a valuable problem. Dissimilarly to existing methods in discrete-time case, a mixed control scheme for each subsystem of continuous-time PWL systems is designed to regulate the state of system (1) to the origin, which can be described as:

Problem 2.1 For any given $x(0) = x_0$, determine each controller u_i , $i \in S$ for each subsystem of continuous-time PWL systems such that the whole system (1) in closed-loop is global asymptotically stable, i.e. $\lim_{t \to \infty} x(t) = 0$.

For problem 2.1, the controller u_i , $i \in S$ is designed for each subsystem with switching different subsystems to guarantee closed-loop stability. In order to develop the controllers that guarantee stability for continuous-time PWL systems, the tool of stability analysis for the class systems should be introduced first.

2.2 Multiple Lyapunov Functions Approach

Lyapunov techniques are useful tools for stability analysis. The basic conceptual idea behind any Lyapunov design is that of “energy shaping”, where an appropriate “energy” function is chosen for the systems, and the controller is designed in a way that enforces the monotonic decay of this function along the trajectories of the closed-loop systems. Therefore, it is quite intuitive to exploit Lyapunov tools to analyze stability of continuous-time PWL systems as a finite collection of continuous-time processes with discrete events that govern the transition between them. One of the main tools is multiple Lyapunov functions (MLFs) (Branicky, 1998). The idea is that even if we have Lyapunov functions for each subsystem individually, we need to impose restrictions on switching to guarantee stability. In this subsection, we briefly review the main idea. For the class of continuous-time PWL systems (1), a family of Lyapunov-like functions $\{V_i : i \in S\}$ can be found such that the value of V_i decreases on each interval when the i -th subsystem is active, i.e.,

$$V_i(x(t_{i_k'})) < V_i(x(t_{i_k})) , \tag{2}$$

for all $i \in S$, $k \in Z^+$. However, such a Lyapunov function for each subsystem can not guarantee the stability of the whole continuous-time PWL systems. The reason is that during the time interval when a particular model is inactive, its energy might be adversely affected by the evolution of the active mode such that at the next time instant when the inactive model is activated, its energy has already exceeded the level of its last interval of activity. A sufficient condition to guarantee Lyapunov stability of the whole PWL systems is that for every subsystem i , the value of V_i at the beginning of each interval on which the i th subsystem is active not exceed the value at the beginning of the previous such interval (El-Farra *et al*., 2004), i.e.,

$$V_i(x(t_{i_k})) \le V_i(x(t_{i_{k-1}})) . \tag{3}$$

The Lyapunov stability of the PWL systems can be guaranteed using (2) and (3).

3. BOUNDED CONTROL OF CONTINUOUS-TIME PWL SYSTEMS

The bounded controller (Lin and Sontag, 1991) was proposed for continuous-time linear systems with the following characteristic: 1) satisfy the constraints of the systems; 2) enforce asymptotic stability of the systems; 3) provide an explicit characterization of the set of admissible initial stable conditions. In this section, we extend the bounded control to PWL systems (1) to guarantee closed-loop asymptotically stability. For i th subsystem, assume a control Lyapunov function of $V_i = x^T P_i x$ exists that satisfies the Riccati equation

$$A_i^T P_i + P_i A_i - P_i B_i B_i^T P_i = -\tilde{Q}_i , \tag{4}$$

for some positive-definite matrix $\tilde{Q}_i$. The following continuous bounded control law for i th subsystem can be constructed:

$$u_i(x) = -2k_i(x)B_i^T P_i x := b_i(x), \quad i = 1,2,\cdots s, \tag{5}$$

where

$$k_i(x) = \frac{L_{fi}V_i + \sqrt{(L_{fi}V_i)^2 + u_{\max}\left\|(L_{gi}V_i)^T\right\|)^4}}{\left\|(L_{gi}V_i)^T\right\|^2 [1 + \sqrt{1 + (u_{\max}\left\|(L_{gi}V_i)^T\right\|)^2}]}. \tag{6}$$

$L_{fi}V_i = x^T(A_i^T P_i + P_i A_i)x$, $(L_{gi}V_i)^T = 2B_i^T P_i x$. For the above controller, a set is described as:

$$\Phi_i(u_{\max}) = \left\{x \in \mathcal{X}_i \left| x^T(A_i^T P_i + P_i A_i)x < 2u_{\max}\left\|B_i^T P_i x\right\|\right.\right\}. \tag{7}$$

whenever the closed-loop state x evolves within $\Phi_i(u_{\max})$, the controller satisfies the state and input constraints and the time-derivate of the Lyapunov function is negative-definite. So as long as the initial state and the state trajectory of the closed-loop system remain within $\Phi_i(u_{\max})$, the asymptotic stability of the constrained closed-loop system can be guaranteed. To ensure this, an subset (preferably the largest) of $\Phi_i(u_{\max})$ should be constructed by using the level sets of V_i:

$$\Omega_i(u_{\max}) = \left\{x \in \mathbb{R}^n \left| x^T P_i x \le c_{i\max}\right.\right\} \subseteq \Phi_i(u_{\max}), \tag{8}$$

where $c_{i\max} > 0$ is the largest number for which all nonzero elements of Ω_i are contained within Φ_i.

In order to design the controllers for continuous-time PWL systems to guarantee the global stability, we need to consider the switching conditions between the constituent subsystems. The theorem 1 below provides a formula for the bounded state feedback controllers for continuous-time PWL system (1) and state switching conditions to guarantee the desired properties.

Theorem 1. For system (1), a family of control Lyapunov functions V_i , $i \in S$ exist. The family of bounded state feedback controllers are described as (5) and (6), with the set of x containing the origin (7) and (8). Given $x(0) \in \Omega_i(u_{\max})$ for some $i \in S$, if at time $t_{i_r}^{out}$ the following conditions hold:

$$x(t_{i_r}^{out}) \in \Omega_j(u_{\max}), \tag{9}$$

$$V_j(x(t_{j_m}^{in})) < V_j(x(t_{j_{m-1}}^{in})), \tag{10}$$

where $t_{i_r}^{out} = t_{j_m}^{in}$, $t_{i_r}^{out}$ is the time at which for the r th time, the i th subsystem is switched out , $t_{j_m}^{in}$ is the time at which for the m th time, the j th subsystem is switched in and $t_{j_{m-1}}^{in}$ is the time at which for the $m-1$ th time, the j th subsystem is switched in, then the state is switched into subsystem j and the origin of the continuous-time PWL closed-loop systems is asymptotically stable by repeated the procedure.

Proof: Consider the system (1) for i th subsystem, under the bounded control law (5) and (6), the time-derivative of the Lypunov function along the closed-loop trajectories is

$$\dot{V}_i = \frac{L_{fi}V_i\sqrt{u_{\max}\left\|(L_{gi}V_i)^T\right\|)^2} - \sqrt{(L_{fi}V_i)^2 + (u_{\max}\left\|(L_{gi}V_i)^T\right\|)^4}}{[1 + \sqrt{1 + (u_{\max}\left\|(L_{gi}V_i)^T\right\|)^2}]} \tag{11}$$

From (11), when the inequality $L_{fi}V_i < 0$ holds, we get $\dot{V} < 0$. When the inequality $0 \le L_{fi}V_i < u_{\max}\left\|(L_{gi}V_i)^T\right\|$ holds, we can get the following inequality:

$$-\sqrt{(L_{fi}V_i)^2 + (u_{\max}\left\|L_{gi}V_i\right\|)^4} < L_{fi}V_i\sqrt{1 + (u_{\max}\left\|(L_{gi}V_i)^T\right\|)^2}. \tag{12}$$

From (11) and (12), $\dot{V} < 0$ is obtained. Therefore, the conclusion can be drawn that whenever the following inequality holds:

$$L_{fi}V_i < u_{\max}\left\|(L_{gi}V_i)^T\right\|, \tag{13}$$

the time-derivative of the Lyapunov function is negative-definite. Furthermore, we find that (13) is consistent with the definition of Φ_i . And Ω_i is the largest subset contained within Φ_i, that is to say any initial state $x(0) \in \Omega_i(u_{\max})$, the inequality (7) and $\dot{V} < 0$ hold for all times until the state is out of Ω_i and subsystem i switches to subsystem j . Then the i th subsystem of continuous-time PWL under the control law of (5) and (6) is asymptotically stable.

The switching of individual subsystem of PWL system (1) is dependent on which region the current state belongs to. Here, we assume $x(0) \in \Omega_i(u_{\max})$. From the above proof, we have $\dot{V} < 0$ as long as subsystem i is to remain active. If at time $t_{i_r}^{out}$ such that $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$ for some $j \in S$, $j \ne i$, the subsystem j is activated, then using the same argument, the corresponding Lyapunov function V_j will decay monotonically. So the constraint (2) of MLF stability is satisfied. Together with (10), we can conclude that the origin of the PWL closed-loop systems under the switching laws of theorem 1 is Lyapunov stable. In order to prove the global asymptotic stability of system (1), we note that the sequence $V_i(x(t_{i_1})), V_i(x(t_{i_2}))\cdots$ is decreasing and positive, and assume the limit of Lyapunov function corresponding to each time switching to i th subsystem is $L \ge 0$, i.e. $\lim_{r\to\infty} V_i(x(t_{i_r})) = L$. Therefore,

$$\lim_{r\to\infty} V_i(x(t_{i_{r+1}})) - \lim_{r\to\infty} V_i(x(t_{i_r})) = 0. \tag{14}$$

In (14), the part of $V_i(x(t_{i_{r+1}})) - V_i(x(t_{i_r}))$ is strictly negative for all nonzero x and zero only when $x = 0$. Furthermore, there exists a $\mathcal{K}$ -function α (i.e., continuous, increasing, and zero at zero) such that:

$$V_i(x(t_{i_{r+1}})) - V_i(x(t_{i_r})) \le -\alpha(\|x(t_{i_r})\|) \le 0 . \quad (15)$$

Together with (15), we have:

$$\lim_{r\to\infty}[-\alpha(\|x(t_{i_r})\|)] = 0 . \quad (16)$$

Therefore, the origin of the PWL closed-loop systems is asymptotically stable.

The bounded controller for continuous-time PWL described in Theorem 1 can guarantee closed-loop stability. However, the performance of the bounded controller is not guaranteed to be optimal. Therefore, we will propose an advanced control algorithm to improve the optimality in the following section.

4. MIXED CONTROL OF CONTINUOUS-TIME PWL SYSTEMS

Currently, MPC is one of the few control methods for handling state and input constraints within an optimal control setting and has been the subject of numerous research studies. Numerous research investigations into the stability properties of MPC have been presented (Mayne *et al.*, 2000). The present work for the stability of MPC for discrete-time PWL systems is to use terminal cost and constraint set approach, which considers the discrete-time PWL systems as the whole one instead of each subsystem. However, it is difficult to extend the method from discrete-time case to continuous-time case. According to above analysis, together with the fact that the performance of the bounded controller is absence of optimality, for continuous-time PWL systems, we propose a mixed control strategy for each subsystem integrating MPC with the bounded control to reconcile the stability and optimality properties of each other. With the switching scheme of different control strategies in each subsystem of PWL systems, the desirable properties are realized, and with the switching scheme of different initial stable regions, the global asymptotical stability is guaranteed.

4.1 MPC for Each Subsystem of PWL Systems

In this subsection, model predictive controller for each subsystem of system (1) is presented. For i th subsystem, MPC at state x and time t can be obtained by solving the following finite horizon optimal control problem on-line:

$$P(x,t): \min\{J(x,t,u_i)\} , \quad (17)$$

$$\text{s.t. } \dot{x} = A_i x + B_i u , \quad (18)$$

where

$$J(x,t,u_i) = \int_t^{t+T_i} [\|x^u(\omega;x,t)\|_Q^2 + \|u(\omega)\|_R^2] d\omega , \quad (19)$$

$x^u(\omega;x,t)$ denotes the solution of (18), due to control u with initial state x at time t, Q and R are strictly positive definite symmetric matrices. The minimizing control $u_i^0(\cdot)$ is then applied to the plant and an implicit MPC law

$$M_i(x) := u_i^o(t;x,t) . \quad (20)$$

4.2 Mixed Control Scheme with Stability Guarantee

For the continuous-time PWL systems (1), we formulate a control strategy merging MPC with bounded control in Theorem 2, which can not only guarantee asymptotic stability of the origin of the closed-loop system starting from any initial condition in Ω_i but also guarantee recovery the optimality when the stability criteria is met by MPC.

Theorem 2. Consider the PWL systems (1), with the initial condition $x(0) = x_o \in \Omega_i(u_{\max})$, where the stability region $\Omega_i(u_{\max})$ was defined under continuous implementation of the bounded controller (5), under the model predictive controller. Let t be such that $t_{i_r}^{in} \le t < t_{i_r}^{out}$ and $t_{j_m}^{in} = t_{i_r}^{out}$. Also let $T_{i_r}^s > 0$ be the earliest time for which the closed-loop state of r th time switching into i th subsystem under MPC satisfies:

$$-\|x(T_{i_r}^s)\|_{\bar{Q}}^2 + \|B_i^T P_i x(T_{i_r}^s)\|^2 + 2x^T(T_{i_r}^s) P_i B_i u_i(T_{i_r}^s) \ge 0 . \quad (21)$$

if at time $t_{j_m}^{in} = t_{i_r}^{out}$, (10) and the following conditions hold:

$$x(t_{i_r}^{out}) \in \Omega_j(u_{\max}) , \quad (22)$$

then the mixed controller in i th subsystem is

$$u_i(t) = \begin{cases} M_i(x(t)) & 0 \le t < T_{i_r}^s \\ b_i(x(t)) & T_{i_r}^s \le t < t_{i_r}^{out} \end{cases} . \quad (23)$$

The mixed control is applied to the PWL system and the procedure is repeated, then the origin of the PWL closed-loop system is guaranteed asymptotically stable.

Proof. The theorem 2 contains three cases:

(1) $T_{i_r}^s$ existed and $x(T_{i_r}^s) \notin \Omega_j(u_{\max})$, we implement MPC (20) at $0 \le t < T_{i_r}^s$ and bounded control (5) until $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$;

(2) no such $T_{i_r}^s$ existed, we implement MPC (20) until $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$;

(3) $T_{i_r}^s = 0$, we implement bounded control (5) until $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$.

For case 1, from the definition of $T_{i_r}^s$, it is clear that if $T_{i_r}^s$ existed, then $\dot{V}(x^M(t)) < 0, \forall 0 \le t < T_{i_r}^s$, where $x^M(t)$ denotes the closed-loop state under MPC at time t, which implies that $x(t) \in \Omega_i(u_{\max})$. This fact together with the continuity of the subsystem implies $x(T_{i_r}^s) \in \Omega_i(u_{\max})$ and $u_i(t) = b_i(x(t))$ after $T_{i_r}^s$. If $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$ at time $t_{i_r}^{out}$, the states switch to subsystem j. From above analysis, there is the conclusion that $\dot{V}_i < 0$, $\forall T_{i_r}^s \le t \le t_{i_r}^{out}$, under bounded control by the proof of theorem 1. In summary, starting from any $x(0) \in \Omega_i(u_{\max})$, we have $\dot{V}_i < 0$, $\forall 0 \le t \le t_{i_r}^{out}$. When the subsystem j is activated at $t \ge t_{i_r}^{out} = t_{j_m}^{in}$, we can get the same analysis as subsystem i. With the constraint (10) and the proof of theorem 1, therefore, the closed-loop PWL system is asymptotically stable.

For case 2, because no such $T_{i_r}^s$ existed, MPC is implemented for subsystem i until $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$, which implies $\dot{V}_i(x^M(t)) < 0, \forall 0 \le t \le t_{i_r}^{out}$. Just as the case Ⅰ, the closed-loop PWL system is also asymptotically stable.

For case 3, using the proof theorem 1, the conclusion of theorem 2 can be proved again. Then the proof of the theorem 2 is completed.

Remark 1 In Theorem 2, the classical MPC formulation is used to implement mixed control strategy. In fact, it is not restricted to classical MPC formulation. The structure can be extended to more advanced version of MPC by appropriate design of the switching algorithm.

Remark 2. In the control strategy of Theorem 2, we need only compute quadratic programming problem for each subsystem instead of MIQP problem for the whole PWL systems. In order to guarantee stability, we need only design initial stable region for each subsystem instead of terminal invariant set for the whole PWL system. The underlying idea of this method realizes the decoupling the stability requirement from optimality. When bounded control is active, the well-defined stability region can safeguard against potential closed-loop instability arising from MPC; when MPC is active, the desired optimal performance under constraint can be satisfied.

The mixed control strategy of Theorem 2 is shown in Fig. 1, where for each subsystem i, controller i, $i \in S$ denotes the mixed controller (23) in Theorem 2.

The following algorithm explains the implementation of the control algorithm of Theorem 2.

Step 1: Given the systems (1) and the constraints on inputs, design the bounded controller for each subsystem and compute the stability region $\Omega_i(u_{\max})$. Given the performance objective and a choice of the horizon length, design the MPC controller.

Step 2: Initialize the systems (1), using the MPC controller at any initial condition $x(0) = x_o \in \Omega_i(u_{\max})$.

Step 3: Monitor the evolution of the closed-loop trajectory by checking the condition (21), until the earliest time $T_{i_r}^s$ is met.

If $x(T_{i_r}^s) \notin \Omega_j(u_{\max})$, switch to bounded control (5) until $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$;

If no such $T_{i_r}^s$ existed, only MPC (20) is implemented for the r th time of the i th subsystem, until $x(t_{i_r}^{out}) \in \Omega_j(u_{\max})$;

If $T_{i_r}^s = 0$, implement the bounded control (5) instead.

Step 4: Consider the switching constraint (10), which requires that when the closed-loop system enters the subsystem j, the value of V_j is less than that the system last entered subsystem j. If the system has never entered subsystem j, set $V_j(x(t_{j_{m-1}}^{in})) = c_{j\max}$.

Step 5: Check, off-line whether the constrained optimization in switching constraint (10) yields a feasible solution. If it does not yield an initial feasible solution in subsystem j, go to next step; else go back to step 3.

Step 6: Implement the bounded controller in the closed-loop, continue to check, off-line, and as frequently as desired, the feasibility the MPC optimization. At the earliest time that a feasible solution is found, switch to MPC, else the bounded controller remains active.

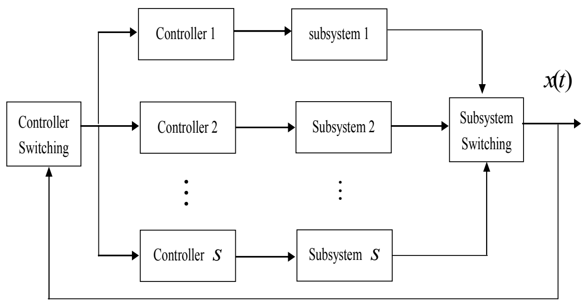

Fig. 1. Structure of control strategy of Theorem 2

5. SIMULATIONS

A 2-tank system (Imura and van der Schaft, 1999) can be described as the following continuous-time PWL systems, where x_i, $i = 1,2$ is the deviation of the water level from the equilibrium state, u_i is the volume of water discharged from the tap i. The logic rule is determined by whether the valve is open or closed. In this section, the presented control strategy of theorem 2 is illustrated by means of this example whose concrete form and parameters are:

$$\dot{x} = \begin{cases} A_1 x + Bu & \begin{bmatrix} 0 & 1 \end{bmatrix} x \le 0 \\ A_2 x + Bu & \begin{bmatrix} 0 & 1 \end{bmatrix} x \ge 0 \end{cases}, \quad (24)$$

where $A_1 = \begin{pmatrix} -1 & -0.2 \\ 0.2 & -1 \end{pmatrix}$, $A_2 = \begin{pmatrix} -1 & 0.2 \\ -0.2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $-4 \le u \le 4$.

For both two subsystems, we consider quadratic Lyapunov functions of the form $x^T P_i x$, with $P_1 = \begin{pmatrix} 1.5973 & -0.0930 \\ -0.0930 & 0.4243 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1.7186 & 0.6230 \\ 0.6230 & 2.3869 \end{pmatrix}$, which can be computed by solving Riccatic equality. The initial stable regions for two subsystems are Ω_1 and Ω_2, shown in Fig. 2. The parameters in the objective function of MPC are chosen as: $Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, R = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. The resulting quadratic program is solved using the MATLABA subroutine QudaProg, and the set of ODE is integrated using the MATLAB solver ODE45. As shown by the dotted curve in Fig. 2, applying the MPC controller from the initial condition $x_0 = \begin{bmatrix} 2 & -1 \end{bmatrix} \in \Omega_1$, the region switching happens until $t_r = 0.74$ s. After this instant MPC is implemented in Ω_2, however, we find that an increase in V_2 at $t_c = 1.25$ s and therefore control switching happens from the MPC controller to the bounded controller in order to preserve closed-loop stability. If only the MPC controller is implemented when the system switches to subsystem 2, the state trajectory can not yield asymptotically stability (dashed line as in Fig. 2). This happens because the Lyapunov function V_2 is not non-increasing in Ω_2.

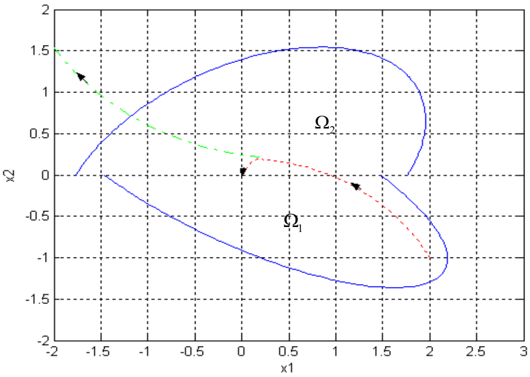

Fig. 2. The closed-loop state trajectory when control strategy of theorem 2 is implemented (dotted line), when only MPC formulation is implemented in Ω_2 (dashed line).

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