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DECENTRALIZED ROBUST CONTROL OF LARGE-SCALE TIME-DELAY SYSTEMS

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Abstract: Decentralized robust controller design problem for large-scale interconnected systems which involve uncertainties in the system matrices and uncertain time-delays is considered. An error bound, which accounts for the neglected interactions between the subsystem models and uncertainties in the interactions, in the subsystem models, and in the time delays is first derived using overlapping decompositions and expansions. A decentralized controller design approach, which uses this bound, is then proposed. The advantage of the proposed approach is that, all uncertainties and neglected dynamics are summarized in one frequency-dependent scalar function and satisfying a simple condition guarantees that the overall closed-loop system under decentralized controllers, which are designed considering local models, is robustly stable. The application of the proposed approach is demonstrated on a flow control problem in a data-communication network.

1. INTRODUCTION

Many physical systems, especially large-scale systems, involve time-delays. The main difficulty with the time-delay systems is that they are infinite-dimensional. A review of various controller design approaches for systems which involve delays may be found in Niculescu (2001). The operator theory approach for the control of infinite-dimensional systems is presented in Curtain and Zwart (1995) and Foias et al. (1996). Toker and Özbay (1995) used this approach to formulate $\mathcal{H}^\infty$-optimal controller design for infinite-dimensional single-input single-output systems. Later, Meinsma and Zwart (2000) used J-spectral factorizations to solve the same problem for systems which involve a single delay. Recently, using decomposition into adobe problems, Meinsma and Mirkin (2005) formulated a solution to the $\mathcal{H}^\infty$-optimal controller design problem for systems which involve multiple delays.

A good example of a large-scale time-delay system is a data-communication network. A data-communication network requires flow control, among other resource management methods, in order to provide good quality of service to its users. The time delays in the network makes the problem of designing a flow controller challenging. An $\mathcal{H}^\infty$-based controller design approach for this problem was proposed in Özbay et al. (1998) by using the design techniques in Toker and Özbay (1995). The implementation of this controller was later illustrated in Özbay et al. (1999). The case of uncertain time-varying multiple time delays was later considered in Quet et al. (2002), where a flow controller which is robust to variations in such delays was designed. The approach of Meinsma and Mirkin (2005) was first considered for the robust flow controller design problem in Ataşlar (2004). Then, in Ünal et al. (2006), this approach was used to obtain an $\mathcal{H}^\infty$-optimal solution to the flow controller design problem.

In this work, we consider robust decentralized controller design for large-scale interconnected time-delay systems. We first use overlapping decompositions and expansions, first introduced by Ikeda and Šiljak (1980), to obtain local models. The approach of overlapping decompositions and expansions has been used successfuly to design decentralized controllers for large-scale systems which have subsystems that are interconnected through certain dynamics (*the overlapping part*). Large flexible structures, Özgüner et al. (1988), interconnected power systems, Šiljak (1978), socio-economic systems, Aoki (1976), freeway traffic regulation systems, Isaksen and Payne (1973), intelligent vehicle-highway systems, Stanković et al. (2000), data-communication networks, Ataşlar and İftar (1999), and manufacturing systems, Aybar and İftar (2002), are typical examples of such systems. Although in most of the references above the time-delays in those systems have been ignored, such large-scale systems usually involve time delays, which may be uncertain. Overlapping decompositions have recently been used for time-delay systems in Bakule et al. (2005), where guaranteed cost control has been studied.

Robustness of decentralized controllers designed using local models for delay-free systems was studied by İftar and Özgüner (1987b), where overlapping decompositions was used to derive error functions to account for neglected interactions between local models. A similar approach was undertaken by İftar and Özgüner (1987a) for interconnected systems. Overlapping decompositions were also used to assess the robustness and suboptimality of decentralized reduced order controllers by İftar and Özgüner (1984) and Özgüner and İftar (1985). Robustness of decentralized control has also been analyzed using the *quasi-block diagonal dominance* of transfer functions under overlapping decompositions by Ohta et al. (1986). A robustness bound for delay-free interconnected systems with uncertain dynamics has been derived by İftar (2004). Using this bound, a decentralized controller design approach, based on local models has also been proposed in İftar (2004).

In the present paper, we extend the approach of İftar (2004) to large-scale interconnected time-delay systems. After obtaining local models by the use of ovelapping decompositions and expansions, a robustness bound which accounts for

(i) the neglected interactions (which may involve time-delays) between the subsystem models,
(ii) uncertainties in the interactions,
(iii) uncertainties in the subsystem models, and
(iv) uncertainties in the time-delays

is derived in Section 2. Using this bound, a decentralized controller design approach is proposed in Section 3, which guarantees the stability of the overall system. In this approach, each local controller is designed independently by considering only the corresponding local nominal model and the robustness bound found in Section 2. An application example is presented in Section 4.

Throughout the paper, $\mathbf{R}$ denotes the set of real numbers, for a positive integer n, $\mathbf{R}^n$ denotes the n-dimensional real vector space, t is the time variable, $\dot{x}(t)$ is the derivative of $x(t)$ with respect to t, I and 0 respectively denote the identity and the zero matrix of appropriate dimensions, and $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ respectively denote the maximum and minimum singular values of the indicated matrices.

2. ROBUSTNESS BOUND FOR DECENTRALIZED CONTROLLER DESIGN

In this section we consider large-scale interconnected time-delay systems. Using overlapping decompositions and expansions, we develop a robustness bound for decentralized controller design for such systems. For notational simplicity, we consider a linear interconnected system with only two subsystems coupled through some dynamic interconnections. This system can be represented as:

$$\dot{x}(t) = A_\Delta x(t) + A^d_\Delta x(t-\tau_x) + B_\Gamma u(t) + B^d_\Gamma u(t-\tau_u) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where, $A_\Delta := A+\Delta$, $A^d_\Delta := A^d+\Delta^d$, $B_\Gamma := B+\Gamma$, $B^d_\Gamma := B^d+\Gamma^d$, where A, A^d, B, and B^d are known matrices and Δ, Δ^d, Γ, and Γ^d are unknown but norm-bounded matrices, representing modeling uncertainties. The output matrix C is also assumed to be known. Furthermore, $\tau_x := h_x+\theta_x$ and $\tau_u := h_u+\theta_u$ are time delays, where h_x and h_u are known nominal delays and θ_x and θ_u are the uncetain parts, which satisfy

$$|\theta_x| \le \bar{\theta}_x \qquad \text{and} \qquad |\theta_u| \le \bar{\theta}_u \quad (3)$$

for some known non-negative bounds $\bar{\theta}_x$ and $\bar{\theta}_u$.

Remark 1: Since for most systems uncertainties and delays can be represented either at the input or at the output, we took the output equation (2) as free of any uncertainties and delays. A similar approach may also be developed when uncertainties and delays are present at the output.

Remark 2: For notational simplicity, we assumed only one dynamic and one input delay. The present approach can easily be extended to the case of multiple delays, in which case $A^d_\Delta x(t-\tau_x)$ and $B^d_\Gamma u(t-\tau_u)$ terms in the right hand side of (1) should respectively be replaced by $\sum_{i=1}^{n_x} A^d_{\Delta i} x(t-\tau^i_x)$ and $\sum_{i=1}^{n_u} B^d_{\Gamma i} u(t-\tau^i_u)$.

The state, $x \in \mathbf{R}^n$, the input, $u \in \mathbf{R}^p$, and the output, $y \in \mathbf{R}^q$, are decomposed as $x = \begin{bmatrix} x_1 \\ x_c \\ x_2 \end{bmatrix}$, $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, where $x_i \in \mathbf{R}^{n_i}$, $u_i \in \mathbf{R}^{p_i}$, and $y_i \in \mathbf{R}^{q_i}$ are, respectively, the state, the input, and the output of the i^{th} subsystem $(i = 1, 2)$ and $x_c \in \mathbf{R}^{n_c}$ is the state for the dynamic interconnections. We assume that the input of the i^{th} subsystem directly effects only the state of the i^{th} subsystem and that the output of the i^{th} subsystem is directly effected by only the state of the i^{th} subsystem $(i = 1, 2)$. Hence, the input and the output matrices have the forms

$$N = \begin{bmatrix} N_1 & 0 \\ 0 & 0 \\ 0 & N_2 \end{bmatrix} \qquad C = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & 0 & C_2 \end{bmatrix}$$

where N stands for B, B^d, Γ, and Γ^d. The partitionings are compatible with the partitionings of x, u, and y. We also partition A, A^d, Δ, and Δ^d accordingly:

$$M = \begin{bmatrix} M_1 & M_{1c} & M_{12} \\ M_{c1} & M_c & M_{c2} \\ M_{21} & M_{2c} & M_2 \end{bmatrix}$$

where M stands for A, A^d, Δ, and Δ^d.

Now, let us use the transformation

$$\hat{x} = Tx = \begin{bmatrix} x_1 \\ x_c \\ \cdots \\ x_c \\ x_2 \end{bmatrix} =: \begin{bmatrix} \hat{x}_1 \\ \cdots \\ \hat{x}_2 \end{bmatrix} \quad (4)$$

to *expand* (Ikeda and Šiljak (1980)) the system (1)–(2) to obtain

$$\dot{\hat{x}}(t) = \hat{A}_\Delta \hat{x}(t) + \hat{A}^d_\Delta \hat{x}(t-\tau_x) + \hat{B}_\Gamma u(t) + \hat{B}^d_\Gamma u(t-\tau_u) \quad (5)$$

$$y(t) = \hat{C}\hat{x}(t) \quad (6)$$

where $\hat{A}_\Delta := \hat{A}+\hat{\Delta}$, $\hat{A}^d_\Delta := \hat{A}^d+\hat{\Delta}^d$, $\hat{B}_\Gamma := \hat{B}+\hat{\Gamma}$, and $\hat{B}^d_\Gamma := \hat{B}^d+\hat{\Gamma}^d$, where

$$\hat{M} := \begin{bmatrix} \hat{M}_1 & \hat{M}_{12} \\ \hat{M}_{21} & \hat{M}_2 \end{bmatrix} \qquad \hat{M}_1 := \begin{bmatrix} M_1 & M_{1c} \\ M_{c1} & M_c \end{bmatrix}$$

$$\hat{M}_{12} := \begin{bmatrix} 0 & M_{12} \\ 0 & M_{c2} \end{bmatrix} \qquad \hat{M}_{21} := \begin{bmatrix} M_{c1} & 0 \\ M_{21} & 0 \end{bmatrix}$$

$$\hat{M}_2 := \begin{bmatrix} M_c & M_{c2} \\ M_{2c} & M_2 \end{bmatrix} \qquad \hat{N} := \begin{bmatrix} \hat{N}_1 & 0 \\ 0 & \hat{N}_2 \end{bmatrix}$$

$$\hat{N}_1 := \begin{bmatrix} N_1 \\ 0 \end{bmatrix} \qquad \hat{N}_2 := \begin{bmatrix} 0 \\ N_2 \end{bmatrix}$$

where, as above, M stands for A, A^d, Δ, and Δ^d and N stands for B, B^d, Γ, and Γ^d. Furthermore,

$$\hat{C} := \begin{bmatrix} \hat{C}_1 & 0 \\ 0 & \hat{C}_2 \end{bmatrix} \qquad \hat{C}_1 := [\, C_1 \;\; 0 \,]$$

$$\hat{C}_2 := [\,0\;\; C_2\,]$$

It can be shown that the *expanded system* (5)–(6) *includes* (Ikeda et al. (1984), Bakule et al. (2000)) the *original system* (1)–(2) and, hence, the two systems have the same input-output map:

$$\hat{G}(s) := \hat{C}\left(sI - \hat{A}_\Delta - e^{-s\tau_x}\hat{A}^d_\Delta\right)^{-1}\left(\hat{B}_\Gamma + e^{-s\tau_u}\hat{B}^d_\Gamma\right)$$
$$= G(s) := C\left(sI - A_\Delta - e^{-s\tau_x}A^d_\Delta\right)^{-1}\left(B_\Gamma + e^{-s\tau_u}B^d_\Gamma\right) \quad (7)$$

At this point, let us introduce the assumed bounds on the norms of the uncertain matrices:

$$\bar{\sigma}(\hat{\Delta}_1) \le \delta_1 \qquad \bar{\sigma}(\hat{\Delta}_2) \le \delta_2 \quad (8)$$
$$\bar{\sigma}(\hat{\Delta}_{12}) \le \delta_{12} \qquad \bar{\sigma}(\hat{\Delta}_{21}) \le \delta_{21} \quad (9)$$
$$\bar{\sigma}(\hat{\Delta}^d_1) \le \delta^d_1 \qquad \bar{\sigma}(\hat{\Delta}^d_2) \le \delta^d_2 \quad (10)$$
$$\bar{\sigma}(\hat{\Delta}^d_{12}) \le \delta^d_{12} \qquad \bar{\sigma}(\hat{\Delta}^d_{21}) \le \delta^d_{21} \quad (11)$$
$$\bar{\sigma}(\Gamma_1) = \bar{\sigma}(\hat{\Gamma}_1) \le \gamma_1 \qquad \bar{\sigma}(\Gamma_2) = \bar{\sigma}(\hat{\Gamma}_2) \le \gamma_2 \quad (12)$$
$$\bar{\sigma}(\Gamma^d_1) = \bar{\sigma}(\hat{\Gamma}^d_1) \le \gamma^d_1 \qquad \bar{\sigma}(\Gamma^d_2) = \bar{\sigma}(\hat{\Gamma}^d_2) \le \gamma^d_2 \quad (13)$$

where δ's and γ's are known non-negative numbers.

We assume that decentralized controllers for each subsystem i $(i = 1, 2)$ is to be designed based on the *local nominal model*:

$$\dot{\hat{x}}_i(t) = \hat{A}_i\hat{x}_i(t) + \hat{A}^d_i\hat{x}_i(t - h_x) + \hat{B}_iu_i(t) + \hat{B}^d_iu_i(t - h_u) \quad (14)$$
$$y_i(t) = \hat{C}_i\hat{x}_i(t) \quad (15)$$

Then the *overall design model* has the transfer function matrix (TFM):

$$G_d(s) = \begin{bmatrix} G_1(s) & 0 \\ 0 & G_2(s) \end{bmatrix} \quad (16)$$

where

$$G_i(s) := \hat{C}_i\left(sI - \hat{A}_i - e^{-sh_x}\hat{A}^d_i\right)^{-1}\left(\hat{B}_i + e^{-sh_u}\hat{B}^d_i\right)$$

is the TFM of the i^{th} local nominal model $(i = 1, 2)$. We now introduce the *multiplicative error matrix*, $E(s)$, between the true TFM, $G(s)$, and the *design TFM*, $G_d(s)$, which satisfies

$$G(s) = G_d(s)\left(I + E(s)\right) \ . \quad (17)$$

The following result gives an upper bound on the norm of $E(j\omega)$.

Lemma 1: Assuming $e_d(\omega) > 0$,

$$\bar{\sigma}(E(j\omega)) \le \frac{e_n(\omega)}{e_d(\omega)} =: e(\omega) \quad (18)$$

where

$$e_n(\omega) := \max\left\{e^{11}_n(\omega), e^{22}_n(\omega)\right\} + \max\left\{e^{12}_n(\omega), e^{21}_n(\omega)\right\}$$

and

$$e_d(\omega) := \underline{\sigma}\left(H(j\omega)\right) - \left[\max\left\{e^{11}_d(\omega), e^{22}_d(\omega)\right\} + \max\left\{e^{12}_d(\omega), e^{21}_d(\omega)\right\}\right]$$

where $H(s) := \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix}$,

$$e^{ii}_n(\omega) := \gamma_i + \bar{\rho}_u(\omega)\bar{\sigma}(\hat{B}^d_i) + \gamma^d_i + e^{ii}_d(\omega)$$
$$e^{ij}_n(\omega) := \left(\bar{\sigma}(\hat{A}_{ij}) + \delta_{ij} + \bar{\sigma}(\hat{A}^d_{ij}) + \delta^d_{ij}\right)g_j(\omega)$$
$$e^{ii}_d(\omega) := \left(\delta_i + \bar{\rho}_x(\omega)\bar{\sigma}(\hat{A}^d_i) + \delta^d_i\right)g_i(\omega)$$
$$e^{ij}_d(\omega) := \left(\delta_{ij} + \bar{\rho}_x(\omega)\bar{\sigma}(\hat{A}^d_{ij}) + \delta^d_{ij}\right)g_j(\omega)$$
$$H_{ii}(s) := \hat{B}_i + e^{-sh_u}\hat{B}^d_i$$

and

$$H_{ij}(s) := -\left(\hat{A}_{ij} + e^{-sh_x}\hat{A}^d_{ij}\right)\left(sI - \hat{A}_j - e^{-sh_x}\hat{A}^d_j\right)^{-1} \times \left(\hat{B}_j + e^{-sh_u}\hat{B}^d_j\right)$$

where

$$g_i(\omega) := \bar{\sigma}\left[\left(j\omega I - \hat{A}_i - e^{-j\omega h_x}\hat{A}^d_i\right)^{-1}\left(\hat{B}_i + e^{-j\omega h_u}\hat{B}^d_i\right)\right]$$

$(i, j = 1, 2,\ i \ne j)$ and

$$\bar{\rho}_v(\omega) := \begin{cases} 2\sin\left(\dfrac{|\omega|\bar{\theta}_v}{2}\right), & |\omega| \le \dfrac{\pi}{\bar{\theta}_v} \\ 2\,, & |\omega| > \dfrac{\pi}{\bar{\theta}_v} \end{cases}, \quad (19)$$

where v stands for x or u.

Proof: Using (7) and (16), $E(s)$ in (17) can be chosen to satisfy

$$\left(sI - \hat{A}_\Delta - e^{-s\tau_x}\hat{A}^d_\Delta\right)^{-1}\left(\hat{B}_\Gamma + e^{-s\tau_u}\hat{B}^d_\Gamma\right)$$
$$= \left(sI - \tilde{A} - e^{-sh_x}\tilde{A}^d\right)^{-1}\left(\hat{B} + e^{-sh_u}\hat{B}^d\right)(I + E(s)) \quad (20)$$

where $\tilde{A} := \begin{bmatrix} \hat{A}_1 & 0 \\ 0 & \hat{A}_2 \end{bmatrix}$ and $\tilde{A}^d := \begin{bmatrix} \hat{A}^d_1 & 0 \\ 0 & \hat{A}^d_2 \end{bmatrix}$. Premultiply both sides of (20) by $\left(sI - \hat{A}_\Delta - e^{-s\tau_x}\hat{A}^d_\Delta\right)$ and rearrange terms to obtain $Q(s) = R(s)E(s)$, where $Q(s) := \begin{bmatrix} Q_{11}(s) & Q_{12}(s) \\ Q_{21}(s) & Q_{22}(s) \end{bmatrix}$ and $R(s) := \begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix}$, where

$$Q_{ii}(s) = \hat{\Gamma}_i + \rho_u(s)\hat{B}^d_i + e^{-s\tau_u}\hat{\Gamma}^d_i + \left(\hat{\Delta}_i + \rho_x(s)\hat{A}^d_i + e^{-s\tau_x}\hat{\Delta}^d_i\right)\left(sI - \hat{A}_i - e^{-sh_x}\hat{A}^d_i\right)^{-1} \times \left(\hat{B}_i + e^{-sh_u}\hat{B}^d_i\right)$$
$$Q_{ij}(s) = \left(\hat{A}_{ij} + \hat{\Delta}_{ij} + e^{-s\tau_x}(\hat{A}^d_{ij} + \hat{\Delta}^d_{ij})\right) \times \left(sI - \hat{A}_j - e^{-sh_x}\hat{A}^d_j\right)^{-1}\left(\hat{B}_j + e^{-sh_u}\hat{B}^d_j\right)$$
$$R_{ii}(s) = \hat{B}_i + e^{-sh_u}\hat{B}^d_i - \left(\hat{\Delta}_i + \rho_x(s)\hat{A}^d_i + e^{-s\tau_x}\hat{\Delta}^d_i\right) \times \left(sI - \hat{A}_i - e^{-sh_x}\hat{A}^d_i\right)^{-1}\left(\hat{B}_i + e^{-sh_u}\hat{B}^d_i\right)$$
$$R_{ij}(s) = -\left(\hat{A}_{ij} + \hat{\Delta}_{ij} + e^{-s\tau_x}(\hat{A}^d_{ij} + \hat{\Delta}^d_{ij})\right) \times \left(sI - \hat{A}_j - e^{-sh_x}\hat{A}^d_j\right)^{-1}\left(\hat{B}_j + e^{-sh_u}\hat{B}^d_j\right)$$

$(i, j = 1, 2,\ i \ne j)$ where $\rho_v(s) := e^{-s\tau_v} - e^{-sh_v}$ (where v stands for x or u). Note that, $|\rho_v(j\omega)| \le \bar{\rho}_v(\omega)$, $\forall\omega$. The result now follows by noting that $\bar{\sigma}(Q(j\omega)) \leq e_n(\omega)$ and $\underline{\sigma}(R(j\omega)) \geq e_d(\omega)$. □

The bound $e(\omega)$ given in (18) can be used as a robustness bound in decentralized controller design for the system (1)–(2), as will be explained in the next section.

Remark 3: The bound (18) is not defined if $e_d(\omega) \leq 0$, i.e., if $\underline{\sigma}(H(j\omega)) \leq \left[\max\left\{e_d^{11}(\omega), e_d^{22}(\omega)\right\} + \max\left\{e_d^{12}(\omega), e_d^{21}(\omega)\right\}\right]$. However, note that the matrix H involves the nominal input matrices on the diagonal blocks (which typically have large norm) and interaction terms (which typically have smaller norm) on the off-diagonal blocks. Furthermore, the terms e_d^{ij} involve bounds on the uncertainties (which are typically small - although the bound $\bar{\rho}_x(\omega)$ may become large (upto 2) for large $|\omega|$ (as compared to $1/\bar{\theta}_x$), the interaction term $g_i(\omega)$ (which multiplies $\bar{\rho}_x(\omega)$) is typically small for large $|\omega|$). Therefore, typically, $\underline{\sigma}(H(j\omega)) > \left[\max\left\{e_d^{11}(\omega), e_d^{22}(\omega)\right\} + \max\left\{e_d^{12}(\omega), e_d^{21}(\omega)\right\}\right]$, and hence $e_d(\omega) > 0$.

3. ROBUST DECENTRALIZED CONTROLLER DESIGN

Suppose that decentralized controllers

$$U_i(s) = -K_i(s)Y_i(s) \ , \qquad i = 1,2 \tag{21}$$

are to be designed for the system (1)–(2). Here, $U_i(s)$ and $Y_i(s)$ respectively denote the Laplace transforms of $u_i(t)$ and of $y_i(t)$ ($i = 1,2$) and $K_i(s)$ is the TFM of the i^{th} decentralized controller.

Here, we propose to design $K_i(s)$ to stabilize and achieve good performance for the local nominal model (14)–(15) and also to satify the following constraint

$$\bar{\sigma}\left(T_i(j\omega)\right) < \frac{1}{e(\omega)} \ , \qquad \forall \omega \in \mathbf{R} \tag{22}$$

where $T_i(s) := G_i(s)K_i(s)\left[I + G_i(s)K_i(s)\right]^{-1}$ is the complementary sensitivity function for the i^{th} local nominal closed-loop system and $e(\omega)$ is given by (18). Then we have the following result.

Theorem 1: Suppose that, for each $i \in \{1,2\}$, the control (21) stabilizes the system (14)–(15) and that (22) is satisfied. Then the true overall closed-loop system, obtained by applying the decentralized controllers (21) to the original system (1)–(2), is stable as long as the uncertainties are bounded as given in (3), (8)–(13).

Proof: The complementary sensitivity function for the overall design model is given by

$$\begin{aligned} T(s) &= G_d(s)K(s)\left[I + G_d(s)K(s)\right]^{-1} \\ &= \begin{bmatrix} T_1(s) & 0 \\ 0 & T_2(s) \end{bmatrix} \end{aligned} \tag{23}$$

where $G_d(s)$ is given by (16) and $K(s) = \begin{bmatrix} K_1(s) & 0 \\ 0 & K_2(s) \end{bmatrix}$.
Stability of the local nominal closed-loop systems, (14)–(15) under controls (21) implies stability of the overall nominal closed-loop system in the expanded space, since the two subsystems are decoupled in the expanded space. Then, due to relation (17), the true overall closed-loop system, (1)–(2) under controls (21), is stable as long as

$$\bar{\sigma}\left(T(j\omega)\right) < \frac{1}{\bar{\sigma}\left(E(j\omega)\right)} \ , \qquad \forall \omega \in \mathbf{R} \tag{24}$$

(e.g., see Zhou et al. (1996)). However, due to the block diagonal structure of $T(s)$, $\bar{\sigma}\left(T(j\omega)\right) = \max\left\{\bar{\sigma}\left(T_1(j\omega)\right)\ ,\ \bar{\sigma}\left(T_2(j\omega)\right)\right\}$. Hence, using (18), (24) is satisfied when the constraint (22) is satisfied for each $i \in \{1,2\}$. □

4. APPLICATION

In this section we apply the above proposed controller design approach to a flow control problem in a data-communication network with two bottleneck nodes. A queue is formed at each bottleneck node, whose dynamics is given by

$$\dot{q}_1(t) = r_1(t - \tau_{r1}) - z(t - \tau_z) - c_1(t) \tag{25}$$

and

$$\dot{q}_2(t) = r_2(t - \tau_{r2}) + z(t - \tau_z) - c_2(t) \tag{26}$$

where $q_i(t)$ is the deviation of the queue length at the i^{th} bottleneck node ($i = 1,2$) from its desired level at time t, $r_i(t)$ is the flow rate command issued at time t for the source feeding the i^{th} bottleneck node (here, for simplicity, we assume a single source for each bottleneck, we also assume that the sources send data at the commanded rate after a certain delay, the controller which issues the rate command r_i is implemented at the i^{th} bottleneck node), $c_i(t)$ is the outgoing flow rate from the i^{th} bottleneck node (i.e., the capacity of the outgoing link) at time t, and $z(t)$ represents the net rate of the flow from the first bottleneck node to the second bottleneck node, which is managed by a *queue balancer* according to the following rule:

$$\dot{z}(t) = \kappa[q_1(t - \tau_{q1}) - q_2(t - \tau_{q2})] \tag{27}$$

where κ is the *balancing constant*. We assume $\kappa = 0.1 + \delta_\kappa$, where δ_κ is the uncertain part which satisfies $|\delta_\kappa| \leq 0.01$. Here, τ_{ri} is the *round trip delay* for the i^{th} bottleneck node (the delay needed for the rate command to reach the source of the i^{th} bottleneck node plus the delay needed for the data to reach the i^{th} bottleneck node), τ_{qi} is the delay needed for the queue information at the i^{th} bottleneck node to reach the queue balancer, and τ_z is the delay needed for the rate information produced by the queue balancer to reach the first (second) bottleneck node plus the delay nedded for the data sent out from the first (second) bottleneck node to reach the second (first) bottleneck node. Although all of these delays can be treated independently (see Remark 2), in order to use the same notation as above, we assume that $\tau_{r1} = \tau_{r2} =: \tau_u$ and $\tau_{q1} = \tau_{q2} = \tau_z =: \tau_x$. Furthermore, we assume that the nominal part of each delay is unity, i.e., $h_x = h_u = 1$, and the bound on the uncertain part is 0.1, i.e., $\bar{\theta}_x = \bar{\theta}_u = 0.1$.

Let us define

$$x = \begin{bmatrix} q_1 \\ z \\ q_2 \end{bmatrix} \qquad u = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \qquad y = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Then the overall system can be represented as in (1)–(2), where $A = \Delta = 0$, $B = \Gamma = \Gamma^d = 0$,

$$A^d = \begin{bmatrix} 0 & -1 & 0 \\ 0.1 & 0 & -0.1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \Delta^d = \begin{bmatrix} 0 & 0 & 0 \\ \delta_\kappa & 0 & -\delta_\kappa \\ 0 & 0 & 0 \end{bmatrix}$$

$$B^d = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The uncertainty bounds in (8)–(13) are then obtained as $\delta_1 = \delta_2 = \delta_{12} = \delta_{21} = \gamma_1 = \gamma_2 = \gamma_1^d = \gamma_2^d = 0$, $\delta_1^d = \delta_2^d = \delta_{12}^d = \delta_{21}^d = 0.01$. The error bound defined in Lemma 1 is then obtained as

$$e(\omega) = \frac{\bar{\rho}(\omega)\sqrt{d(\omega)} + (\bar{\rho}(\omega) + 0.12)\sqrt{\omega^2 + 0.01}}{\sqrt{\omega^2 + d(\omega)} - (1.1\bar{\rho}(\omega) + 0.02)\sqrt{\omega^2 + 0.01}}$$

where $d(\omega) := \omega^4 - 0.2\omega^2\cos(2\omega) + 0.01$ and $\bar{\rho}(\omega)$ is given by the right hand side of (19) with $\bar{\theta}_v = 0.1$.

Following the steps in Section 2, the local nominal models, on which the controller design is to be based, are obtained as

$$\dot{\hat{x}}_1(t) = \begin{bmatrix} 0 & -1 \\ 0.1 & 0 \end{bmatrix} \hat{x}_1(t-1) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (r_1(t-1) - c_1(t))$$
$$y_1(t) = [\,1 \;\; 0\,]\,\hat{x}_1(t)$$

and

$$\dot{\hat{x}}_2(t) = \begin{bmatrix} 0 & -0.1 \\ 1 & 0 \end{bmatrix} \hat{x}_2(t-1) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (r_2(t-1) - c_2(t))$$
$$y_2(t) = [\,0 \;\; 1\,]\,\hat{x}_2(t)$$

The TFMs (from r_i to y_i) for these systems are then obtained as

$$G_1(s) = G_2(s) = \frac{se^{-s}}{s^2 + 0.1e^{-2s}}$$

We note that the local systems are unstable ($s^2 + 0.1e^{-2s} = 0$ for $s = 0.095 \pm j0.281$). Both of these systems can be stabilized, however, by a static feedback:

$$r_i(t) = -kq_i(t)\ , \qquad i = 1, 2 \tag{28}$$

by choosing k in the range $0.20 < k < 1.64$. Within this range, however, condition (22) is satisfied only for $k < 1.2$. $\bar{\sigma}(T_1(j\omega)) = \bar{\sigma}(T_2(j\omega))$ is plotted for $k = 0.2$, $k = 0.7$, and $k = 1.2$, together with $1/e(\omega)$ in Fig. 1. By Theorem 1, applying the local controls (28) to the system described by (25)–(27), where $0.20 < k < 1.20$, robustly stabilizes the overall system for all uncertainties satisfying the above introduced bounds. By examining Figure 1, a smooth response (for the local nominal models, if not for the actual system) is also obtained by choosing $k = 0.7$.

5. CONCLUSIONS

Robust decentralized controller design for large-scale interconnected time-delay systems has been considered. Using overlapping decompositions and expansions, a robustness bound, to account for (i) the neglected interactions between the subsystem models, (ii) uncertainties in the interactions, (iii) uncertainties in the subsystem models, and (iv) uncertainties in the time delays, has been derived. A decentralized controller design approach, which uses this bound, has then been propsed. The advantage of the proposed approach is that, all uncertaities and neglected dynamics are summarized in one frequency-dependent scalar function ($e(\omega)$) and satisfying a simple condition (22) guarantees that the overall closed-loop system under decentralized controllers, which are designed considering local models, is robustly stable.

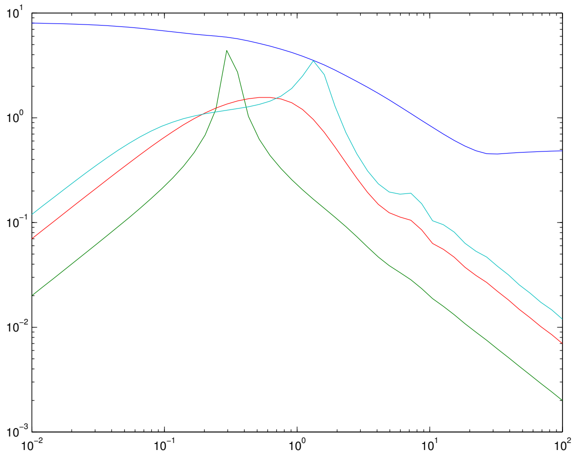

Fig. 1. Plot of $1/e(\omega)$ (in blue - plot with highest low frequency gain) and plots of $\bar{\sigma}(T_1(j\omega)) = \bar{\sigma}(T_2(j\omega))$ for $k = 0.2$ (in green - plot with lowest low frequency gain), $k = 0.7$ (in red - plot with second lowest low frequency gain), and $k = 1.2$ (in cyan - plot with third lowest low frequency gain) vs. ω.

Although we have considered only one input and one output delay, extension to the case of multiple delays is straight forward. Similarly, although we have considered interconnected systems with only two subsystems, the results can directly be extended to interconnected systems with more subsystems. Furthermore, a similar approach can be undertaken for different types of interconnections, such as systems interconnected through inputs and/or outputs (e.g., see İftar and Özgüner (1987b)). It is also possible to consider overlapping decompositions on input and output spaces besides the state space İftar (1993). Extensions to time-varying systems and to systems with time-varying delays are subjects for future research.

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