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$\mathscr{H}_2/\mathscr{H}_\infty$ Multiobjectives for Fault Detection in Uncertain Polytopic Systems ⋆

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Abstract: This paper presents a novel technique for robust fault detection, based on a modified $\mathscr{H}_2/\mathscr{H}_\infty$ performance condition, which is described as LMI. Some theoretical results are shown in order to synthesize the residual generation scheme, for systems subjected to parametric uncertainty. The uncertainty parameters are supposed to belong to a polytope. The extended $\mathscr{H}_2/\mathscr{H}_\infty$ conditions are obtained by means of the well known projection lemma. Fault detection and isolation are done by using a filters bank (i.e. multifiltering) based on Luenberger's observer and one filter is obtained for each fault. Performance of the proposed synthesis technique is illustrated by a numerical example.

1. INTRODUCTION

Fault detection and isolation (FDI) systems, have been an active research field from some decades ago. This fact is enforced by the necessity of safer operation conditions, which must be guaranteed for any dynamical system. Basically a FDI system is composed by three main parts: a) A residual generation scheme, b) A residual evaluation component and c) A decision mechanism. The most critical step in a fault detection procedure, the residual generation, provides the basis for the decision taken for any other mechanism.

Robust fault detection problem, is closely related with robust filtering problem. In both cases a state estimation is used from control and output signals, and this estimate is used for producing fault residuals in any way. In recent years, a lot of research have been dedicated to robust filtering for polytopic systems (see Duan et al. (2006) and references therein), and there are some recent works treating fault diagnosis using this framework as Mazars et al. (2007).

A popular approach for residual generation is based on Luenberguer type observers proposed by Beard and Jones, which provides a useful framework for LTI systems. The main drawback with this approach is the determinism, which can not handle any perturbation or uncertainty in the used model. However, some modifications to the original observer may be done, in such a way that new robustness and performance conditions may be satisfied. Some approaches have been presented by Frisk and Nielsen (2006); Rodrigues et al. (2005) to overcome the robustness problems in residual generation, as well as Khosrowjerdi et al. (2004) devoted to $\mathscr{H}_2/\mathscr{H}_\infty$ fault detection. Some recent papers have tackled the problem of uncertainty systems, based on similar techniques to the ones presented in this paper Casavola et al. (2007); Weng et al. (2007). In Bokor and Balas (2004) a formal review and solution for linear parameter-varying (LPV) systems is presented.

This paper presents an approach based on convex optimization, using an affine representation by LMIs of polytopic systems. In other words, the design of a FDI filter which can detects any fault, preserving some $\mathscr{H}_2/\mathscr{H}_\infty$ performance, irrespective of system parameters uncertainty.

The rest of the paper is organized as follows. Section 2 presents some preliminary results of the filter design, providing extended versions for the $\mathscr{H}_2$ and $\mathscr{H}_\infty$ LMI conditions. Section 3 presents the solution for synthesis of fault diagnosis systems. Finally a numerical example and some concluding remarks are presented.

2. PRELIMINARIES

In this section some preliminary facts are given, in order to determine additional performance conditions for linear systems. Consider the following continuous time linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1}$$

where, $x(t) \in \mathbb{R}^n$ are states, $u(t) \in \mathbb{R}^m$ are control signals and $y(t) \in \mathbb{R}^p$ are measurements. Matrices A, B, C, D are well known and with proper dimensions.

Additionally, exist some modifications to classical results of robust control theory like improved versions to Bounded Real Lemma Shaked (2001); He et al. (2005), or for $\mathscr{H}_2$ performance Apkarian et al. (2001). Next, some of these approaches are presented as a basis for developments which will be presented later in this paper.

Lemma 1. (Extended $\mathscr{H}_2$ performance). Consider system (1) with $D = 0$. The following statements, with $P = P^T > 0$ are equivalent

i) A is stable and $\left\|C(sI - A)^{-1}B\right\|_2^2 < \mu$.

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ii) There exist P and Z, such that

$$\begin{bmatrix} A^T P + PA & PB \\ B^T P & -\mu I \end{bmatrix} < 0, \quad \begin{bmatrix} P & C^T \\ C & Z \end{bmatrix} > 0, \qquad (2)$$

$\text{tr}(Z) < 1$

iii) There exist P, Z and G such that

$$\begin{bmatrix} -(G+G^T) & G^T A + P & G^T B & G^T \\ A^T G + P & -P & 0 & 0 \\ B^T G & 0 & -\mu I & 0 \\ G & 0 & 0 & -P \end{bmatrix} < 0, \qquad (3)$$

$$\begin{bmatrix} P & C^T \\ C & Z \end{bmatrix} > 0, \quad \text{tr}(Z) < 1 \qquad (4)$$

iv) There exist P, Z and G, such that

$$\begin{bmatrix} -(G+G^T) & G^T A + P + G^T & G^T B \\ A^T G + P + G & -2P & 0 \\ B^T G & 0 & -\mu I \end{bmatrix} < 0, \qquad (5)$$

$$\begin{bmatrix} P & C^T \\ C & Z \end{bmatrix} > 0, \quad \text{tr}(Z) < 1 \qquad (6)$$

Proof. The equivalence between the three first statements has been shown in theorem 3.3 of Apkarian et al. (2001) and the equivalence between 2 and 4 is shown in Wei (2003).

Remark 2. It is known that more conservative results are obtained (for example in the case of polytopic uncertainty), when there exist relationships between system matrix and the Lyapunov matrix Oliveira et al. (1999). This result solves that problem, decoupling the Lyapunov matrix and the system dynamic matrix. Additionally, statement 4 in Lemma 1, provides a smaller representation of the $\mathscr{H}_2$ performance condition given by Apkarian et al. (2001). Further details on this lemma can be found in Wei (2003).

As in the case of $\mathscr{H}_2$ performance condition given before, there are some attempts for the improvement of $\mathscr{H}_\infty$ performance, next one of them is shown.

Lemma 3. (Extended $\mathscr{H}_\infty$ performance). Consider system (1). The following statements, with $P = P^T > 0$ and matrix G are equivalent

i) A is stable and $\left\| C(sI - A)^{-1}B + D \right\|_\infty < \gamma$.

ii) There exist P, such that

$$\begin{bmatrix} A^T P + PA & PB & C^T \\ B^T P & -\gamma^2 I & D^T \\ C & D & -I \end{bmatrix} < 0. \qquad (7)$$

iii) There exist P and G such that, for $\tau \gg 1$

$$\begin{bmatrix} -(G+G^T) & G^T A + P + \tau G^T & 0 & G^T B \\ A^T G + P + \tau G & -2\tau P & C^T & 0 \\ 0 & C & -I & D \\ B^T G & 0 & D^T & -\gamma^2 I \end{bmatrix} < 0. \qquad (8)$$

Proof. Conditions 1 and 2 are the well known *Bounded Real Lemma*. Equivalence between 2 and 3 can be seen in Wei (2003).

Next section provides some conditions for fault diagnosis and additionally gives a synthesis method to obtain the FDI filters.

3. ROBUST FAULT DIAGNOSIS FILTERS (RFDI)

As it is well known, fault diagnosis systems are a very important research topic in applied control theory. The most common way for residual generation consists of a Luenberger based observer, which produces an estimate of the system states. In the case of fault presence, these estimated states will be different from the real ones, and the estimation dynamics will not tend asymptotically to zero. This difference constitutes the fault residual. All this is true, in the case of a perfect knowledge of the system and a perturbation free scenario. When any type of noise exists in a well known system, it is necessary to consider a bound for the residuals from which it is considered that the fault exists. However, this is another hypothetical framework for systems where the parameters change quickly, or there are unknown parameters, the fault diagnosis problem still remains open. Then, an extension to the classical Luenberger approach is presented, in order to overcome some of the problems just mentioned by means of a polytopic representation of the system.

Consider the following linear continuous time system

$$\begin{aligned} \dot{x}(t) &= A(\alpha)x(t) + B_1(\alpha)w(t) + B_2(\alpha)u(t) \\ z(t) &= C_1(\alpha)x(t) \\ y(t) &= C_2(\alpha)x(t), \end{aligned} \qquad (9)$$

where $x(t) \in \mathbb{R}^n$ are states, $w(t) \in \mathbb{R}^q$ are unknown perturbations, $u(t) \in \mathbb{R}^m$ are control inputs, $y(t) \in \mathbb{R}^p$ are measurements and $z(t) \in \mathbb{R}^s$ are controlled outputs. Additionally, the system matrices belong to a convex polytopic set defined as

$$\Omega = \Big\{ \big(A(\alpha), B_1(\alpha), B_2(\alpha), C_1(\alpha), C_2(\alpha)\big) \Big| \big(A(\alpha), B_1(\alpha), B_2(\alpha), C_1(\alpha), C_2(\alpha)\big) = \sum_{i=1}^{N} \alpha_i \Big(A^{(i)}, B_1^{(i)}, B_2^{(i)}, C_1^{(i)}, C_2^{(i)}\Big), \quad \alpha_i \geq 0, \quad \sum_{i=1}^{N} \alpha_i = 1 \Big\}. \qquad (10)$$

Now, for making fault diagnosis, adding another term to (9), the fault diagnosis model is given by

$$\begin{aligned} \dot{x}(t) &= A(\alpha)x(t) + B_1(\alpha)w(t) + B_2(\alpha)u(t) + \sum_{k=1}^{M} L_k \nu_k(t) \\ z(t) &= C_1(\alpha)x(t) \\ y(t) &= C_2(\alpha)x(t), \end{aligned} \qquad (11)$$

where $\nu_k \in \mathbb{R}^{n_f}$ corresponds to *fault modes*, and L_k are called *fault signatures*, which are assumed known.

From the classical approach for fault detection, it can be done by a Luenberger's observer, given by

$$\begin{aligned} \dot{\hat{x}}(t) &= A^{(i)}\hat{x}(t) + B_2^{(i)}u(t) + \mathcal{D}\big(y(t) - C_2^{(i)}\hat{x}(t)\big) \\ \hat{z}(t) &= C_1^{(i)}\hat{x}(t), \end{aligned} \qquad (12)$$

where, $\hat{x}(t)$ y $\hat{z}(t)$ represent the estimated states and controlled outputs respectively, both with appropriate dimensions, and $\mathcal{D}$ is the estimator gain which must be designed. Defining an estimation error as

$$e(t) = x(t) - \hat{x}(t), \qquad (13)$$

thus, the estimation dynamic is given by

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t)\\ &= \big(A^{(i)} - \mathcal{D}C_2^{(i)}\big)e(t) + B_1^{(i)}w(t) + \sum_{k=1}^{M} L_k\nu_k(t).\end{aligned} \tag{14}$$

Additionally, the output prediction error reads

$$e_z(t) = C_1^{(i)}x(t) - C_1^{(i)}\hat{x}(t) = C_1^{(i)}e(t). \tag{15}$$

Before of continue, some conditions for fault detection existence must be given.

3.1 RFDI conditions

For a successful fault diagnosis, some conditions must be satisfied. Those conditions are well known in the case of LTI systems (see Massoumnia (1986) and references therein), now an extension will be given for polytopic uncertainty systems.

Consider the system

$$\begin{aligned}\dot{x}(t) &= A(\alpha)x(t) + B(\alpha)u(t) + \sum_{i=1}^{M} L_i\nu_i(t)\\ y(t) &= C(\alpha)x(t) + D(\alpha)u(t),\end{aligned} \tag{16}$$

the system matrices belong to a convex polytopic set defined as

$$\begin{aligned}\Omega = \Bigg\{ &\big(A(\alpha), B(\alpha), C(\alpha), D(\alpha)\big) \Big| \big(A(\alpha), B(\alpha), C(\alpha), D(\alpha)\big)\\ = &\sum_{i=1}^{N} \alpha_i \Big(A^{(i)}, B^{(i)}, C^{(i)}, D^{(i)}\Big), \alpha_i \geq 0,\ \sum_{i=1}^{N} \alpha_i = 1 \Bigg\}.\end{aligned} \tag{17}$$

Detectability condition: Detectability condition, is based on the fact that faults are not in the unobservable subspace given by

$$\mathbb{U}_{os} := \bigcup_{i=1}^{N} \left[\bigcap_{k=0}^{n-1} \ker\big(C^{(i)}A^{(i)k}\big) \right]. \tag{18}$$

Taking this in consideration, detectability condition may be defined.

Theorem 4. Let $\mathbb{W}_{L_i} = \mathrm{Im}(L_i), i = 1, \ldots, M$ be the fault signature maps. For system (16), the i-th fault is detectable if

$$\mathbb{W}_{L_i} \bigcap \mathbb{U}_{os} = \{0\}, \quad i = 1, \ldots, M. \tag{19}$$

Proof. The proof is an extension of the results in Massoumnia (1986); Ríos-Bolívar (2001). If $\mathcal{O}(A^{(i)}, B^{(i)})$ is the observability matrix of the system (16), such as is defined in Angelis (2001), then $\mathbb{R}^n = \mathrm{Im}(\mathcal{O}) \oplus \mathbb{U}_{os}$. Thus, $\mathrm{Im}(L_i) = \mathrm{Im}(\mathcal{O}L_i) \oplus \ker\big(\mathcal{O}L_i\big)$. If condition (19) is satisfied, then $\mathbb{W}_{L_i} = \mathrm{Im}(\mathcal{O}L_i)$, which allows the fault propagation on the output subspaces.

Separability condition: Separability condition, is based on the fact that the mapped subspaces by each fault over the system output must be independent. Next result summarizes this idea.

Theorem 5. Let $\mathbb{W}_{L_i} = \mathrm{Im}(L_i)$, $i = 1, \ldots, k$ be the fault signature maps. Consider the diagnosis model defined by (16) and let $\mathcal{O}(A^{(i)T}, B^{(i)T})$ be the observability matrix Angelis (2001). Faults are separable if each one of the fault signature on the estimation output are isolated, i.e.,

$$\mathcal{O}\mathbb{W}_{L_i} \bigcap \sum_{i \neq j}^{M} \mathcal{O}\mathbb{W}_{L_j} = \{0\}, \quad i, j = 1, \ldots, k. \tag{20}$$

Proof. In the same way for the detectability condition, the demonstration is an extension of the results in Massoumnia (1986); Ríos-Bolívar (2001). The separability condition establishes that the fault signatures are mapped to sub-spaces of the observability space, which are discernible in the output space. Thus, the faults can be assigned to some particular address on the outputs. From this condition, it is possible to establish a performance index for each fault signatures, then multiobjective performance indexes can be obtained.

An additional subject must be treated now, in order to separate residuals produced by each fault. In general, if only one FDI filter is used, all the residual information will be contained in its single output, and with the purpose of identify or separate each individual fault residual, some conditions must be imposed in the design process, for example, geometrical conditions. Here, a filters bank scheme is used, providing a more flexible design process.

3.2 Multifiltering scheme

For fault isolation, a slight modification to this design must be done producing a multifiltering scheme used by Ríos-Bolívar and Acuña (2007), in such a way that fault separation can be obtained, and the effect of exogenous noise can be attenuated.

Using the multifiltering approach, each filter is designed in order to satisfy different requisites, producing a residual signal isolated from the other ones. From (12), the filters bank is given by

$$\begin{aligned}\dot{\hat{x}}_j(t) &= A^{(i)}\hat{x}_j(t) + \mathcal{B}_j^{(i)}u(t) + \mathcal{D}_j\big(y(t) - C_2^{(i)}\hat{x}_j(t)\big)\\ \hat{z}_j(t) &= C_1^{(i)}\hat{x}_j(t),\end{aligned} \tag{21}$$

where, $\hat{x}_j(t)$ and $\hat{z}_j(t)$ represent the estimated states and controlled outputs by the j-th filter, and $\mathcal{D}_j$ is the j-th filter gain. Thus, the estimation dynamic is given by

$$\left.\begin{aligned}\dot{e}_j(t) &= \mathcal{A}_j e_j(t) + \mathcal{B}_j^{(i)}\tilde{w}_j(t) + L_j\nu_j(t)\\ e_{zj} &= C_1^{(i)}e_j(t),\end{aligned}\right. \quad j = 1, \ldots, M, \tag{22}$$

where,

$$\mathcal{A}_j^{(i)} = A^{(i)} - \mathcal{D}_jC_2^{(i)}, \tag{23}$$

$$\mathcal{B}_j^{(i)} = \Big[B_1^{(i)}\ L_{j0}\Big], \tag{24}$$

$$\tilde{w}_j(t) = \big[w(t)^T\ \nu_{j0}^T(t)\big]^T. \tag{25}$$

$\nu_j(t)$ and L_j are the fault modes and signatures, for the j-th filter (the currently designed). The rest of fault modes and signatures, i.e., $\nu_{j0}(t)$ and L_{j0}, are included inside the extended perturbations vector $\tilde{w}_j(t) \in \mathbb{R}^{n_f+q-1}$, and map $\mathcal{B}_j^{(i)}$ respectively. This is made in order to minimize the effect of other faults, together with the external perturbations in the same optimization process.

The solution for the design problem is provided in terms of LMIs, the main result is given next.

Theorem 6. Consider system (11) on the polytope (10). A FDI filter for the j-th fault of the form (21), which guarantees a suboptimal $\mathscr{H}_2$ performance for (22), i.e., $\left\|C_1^{(i)}\left(sI-\mathcal{A}\right)^{-1}\mathcal{B}_j^{(i)}\right\|_2^2$, can be obtained from the following optimization problem:

$$\min_{\substack{G_j,\,Q_j,\,Z_j,\,P_j^{(i)}\\ i=1,\ldots,N,\,j=1,\ldots,M.}} \quad \operatorname{tr}(Z_j)$$

s.t.

$$\begin{bmatrix} -G_j-G_j^T & G_j^TA_j^{(i)}-Q_jC_2^{(i)}+P_j^{(i)}+G_j^T & G_j^T\mathcal{B}_j^{(i)T}\\ \star & -2P_j^{(i)} & 0\\ \star & \star & -I\end{bmatrix}<0 \tag{26}$$

$$\begin{bmatrix} P_j^{(i)} & \star\\ C_1^{(i)} & Z_j\end{bmatrix}>0 \tag{27}$$

for $i=1,\ldots,N, j=1,\ldots,M$, where $G_j\in\mathbb{R}^{n\times n}, Q_j\in\mathbb{R}^{n\times p}, Z_j\in\mathbb{R}^{s\times s}$, and $P_j^{(i)}=P_j^{(i)T}>0\in\mathbb{R}^{n\times n}$. Thus, the estimator gain is given by $\mathcal{D}_j=(G_j^T)^{-1}Q_j$.

Proof. By variable change $Q_j=G_j^T\mathcal{D}_j$, (26) can be rewritten as

$$\begin{bmatrix} -G_j-G_j^T & G_j^T\left(A_j^{(i)}-\mathcal{D}_jC_2^{(i)}\right)+P_j^{(i)}+G_j^T & G_j^T\mathcal{B}_j^{(i)T}\\ \star & -2P_j^{(i)} & 0\\ \star & \star & -I\end{bmatrix}<0,$$

and by Lemma 1, the conclusion follows.

Remark 7. Note that, in this optimization problem, a parameter dependent Lyapunov function $P(\alpha)=\sum_{i=1}^N\alpha_iP_i$ is used, just as was proposed by Oliveira et al. (1999) and has been recently used in some papers in robust filtering. This fact appears as an alternative to the conservative case of a fixed Lyapunov matrix, i.e., $P^{(i)}=P$. In this way, a Lyapunov function is obtained for each vertex in the polytope, without forcing only one Lyapunov matrix for all the system.

4. FDI DESIGN: A $\mathscr{H}_2/\mathscr{H}_\infty$ APPROACH

This section is devoted to provide a synthesis procedure for FDI filters. In this case, an extended alternative for the result given in the last section is done, considering measurements contaminated by noise and additionally the requisite of independence between faults and perturbations is avoided.

Consider the diagnosis model given by

$$\begin{aligned}\dot{x}(t)&=A(\alpha)x(t)+B_1(\alpha)w(t)+B_2(\alpha)u(t)+\sum_{k=1}^{M}L_k\nu_k(t)\\ z(t)&=C_1(\alpha)x(t)\\ y(t)&=C_2(\alpha)x(t)+D(\alpha)w(t),\end{aligned} \tag{28}$$

which belongs to the polytope

$$\Omega=\Big\{\big(A(\alpha),B_1(\alpha),B_2(\alpha),C_1(\alpha),C_2(\alpha),D(\alpha)\big)\Big|\big(A(\alpha),B_1(\alpha),B_2(\alpha),C_1(\alpha),C_2(\alpha),D(\alpha)\big)=\sum_{i=1}^N\alpha_i\Big(A^{(i)},B_1^{(i)},B_2^{(i)},C_1^{(i)},C_2^{(i)},D^{(i)}\Big),\alpha_i\geq 0,\quad \sum_{i=1}^N\alpha_i=1\Big\}. \tag{29}$$

Now, let the system (21) be, a state estimators bank used for fault detection. The estimation dynamic is given by

$$\begin{aligned}\dot{e}_j(t)&=\mathcal{A}_je_j(t)+\mathcal{B}_j^{(i)}w(t)+\sum_{k=1}^{M-1}L_{k,j}\nu_{k,j}(t)+L_j\nu_j(t)\\ e_{zj}&=C_1^{(i)}e_j(t),\quad j=1,\ldots,M,\end{aligned} \tag{30}$$

where,

$$\mathcal{A}_j^{(i)}=A^{(i)}-\mathcal{D}_jC_2^{(i)}, \tag{31}$$

$$\mathcal{B}_j^{(i)}=B_1^{(i)}-\mathcal{D}_jD^{(i)}, \tag{32}$$

$\nu_j(t)$ are fault modes associated to the currently designed filter, and $\nu_k(t)$ are the rest of fault modes. Same indications for L_j, L_k.

In order to obtain fault separation and noise rejection at the same time, a multiobjective approach is proposed defining two channels:

1. Fault separation is done by minimizing ∞-norm of system $H_{\nu_{k,j}\to e_{zj}}(s)=C_1^{(i)}\left(sI-\mathcal{A}_j^{(i)}\right)^{-1}L_{k,j}$, i.e., $\left\|H_{\nu_{k,j}\to e_{zj}}(s)\right\|_\infty<\gamma_j$.
2. Noise rejection is done by minimizing 2-norm of system $H_{w\to e_{zj}}(s)=C_1^{(i)}\left(sI-\mathcal{A}_j^{(i)}\right)^{-1}\mathcal{B}_j$, i.e., $\left\|H_{w\to e_{zj}}(s)\right\|_2<\mu_j$.

Taking this in consideration, next result summarizes the solution for multiobjective FDI filters bank synthesis.

Theorem 8. Consider system (28) on the polytope (29). A FDI filter for the j-th fault of the form (21), which guarantees a suboptimal $\mathscr{H}_2/\mathscr{H}_\infty$ performance for (30), satisfying the former conditions, can be obtained from the following optimization problem:

$$\min_{\substack{G_j,\,K_j,\,Z_j,\,P_{2,j}^{(i)},\,P_{\infty,j}^{(i)},\,\gamma_j^2\\ i=1,\ldots,N,\,j=1,\ldots,M.}} \quad \operatorname{tr}(Z_j)+\gamma_j^2$$

s.t.

$$\begin{bmatrix} -G_j-G_j^T & \Psi_{2,j}^{(i)} & G_j^TB_1^{(i)}-K_jD^{(i)}\\ \star & -2P_{2,j}^{(i)} & 0\\ \star & \star & -I\end{bmatrix}<0 \tag{33}$$

$$\begin{bmatrix} P_{2,j}^{(i)} & \star\\ C_1^{(i)} & Z_j\end{bmatrix}>0 \tag{34}$$

$$\begin{bmatrix} -G_j-G_j^T & \Psi_{\infty,j}^{(i)} & 0 & G_j^TL_{k,j}\\ \star & -2\tau_jP_{\infty,j}^{(i)} & C_1^{(i)T} & 0\\ \star & \star & -I & 0\\ \star & \star & \star & -\gamma_j^2I\end{bmatrix}<0, \tag{35}$$

where

$$\Psi_{2,j} = G_j^T A - K_j C_2 + P_{2,j}^{(i)} + G_j^T$$
$$\Psi_{\infty,j} = G_j^T A - K_j C_2 + P_{\infty,j}^{(i)} + \tau_j G_j^T,$$

for $i = 1, \ldots, N, j = 1, \ldots, M$, where $G_j \in \mathbb{R}^{n \times n}, K_j \in \mathbb{R}^{n \times p}, Z_j \in \mathbb{R}^{s \times s}$, $P_j^{(i)} = P_j^{(i)T} > 0 \in \mathbb{R}^{n \times n}$ and $\tau_j >> 1$. The estimator gain is given by $\mathcal{D}_j = (G_j^T)^{-1} K_j$.

Proof. The proof is based on variable changes and application of lemmas 1 and 3.

Remark 9. In classical multiobjective approaches, expressions like $A^T P + PA$ involve products between Lyapunov matrices and controller design matrices, in order to guarantee problem convexity it is necessary to make a fundamental assumption, enforcing all the specifications by only one Lyapunov function. In this case, due to the used extended versions of $\mathscr{H}_2/\mathscr{H}_\infty$, there are not any product involving Lyapunov matrices, and the design matrix K_j does not depend on this last one, avoiding the necessity of using the same Lyapunov matrix for all the specifications. Indeed, this fact is very important because not only there is a parameter dependent Lyapunov matrix for each filter, but there is a Lyapunov function for each performance specification as well.

5. NUMERICAL EXAMPLE

In order to illustrate the results provided before, a numerical example is shown. Consider the mass-spring system given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1}{m_1} & \dfrac{k_1}{m_1} & -\dfrac{b_1}{m_1} & \dfrac{b_1}{m_1} \\ \dfrac{k_1}{m_2} & -\dfrac{(k_1+k_2)}{m_2} & \dfrac{b_1}{m_2} & -\dfrac{(b_1+b_2)}{m_2} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.5 & 0 \\ 1 & 0 \\ 0.1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{m_1} & 0 \\ 0 & \dfrac{1}{m_2} \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \\ 0 & 0.1 \\ 0 & 0 \end{bmatrix},$$

$$C_1 = C_2 = I_{4\times 4},$$

with constants $m_2 = 2, b_1 = 0.2, b_2 = 0.1$. Parameters m_1, k_1 and k_2 are defined as $m_1 \in [0.9, 1.1], k_1 \in [0.9, 1.1]$ and $k_2 \in [3.6, 4.4]$. As can be seen, the system does not have an affine representation, then some variable changes must be carried out in order to overcome this problem.

Fault signatures are given by the map

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^T.$$

Filter gains, are obtained by Theorem 8.

$$\mathcal{D}_1 = \begin{bmatrix} 406.5438 & 146.0345 & -406.4529 & 15.7237 \\ 239.505\times10^{3} & 107.628\times10^{6} & -239.505\times10^{3} & -15.563\times10^{3} \\ 75.9693 & 28.3565 & -75.8568 & 4.3013 \\ 909.6072\times10^{-3} & -2.6537 & -908.4968\times10^{-3} & 8.7552 \end{bmatrix}$$

$$\mathcal{D}_2 = \begin{bmatrix} 279.1337\times10^{6} & 50.3397\times10^{6} & -279.1337\times10^{6} & 12.1540\times10^{6} \\ 168.4785 & 122.9829 & -168.5147 & 12.1007 \\ 153.1174 & 111.3985 & -153.0851 & 12.5306 \\ 1.4556 & -1.3987 & -1.4565 & 8.7360 \end{bmatrix}.$$

It is worth notting that here two objectives were optimized at one time, that is, two different performance measurements were taken into account. By one hand, a $\mathscr{H}_2$ performance was associated to noise signals, this with the aim of detect fault presence. On the other hand, in order to obtain fault isolation a $\mathscr{H}_\infty$ measure was minimized, providing a separation scheme defined by constant bounds.

Simulation results are obtained using MATLAB and SEDUMI solver, written by Sturm (1999). Simulation is distributed in 16 stages, each one of 200 s; each stage shows a different system representing parameter changes using each polytope vertex. In the same way, in each stage, different fault conditions are represented. The purpose of this configuration is to simulate a parameter changing system and verify if the FDI filters produce isolated residuals for each fault mode. Fault modes are produced emulating some common faulty scenarios: intermittent, abrupt and incipient faults (sawtooth and square signals).

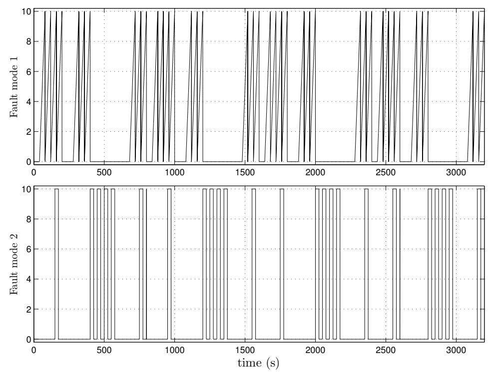

Fig. 1. Fault modes.

The Fig. 1 shows the fault modes. Fault mode 1 appears at 40 s, 280 s and 680 s , as long as, fault mode 2 appears at 150 s, 400 s and 750 s (same sequences each 800 s).

The Fig. 2 shows the system outputs. As can be seen, all the information about fault occurrence is contained in this signal, and must be separated by any procedure.

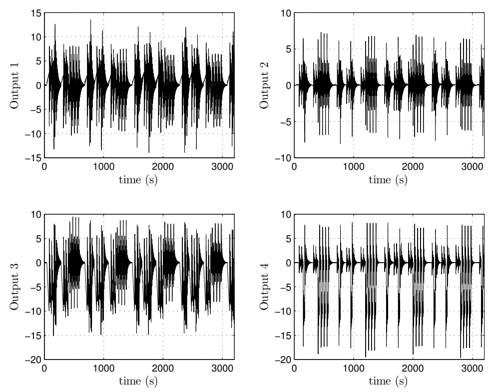

Fig. 2. System outputs.

The Fig. 3 shows the generated residuals. The upper part shows the residuals produced by filter 1, representing fault mode 1. As can be seen, only in the cases where the fault mode appears, a residual signal is generated, providing necessary information for next stages of fault detection, as a residual evaluator or a decision logical system.

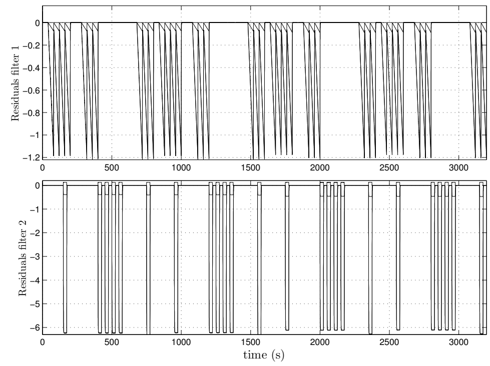

Fig. 3. Residuals produced by FDI filters.

Same result can be observed in the lower part of Fig. 3, where residuals generated by filter 2 are shown. As in the first case, residual signals only appear when the associated fault to this filter is present.

It is worth noting that in each simulation stage, a different system is used, provided by the operation limits of parameters k_1, k_2 and m_1. In this simulation each 200 s a different system is affected by any fault, generating the corresponding residual signal for each one of them.

6. CONCLUDING REMARKS

In this paper, a FDI system for polytopic uncertainty systems has been presented. Some theoretical results for fault diagnosis and for FDI filters design have been given. All the design results are provided by means of LMI, which ensures fast solutions for the problem, and possible extensions in several directions, for example, a multiobjective framework.

On the other hand, some conditions in terms of detectability and separability for fault diagnosis of polytopic systems, have been done, as an extension for the LTI case.

Fault separation was made by a multifiltering scheme, where one filter for each fault was designed, taking the faults as perturbations and minimizing a $\mathscr{H}_2/\mathscr{H}_\infty$ performance measure.

An important issue shown through this paper, is the fact of conservativeness reduction by means of alternative versions of classical results on robust control, as well as parameter dependent Lyapunov functions. This fact is very important because the smallest conservativeness is, the best fault separation or noise attenuation can be obtained.

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