

Efficient Branch and Bound Methods for Pairing Selection

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Abstract: During the past few decades, a number of methods for selection of input-output pairings for decentralized control have been proposed. Most of these available methods require evaluation of every alternative in order to find the optimal pairings. As the number of alternatives grows rapidly with problem size, pairing selection through an exhaustive search can be computationally forbidding for large-scale process. In this paper, we present novel branch and bound (BAB) approaches for pairing selection using relative gain array and μ -interaction measure as the selection criteria to overcome this difficulty. We demonstrate the computational efficiency of the proposed BAB approaches by applying them to randomly generated matrices as well as to the Tennessee Eastman benchmark example.

1. INTRODUCTION

For control of large-scale processes, use of decentralized controllers is predominant in process industries. The design of a decentralized controller involves selection of appropriate input-output pairing and subsequently tuning parameters of the controller. For appropriate pairing selection, a number of criterion are available in the literature [Van de wal, 1994, Skogestad and Postlethwaite, 2005]. A common feature of these available pairing selection methods is that every alternative needs to be evaluated individually. For a process with n outputs and n inputs, the total number of pairing alternatives is $n!$. For large-scale processes (*e.g.* pulp and paper mill [Castro and Doyle, 2002], and vinyl acetate monomer plant [Chen et al., 2003]), pairing selection through an exhaustive search can be computationally forbidding. This paper addresses this combinatorial issue arising in pairing selection.

The computational expense for pairing selection can be reduced by using optimization based approaches. In these approaches, the binary variables related to the selection of pairings are relaxed as continuous variables. Depending on the choice of pairing selection criterion, the relaxed problem can be a mixed integer linear program (MILP) [Kookos and Perkins, 2001] in some cases. In general, however, solution of a mixed integer nonlinear program (MINLP) is required upon relaxation. Such schemes have been used widely in the literature dealing with integrated design and control of chemical processes [Luyben and Floudas, 1994, Sakizlis et al., 2004]. The MINLP based approaches usually result in non-convex optimization problems and thus global optimality of the selected pairing cannot be guaranteed.

In this paper, we propose an efficient branch and bound (BAB) approach for the pairing selection problem. In comparison to an exhaustive search, a BAB approach gains its efficiency by pruning branches of the solution tree, which cannot lead to the optimal solution. Note that the BAB is also used by many optimization algorithms for solving the MILPs and MINLPs. The BAB method proposed in this paper, however, does not require solving relaxed optimization problems. Instead, monotonicity arguments are used for pruning purposes. The usefulness of BAB approaches for some other problems arising in control structure design has been demonstrated by the authors earlier [Cao et al., 1998, Cao and Saha, 2005, Kariwala and Skogestad, 2006, Cao and Kariwala, 2008].

The proposed method is general and can be applied to most of the available pairing selection criteria. In this paper, we discuss pairing selection based on relative gain array (RGA) [Bristol, 1966]. As shown by Kookos and Perkins [2001], this problem can be posed as an MILP. The advantage of the proposed approach is that it can also handle selection criteria which do not lead to MILP formulations. To emphasize this issue, we show the application of proposed approach for pairing selection based on μ -interaction measure (μ -IM) [Grosdidier and Morari, 1986], where μ -IM is seen as a measure of generalized diagonal dominance [Kariwala et al., 2003]. Numerical examples demonstrate that both criteria can be combined with the BAB approaches to efficiently solve pairing selection problems for large-scale processes. Furthermore, we also point out the possible paths, which can be undertaken to apply the proposed approach to other selection criteria such as ν -gap metric [Samyudia et al., 1995], Hankel interaction measure [Wittenmark and Salgado, 2002] and effective RGA [Xiong et al., 2006].

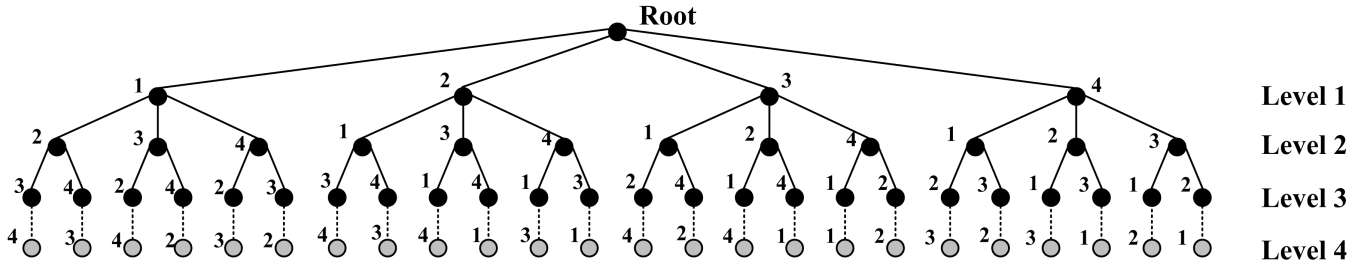


Fig. 1. Solution tree for selecting pairings for a system with 4 outputs and 4 inputs

The rest of this paper is organized as follows: the general BAB framework used in this work is proposed in Section 2. In Sections 3 and 4, the BAB approach is customized for pairing selection using RGA and μ -IM as the selection criteria, respectively. The efficiency of the BAB approaches is demonstrated in Section 5 through some numerical tests and the work is concluded in Section 6.

2. BRANCH AND BOUND METHOD

In this paper, $\{a, b\}$ denotes an unordered set, while (a, b) denotes an ordered set, both consisting of the elements a and b . Note that $\{a, b\} = \{b, a\}$, but $(a, b) \neq (b, a)$. We define N_n as the set of first n natural numbers, *i.e.* $N_n = \{1, 2, \dots, n\}$, where the subscript n is explicitly used to denote the size of the set. For a given set X_n , $\mathbb{P}(X_n)$ represents the ensemble of all possible permutations of the elements of X_n . For $n = 2$, $\mathbb{P}(N_2) = \{(1, 2), (2, 1)\}$. An element of $\mathbb{P}(N_n)$ is represented as P_n .

$G(s)$ denotes the transfer function matrix relating the inputs u and outputs y of the process. The transfer function matrix $G(s)$ evaluated at the frequency ω is represented as $G(j\omega) \in \mathbb{C}^{n \times n}$ and the steady-state gain matrix as $G \in \mathbb{R}^{n \times n}$. For $P_n \in \mathbb{P}(N_n)$, G_{*P_n} or simply G_{P_n} denotes the permuted matrix G with columns indexed by P_n . For given P_n , it is considered that the pairings are selected on the diagonal elements of G_{P_n} . For example, for $n = 2$ the selected pairing are $\{y_1 - u_1, y_2 - u_2\}$ or $\{y_1 - u_2, y_2 - u_1\}$, when P_2 is $(1, 2)$ or $(2, 1)$, respectively. Thus, the pairing selection can be seen as finding the optimal set P_n^{opt} by solving the following optimization problem

$$\min_{P_n \in \mathbb{P}(N_n)} J(P_n) \quad (1)$$

$$\text{s.t. } L_i(P_n) \geq 0; \quad i = 1, 2, \dots, \ell \quad (2)$$

where J is the pairing selection criterion and L_i , $i = 1, \dots, \ell$ denotes a set of inequality constraints. In general, a combinatorial optimization problem can also have equality and logical constraints. Such constraints are not considered in this paper.

The implementation of BAB schemes requires a solution tree containing all possible alternatives. For pairing selection, the solution tree for $n = 4$ is shown in Figure 1. The tree has $(n + 1)$ levels, where level i corresponds to i^{th} output y_i (except level 0 corresponding to the empty root node). A node at level i represents a partially assigned pairing, where the label of the node denotes the input with which y_i is paired. At level i , a node has $(n - i)$ sub-nodes and the pairings assigned at a node are passed to all its sub-nodes. The tree has $n!$ terminal nodes (marked by

grey circles in Figure 1), which represent different pairing alternatives P_n .

For ease of notation in the subsequent discussion, we introduce the concepts of fixed and candidate sets.

Definition 1. For a node P_f , $f \leq n$, the fixed set F_f is an ordered set and is same as the node itself.

Definition 2. The candidate set $C_c = N_n \setminus F_f$ is an unordered set, whose elements can be freely chosen to append P_f .

Based on these definitions, the solution tree is branched as follows:

Definition 3. A node P_f with candidate set C_c has $c = (n - f)$ branches. The fixed and candidate sets of the i^{th} sub-node, $i = 1, 2, \dots, c$ are defined as

$$F_{f+1}^i = (F_f, c_i) \quad (3)$$

$$C_{c-1}^i = C_c \setminus c_i \quad (4)$$

where c_i is the i^{th} element of C_c .

To illustrate the concepts of fixed and candidate sets, consider the leftmost node on level 2. The fixed and candidate sets of this node are $F_2 = (1, 2)$ and $C_2 = \{3, 4\}$, respectively. Similarly, the fixed and candidate sets of the adjacent node with label 3 are $F_2 = (1, 3)$ and $C_2 = \{2, 4\}$. It can be easily established that the symmetric solution tree branched based on Definition 3 is non-redundant and, every terminal node belongs to one and only one branch.

One should note that the number of nodes in the solution tree for pairing selection is much more than the number of pairing alternatives $n!$. A BAB method gains its efficiency by pruning branches of the tree that cannot lead to the optimal solution. To illustrate this, consider a node P_f with fixed set F_f and candidate set C_c . Then, the ensemble of all n -element ordered sets that can be obtained by expanding P_f is given as

$$\mathcal{S} = \{(F_f, P_{n-f}) | P_{n-f} \in \mathbb{P}(C_c)\} \quad (5)$$

For example, for a node with $F_2 = (2, 4)$ and $C_2 = \{1, 3\}$, $\mathcal{S} = \{(2, 4, 1, 3), (2, 4, 3, 1)\}$. Let $\underline{J}(\mathcal{S})$ be a lower bound of J calculated over all the elements of \mathcal{S} ,

$$\underline{J}(\mathcal{S}) \leq J(P_n) \quad \forall P_n \in \mathcal{S} \quad (6)$$

If B is a known upper bound on $J^{\text{opt}}(P_n)$ and $\underline{J}(P_n) > B$, then it follows that

$$\underline{J}(\mathcal{S}) > J^{\text{opt}}(P_n) \quad (7)$$

Then the set $\mathcal{S} \subset \mathbb{P}(N_n)$ cannot contain the optimal solution, hence, none of the pairing alternatives contained in \mathcal{S} need to be evaluated. Similarly, let $\bar{L}_i(\mathcal{S})$ denote an upper bound of $L_i(P_n)$ calculated over all the elements of \mathcal{S} . Then the node P_f can be pruned, if $\bar{L}_i(\mathcal{S}) < 0$, $i = 1, 2, \dots, \ell$. Whenever the BAB method encounters a terminal node, which gives a lower criterion value than B and satisfies (2), the bound B is updated. This way the optimal solution can be obtained without evaluating all the alternatives.

Remark 4. For a node P_{n-1} on level $(n-1)$, the cardinality of the candidate set is 1. This implies that the only terminal set that can be reached from P_{n-1} is $P_n = (F_{n-1}, C_1)$. Thus, instead of evaluating nodes on level $(n-1)$, one may directly consider the terminal node P_n . For feature subset selection using BAB approaches, a similar recommendation has been made by Yu and Yuan [1993].

Remark 5. Instead of finding the optimal solution only, the BAB method can easily be modified to find the best q solutions. This can be achieved by replacing the scalar bound B with a q -element vector of bounds and always using the vector element with maximum value (\bar{B}) for pruning purposes. Whenever a feasible terminal node with criterion value lower than \bar{B} is found, \bar{B} in the vector of bounds is replaced by the criterion value of the new terminal node and \bar{B} is recalculated from the updated vector.

3. RELATIVE GAIN ARRAY

RGA is one of the simplest and most popular tools for pairing selection. For a steady-state gain matrix $G \in \mathbb{R}^{n \times n}$, the RGA is defined as [Bristol, 1966]

$$\Lambda(G) = G \circ G^{-T} \quad (8)$$

where \circ is the element-wise or Hadamard product and G^{-T} is transpose of the inverse of G . Though originally defined for steady-state gain matrices, frequency dependent RGA can be calculated similarly by replacing G with $G(j\omega)$ in (8) [Hovd and Skogestad, 1992]. The following rules are often used for selecting pairings based on RGA [Skogestad and Postlethwaite, 2005]:

- (1) Avoid pairings on the negative elements of RGA evaluated at steady-state.
- (2) Prefer pairings on the elements of RGA, evaluated at (expected) bandwidth frequency ω_B , such that the permuted RGA matrix with selected pairings along the diagonal is close to the identity matrix.

The first rule is a necessary condition for integrity of the closed-loop system against loop failure [Grosdidier et al., 1985]. The second rule is based on interpreting RGA as an interaction measure. Although the theoretical foundations of the second rule are weak for processes with more than 3 inputs and 3 outputs [Skogestad and Postlethwaite, 2005], it has been proven to be beneficial for screening pairing alternatives in the past. For selecting pairings according to the second rule, Skogestad and Postlethwaite [2005] have introduced the RGA-number defined as

$$\text{RGA-number}(P_n) = \|\Lambda(G_{P_n}(j\omega_B)) - I\|_{\text{sum}} \quad (9)$$

where $\|\cdot\|_{\text{sum}}$ is the sum-norm, computed as the sum of the absolute values of all the elements of a matrix.

We recall that RGA for a permuted matrix results in similar permutation in RGA itself, *i.e.* $\Lambda(G_{P_n}(j\omega_B)) = \Lambda_{P_n}(G(j\omega_B))$ [Bristol, 1966]. In the subsequent discussion, we drop the argument of Λ for notational simplicity. We only consider RGA-number evaluated at the steady-state for pairing selection, but the proposed results hold at any arbitrary frequency. Furthermore,

$$\|\Lambda - I\|_{\text{sum}} = \|\Lambda\|_{\text{sum}} + \text{trace}(|\Lambda - 1_{nn}| - |\Lambda|)$$

where $|\cdot|$ represents element by element absolute values and 1_{nn} is an $n \times n$ matrix of 1's. Let $M = |\Lambda - 1_{nn}| - |\Lambda|$. Since $\|\Lambda\|_{\text{sum}}$ is constant and $M_{P_n} = |\Lambda_{P_n} - 1_{nn}| - |\Lambda_{P_n}|$, the optimization problem for selecting pairings based on RGA can be stated as

$$\min_{P_n \in \mathbb{P}(N_n)} \text{trace}(M_{P_n}) \quad (10)$$

$$\text{s.t. } [\Lambda_{P_n}]_{ii} > 0; \quad i = 1, 2, \dots, n \quad (11)$$

The key for applying BAB method for pairing selection is derivation of a tight lower bound on the selection criterion over the set of all pairing alternatives that can be reached from a node representing partially assigned pairings. Such a lower bound is proposed in the next proposition.

Proposition 6. For a node P_f with fixed set F_f and candidate set C_c , let \mathcal{S} in (5) represent the ensemble of all n -element ordered sets that can be obtained by expanding P_f . For $P_n \in \mathcal{S}$, let $M_{P_n} = |\Lambda_{P_n} - 1_{nn}| - |\Lambda_{P_n}|$ be partitioned as

$$M_{P_n} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (12)$$

where $M_{11} \in \mathbb{R}^{f \times f}$ and $M_{22} \in \mathbb{R}^{c \times c}$ correspond to the assigned and unassigned pairings, respectively. Define c -dimensional vectors r and t with elements

$$r_i = \min_j [M_{22}]_{ij} \quad \text{and} \quad t_j = \min_i [M_{22}]_{ij} \quad (13)$$

Then,

$$\min_{P_n \in \mathcal{S}} \|\Lambda_{P_n} - I\|_{\text{sum}} \geq \|\Lambda\|_{\text{sum}} + \text{trace}(M_{11}) + \max \left(\sum_{i=1}^c r_i, \sum_{j=1}^c t_j \right) \quad (14)$$

The BAB method can be applied for pairing selection based on RGA using the bound in Proposition 6. Note that the constraint on the sign of relative gains provides another alternative for pruning nodes. This constraint only needs to be checked for the element being moved from C_c to F_f during the expansion of node using branching rule in Definition 3. By deriving lower bounds similar to (14), the BAB approach can also be used for pairing selection using Hankel interaction measure [Wittenmark and Salgado, 2002] and variants of RGA, such as effective RGA [Xiong et al., 2006].

The reader should note that the optimization problem for pairing selection using RGA can also be cast as an MILP [Kookos and Perkins, 2001]. The original formulation proposed by Kookos and Perkins [2001] was for non-squared systems. For square systems, as considered in this paper, the formulation of Kookos and Perkins [2001]

is not very efficient (details not shown due to lack of space). Furthermore, the general BAB framework can also handle selection criterion, which cannot be represented as an MILP, as discussed next.

4. μ -INTERACTION MEASURE

The individual loops of a decentralized controller are often designed independent of each other. For assessing the feasibility of stabilizing the closed-loop system through the independent design method, Grosdidier and Morari [1986] proposed the μ -interaction measure (μ -IM). To present this method, let the columns of the transfer matrix $G(s)$ be permuted such that the chosen pairings lie along the diagonal and $\tilde{G}(s)$ represent the matrix consisting of the diagonal elements of $G(s)$. Then, the diagonal controller $K(s)$ stabilizing $\tilde{G}(s)$ also stabilizes $G(s)$, if [Grosdidier and Morari, 1986]

$$\bar{\sigma}(\tilde{T}(j\omega)) < \mu_{\Delta}^{-1}(E(j\omega)) \quad \forall \omega \in \mathbb{R} \quad (15)$$

where μ denotes the structured singular value [Skogestad and Postlethwaite, 2005], which is computed with a diagonally structured Δ , $\tilde{T}(s) = \tilde{G}K(s)(I + \tilde{T}K(s))^{-1}$ and

$$E(s) = (G(s) - \tilde{G}(s))\tilde{G}(s)^{-1} \quad (16)$$

The condition in (15) is called μ -IM. This powerful result allows the designer to impose restrictions on the individual controllers, but still design the controller solely based on $\tilde{G}(s)$ such that closed loop stability is ensured. We note that $E(s)$ is independent of the controller, but depends on the selected pairings. If the pairings are chosen such that $\mu_{\Delta}(E(j\omega))$ is small at all frequencies, the restrictions on decentralized controller synthesis using independent design method is minimum. $\mu_{\Delta}(E(j\omega))$ can also be seen as a measure of generalized diagonal dominance of the matrix $G(j\omega)$, where $G(j\omega)$ is said to be generalized diagonally dominant if [Skogestad and Postlethwaite, 2005]

$$\mu_{\Delta}(E(j\omega)) < 1 \quad (17)$$

If a pairing exist for which (17) holds, this pairing can be easily found using iterative RGA [Skogestad and Postlethwaite, 2005]. In absence of existence of such a pairing, a BAB method can be used to find the pairing for which $G(j\omega)$ is most diagonally dominant. In this paper, we consider pairing selection by minimizing $\mu_{\Delta}(E)$, but the proposed results hold for any other frequency. As the exact computation of μ is computationally intractable, we instead minimize the upper bound on μ (denoted as $\bar{\mu}$) obtained through D -scaling method [Skogestad and Postlethwaite, 2005]. Specifically, the following optimization problem is considered

$$\min_{P_n \in \mathbb{P}(N_n)} \bar{\mu}_{\Delta}(G_{P_n}(I \circ G_{P_n})^{-1} - I) \quad (18)$$

$$\text{s.t. } [\Lambda_{P_n}]_{ii} > 0; \quad i = 1, 2, \dots, n \quad (19)$$

A lower bound of (18) for the application of BAB method is derived next.

Proposition 7. For a node P_f with fixed set F_f and candidate set C_c , let \mathcal{S} in (5) represent the ensemble of all

n -element ordered sets that can be obtained by expanding P_f . For $P_n \in \mathcal{S}$, consider that $E = G_{P_n}(I \circ G_{P_n})^{-1} - I$ is decomposed as

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \quad (20)$$

where $E_{11} \in \mathbb{R}^{f \times f}$ and $E_{22} \in \mathbb{R}^{c \times c}$. Then

$$\min_{P_n \in \mathcal{S}} \bar{\mu}_{\Delta}(E) \geq \rho(E_{11}) \quad (21)$$

In comparison to $\rho(E_{11})$, $\bar{\mu}_{\Delta_1}(E_{11})$ provides a tighter lower bound on $\bar{\mu}_{\Delta}(E)$, where Δ_1 is $f \times f$ diagonal matrix. The computation of $\bar{\mu}_{\Delta_1}(E_{11})$, however, is costly. As a BAB method spends most of its time in evaluating nodes that cannot lead to the optimal solution, we use the computationally cheaper albeit weaker lower bound $\rho(E_{11})$ in this paper. With this lower bound, the computation of $\bar{\mu}$ is only required at the terminal nodes. Note that in (21), E_{11} corresponds to the assigned pairings. A similar idea can be used for deriving the lower bound for pairing selection based on ν -gap metric [Samyudia et al., 1995] using BAB approach.

5. NUMERICAL TESTS

In this section, we demonstrate the efficiency of the developed BAB methods through numerical examples. First, we use randomly generated matrices to check the average performance of these BAB algorithms. Subsequently, these methods are used for selecting pairings for the Tennessee Eastman benchmark problem [Downs and Vogel, 1993]. All these tests are carried out on a notebook with Intel[®] Core[™] Duo Processor T2400 (1.83 GHz, 2MB RAM) using MATLAB[®] 7.0.

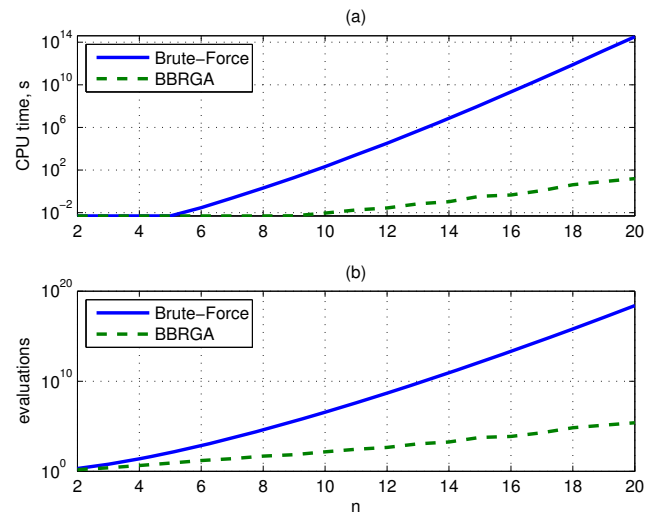


Fig. 2. Random test for pairing $n \times n$ systems based on the RGA-number, (a) computation time against n and (b) number of nodes evaluated against n ;

5.1 Randomly generated matrices

The efficiency of the BAB pairing approaches are firstly examined through some random tests. The RGA-number

based BAB algorithm (BBRGA) is applied to 100 randomly generated $n \times n$ matrices for every n ranging from 2 to 20. The average computation times and the average number of node evaluations required by BBRGA are shown in Figure 2. For comparison, the estimated computation time and the number of node evaluations ($n!$) required by a brute-force approach are also shown in Figure 2. For each n , the computation time required for a brute-force approach is estimated by multiplying $n!$ with the time required for evaluating the RGA-number of an $n \times n$ matrix (averaged over 10,000 instances).

It can be seen from Figure 2 that the BAB approach can easily handle matrix sizes as large as $n = 20$ within 1 minute. The brute-force based approach can only deal with matrix sizes up to $n = 9$ in the same amount of time. For $n > 8$, BBRGA is at least 100 times more efficient (in terms of both computation time and number of nodes evaluated) than the brute-force search.

A similar test is carried out for μ -IM based pairing selection. The BAB based approach (BBMU) is applied to 100 randomly generated $n \times n$ matrices for every n ranging from 2 to 10. The average computation time and number of node evaluations of BBMU are compared with the brute-force approach in Figure 3. For the brute-force approach, the computation time for every n is estimated as the product of $n!$ and the time required for evaluating $\bar{\mu}$ of an $n \times n$ matrix (averaged over 10 instances).

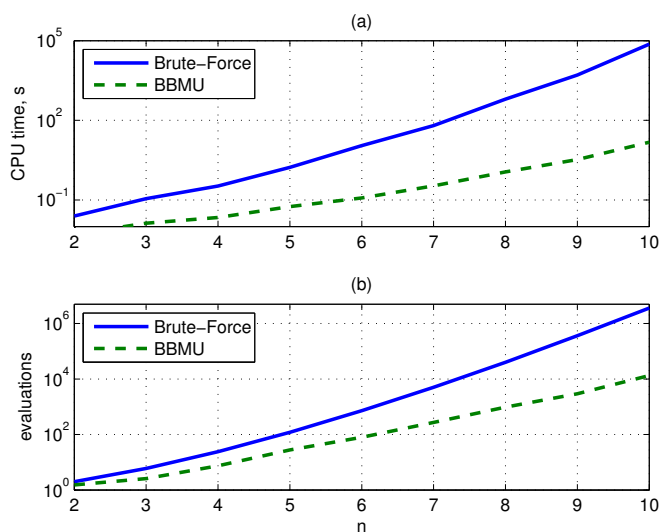


Fig. 3. Random test for pairing $n \times n$ systems based on the μ -IM, (a) computation time against n and (b) number of nodes evaluated against n ;

The lower bound on $\bar{\mu}(E)$ in (21) is loose. Therefore, the node evaluation improvement of BBMU over the brute-force search is not as large as that achieved by BBRGA. However, calculation of the lower bound $\rho(E_{11})$ is much faster than direct evaluation of $\bar{\mu}(E)$. Therefore, the improvement in computation time is still significant. Overall, BBMU is at least 100 times faster than the brute-force search for $n > 6$. Efforts are currently being made to find a tighter and computationally cheaper lower bound

on $\bar{\mu}(E)$ in order to improve the efficiency of the BAB approach further.

5.2 Tennessee Eastman benchmark problem

Finally, the RGA-based approach is applied to the Tennessee Eastman (TE) benchmark problem [Downs and Vogel, 1993], which has 12 manipulated variables and 22 (non-analyzer) measurements. For this process, McAvoy and Ye [1994] selected 10 out of 22 measurements to form the inner loops of cascade controllers. In addition, 3 measurements were used to stabilize the system. Finally, a 9×9 steady-state gain matrix corresponding to the outputs and inputs shown in Table 1 was provided for pairing selection.

Table 1. Input and output variables

Input	Description	Output	Description
u_1	A feed s.p.	y_1	Reactor feed
u_2	D feed s.p.	y_2	Reactor temp.
u_3	C feed s.p.	y_3	Reactor pressure
u_4	Purge s.p.	y_4	Separator temp.
u_5	Steam s.p.	y_5	Stripper temp.
u_6	Reactor cooling s.p.	y_6	Recycle flow
u_7	Separator cooling s.p.	y_7	Compressor power
u_8	Recycle valve	y_8	Separator pressure
u_9	Agitator speed	y_9	Stripper pressure

This problem has 362,880 pairing alternatives. BBRGA is able to find the best 50 pairing alternatives in less than 0.1 seconds with evaluation of only 843 nodes. 28 among these 50 pairing alternatives have the same minimum RGA-number value, 47237.6359. We also use BBMU, which finds 50 best pairing alternatives with positive RGA elements in about 10 seconds with evaluation of 4787 nodes. The 10 best pairing alternatives found using BBMU and their corresponding $\bar{\mu}(E)$ values are shown in Table 2.

Table 2. Best pairing alternatives using BBMU

$\bar{\mu}(E)$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
3.9635	u_7	u_6	u_4	u_2	u_5	u_9	u_8	u_1	y_3
3.9890	u_7	u_6	u_4	u_2	u_5	u_9	u_8	u_3	u_1
3.9988	u_3	u_6	u_4	u_9	u_5	u_7	u_8	u_1	u_2
4.0203	u_3	u_6	u_4	u_9	u_5	u_7	u_8	u_2	u_1
4.0871	u_7	u_6	u_4	u_9	u_5	u_2	u_8	u_1	u_3
4.1125	u_7	u_6	u_4	u_9	u_5	u_2	u_8	u_3	u_1
4.3469	u_7	u_6	u_4	u_3	u_5	u_9	u_8	u_1	u_2
4.3676	u_7	u_6	u_4	u_3	u_5	u_9	u_8	u_2	u_1
4.4243	u_3	u_6	u_4	u_2	u_5	u_9	u_8	u_7	u_1
4.4246	u_3	u_6	u_4	u_2	u_5	u_9	u_8	u_1	u_7

For this process, McAvoy and Ye [1994] used some intuitive knowledge to reduce the pairing selection problem to 36 cases of 4×4 and 5×5 systems. Among these cases, 4 schemes, which are shown in Table 3, were selected for detailed comparison. It seems to us that such an approach was undertaken due to the lack of availability of tools to handle large-scale processes. The promising pairing alternatives found in this paper cannot be directly compared with the pairings suggested by McAvoy and Ye [1994], as some pairing information is unavailable. It is interesting to note, however, that among the 10 best pairing alternatives found using BBMU, all contain

scheme 1, 6 of them include scheme 2 and 2 contain scheme 4. In comparison, no match is seen between the 28 pairings with minimum RGA-number and the suggestions made by McAvoy and Ye [1994]. One apparent reason for this discrepancy is that for pairing selection the use of RGA-number evaluated at bandwidth frequency has been recommended and not at steady-state [Skogestad and Postlethwaite, 2005]. Overall, it is demonstrated that the pairing selection problem for a large-scale process can be solved easily with the tools developed in this work.

Table 3. Pairing alternatives recommended by McAvoy and Ye [1994]

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
Scheme 1	n/a	u_6	u_4	n/a	u_5	n/a	u_8	n/a	n/a
Scheme 2	u_7	u_6	u_4	n/a	u_5	n/a	u_8	n/a	n/a
Scheme 3	n/a	u_6	u_4	u_7	u_5	n/a	u_8	n/a	n/a
Scheme 4	n/a	u_6	u_4	n/a	u_5	u_7	u_8	n/a	n/a

6. CONCLUSIONS

A branch and bound framework for input-output pairing selection is developed. The generality and efficiency of this framework is demonstrated using two different pairing selection criteria, *i.e.* the RGA-number and the μ -IM, where a reduction of several orders of magnitude in solution time is seen over brute-force search. Future work will focus on development of tighter lower bounds and alternate pruning strategies in order to improve the efficiency further. To this end, the pairing selection problem has many similarities to the well-studied Traveling Salesman Problem (TSP); (see *e.g.* [Gutin and Punnen, 2002]). We will aim at adapting some of the pruning and branching strategies used for solving the TSP problem using BAB algorithms for the pairing selection problem.

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