Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Guidance and Robust Gyromoment Precise Attitude Control of Agile Observation Spacecraft ⋆

Ye. Somov [*,‡] **S. Butyrin** [*,‡] **S. Somov** [†,*]

[*] *Samara Scientific Center, Russian Academy of Sciences, Samara, 443001 Russia (e-mail: e_somov@mail.ru)*
[†] *Korolev Samara State Aerospace University, Samara, 443086 Russia (e-mail: s_somov@mail.ru)*
[‡] *State Research & Production Rocket-Space Center "TsSKB-Progress", Federal Space Agency, Samara, 443009 Russia (e-mail: somov@pochta.ru)*

Abstract: Some problems on guidance and robust gyromoment attitude control of agile spacecraft for remote sensing the Earth surface are considered. Elaborated methods for dynamic research of the spacecraft programmed angular motion at principle modes under external and parametric disturbances, partial discrete measurement of the state and digital control of the gyro moment cluster by the few-excessive gyrodine schemes, are presented.

1. INTRODUCTION

The dynamic requirements to the attitude control systems (ACSs) for remote sensing spacecraft (SC) are:

- guidance the telescope's line-of-sight to a predetermined part of the Earth surface with the scan in designated direction;
- stabilization of an image motion at the onboard optical telescope focal plane.

Moreover, these requirements are expressed by rapid angular manoeuvering and spatial compensative motion with a variable vector of angular rate. Increased requirements to such information satellites (lifetime up to 10 years, exactness of spatial rotation manoeuvers with effective damping the SC flexible structure oscillations, robustness, fault-tolerance as well as to reasonable mass, size and energy characteristics) have motivated intensive development the gyro moment clusters (GMCs) based on excessive number of gyrodines (GDs) — single-gimbal control moment gyros. Mathematical aspects of the SC nonlinear gyromoment control were represented in a number of research works, see Junkins and Turner [1986], Hoelscher and Vadali [1994] et al. including authors' papers. The paper suggests new results on guidance and nonlinear robust gyromoment attitude control of the agile observation spacecraft.

2. MATHEMATICAL MODELS

Let us introduce the inertial reference frame (IRF) $\mathbf{I}_\oplus$ ($\mathrm{O}_\oplus \mathrm{X}_\mathrm{e}^\mathrm{I}\mathrm{Y}_\mathrm{e}^\mathrm{I}\mathrm{Z}_\mathrm{e}^\mathrm{I}$), the geodesic Greenwich reference frame (GRF) $\mathbf{E}_\mathrm{e}$ ($\mathrm{O}_\oplus\mathrm{X}^\mathrm{e}\mathrm{Y}^\mathrm{e}\mathrm{Z}^\mathrm{e}$) which is rotated with respect to the IRF by angular rate vector $\boldsymbol{\omega}_\oplus \equiv \boldsymbol{\omega}_\mathrm{e}$ and the geodesic horizon reference frame (HRF) $\mathbf{E}_\mathrm{e}^\mathrm{h}$ ($\mathrm{C}\,\mathrm{X}_\mathrm{c}^\mathrm{h}\mathrm{Y}_\mathrm{c}^\mathrm{h}\mathrm{Z}_\mathrm{c}^\mathrm{h}$) with origin in a point C and ellipsoidal geodesic coordinates altitude H_c, longitude L_c and latitude B_c. There are standard defined the body reference frame (BRF) $\mathbf{B}$ ($\mathrm{O}xyz$) with origin in the SC mass center O, the orbit reference frame (ORF) $\mathbf{O}$ ($\mathrm{O}x^\mathrm{o}y^\mathrm{o}z^\mathrm{o}$), the optical telescope (sensor) reference frame (SRF) $\boldsymbol{\mathcal{S}}$ ($\mathrm{O}x^\mathrm{s}y^\mathrm{s}z^\mathrm{s}$) and the image field reference frame (FRF) $\boldsymbol{\mathcal{F}}$ ($\mathrm{O}_i x^i y^i z^i$) with origin in center O_i of the telescope focal plane $y^i\mathrm{O}_i z^i$. The BRF attitude with respect to the IRF is defined by quaternion $\boldsymbol{\Lambda}_\mathrm{I}^b \equiv \boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda}), \boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$. Let us vectors $\boldsymbol{\omega}(t)$, $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are standard denotations of the SC body vector angular rate, the SC mass center's position and progressive velocity with respect to the IRF. Further the symbols $\langle\cdot,\cdot\rangle$, $\times$, $\{\,\cdot\,\}$, $[\,\cdot\,]$ for vectors and $[\mathbf{a}\times]$, $(\cdot)^\mathrm{t}$ for matrixes are conventional denotations. The GMC's angular momentum (AM) vector $\boldsymbol{\mathcal{H}}$ have the form $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = \mathrm{h}_g \sum \mathbf{h}_p(\beta_p)$, there h_g is constant own AM value for each GD $p = 1, \ldots m \equiv 1 \div m$ with the GD's AM unit $\mathbf{h}_p(\beta_p)$ and vector-column $\boldsymbol{\beta} = \{\beta_p\}$. Within precession theory of the control moment gyros, for a fixed position of the SC flexible structures with some simplifying assumptions and for $t \in \mathrm{T}_{t_0} = [t_0, +\infty)$ a SC angular motion model is appeared as

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda}\circ\boldsymbol{\omega}/2; \quad \mathbf{A}^o\{\dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}\} = \{\mathbf{F}^\omega, \mathbf{F}^q\}, \qquad (1)$$

where $\boldsymbol{\omega} = \{\omega_i, i = x, y, z \equiv 1 \div 3\}$, $\mathbf{q} = \{q_j, j = 1 \div n^q\}$,

$$\mathbf{F}^\omega = \mathbf{M}^\mathrm{g} - \boldsymbol{\omega}\times\mathbf{G} + \mathbf{M}_d^o(t, \boldsymbol{\Lambda}, \boldsymbol{\omega}) + \mathbf{Q}^o(\boldsymbol{\omega}, \dot{\mathbf{q}}, \mathbf{q});$$

$$\mathbf{F}^q = \{-((\delta^q/\pi)\Omega_j^q \dot{q}_j + (\Omega_j^q)^2 q_j) + \mathrm{Q}_j^q(\boldsymbol{\omega}, \dot{q}_j, q_j)\};$$

$$\mathbf{A}^o = \begin{bmatrix} \mathbf{J} & \mathbf{D}_q \\ \mathbf{D}_q^\mathrm{t} & \mathbf{I} \end{bmatrix}; \quad \begin{array}{l} \mathbf{G} = \mathbf{G}^o + \mathbf{D}_q\dot{\mathbf{q}}; \quad \mathbf{M}^\mathbf{g} = -\dot{\boldsymbol{\mathcal{H}}} = -\mathbf{A}_\mathrm{h}(\boldsymbol{\beta})\dot{\boldsymbol{\beta}}; \\ \mathbf{G}^o = \mathbf{J}\,\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}); \ \mathbf{A}_\mathrm{h}(\boldsymbol{\beta}) = \partial\mathbf{h}(\boldsymbol{\beta})/\partial\boldsymbol{\beta}, \end{array}$$

vector-column $\mathbf{M}_d^o(\cdot)$ presents an external torque disturbance, and $\mathbf{Q}^o(\cdot)$, $\mathrm{Q}_j^q(\cdot)$ are nonlinear continuous functions.

The GMC torque vector $\mathbf{M}^\mathbf{g}$ is presented as follows:

$$\mathbf{M}^\mathbf{g} = \mathbf{M}^\mathbf{g}(\boldsymbol{\beta}, \dot{\boldsymbol{\beta}}) = -\dot{\boldsymbol{\mathcal{H}}} = -h_g\mathbf{A}_\mathrm{h}(\boldsymbol{\beta})\,\mathbf{u}^\mathrm{g}; \quad \dot{\boldsymbol{\beta}} = \mathbf{u}^\mathrm{g}. \quad (2)$$

Here $\mathbf{u}^\mathrm{g} = \{\mathrm{u}_p^\mathrm{g}\}$, $\mathrm{u}_p^\mathrm{g}(t) = a^\mathrm{g}\,\mathrm{Zh}[\mathrm{Sat}(\mathrm{Qntr}(u_{pk}^g, d^g), \bar{\mathrm{u}}_\mathrm{g}^\mathrm{m}), T_u]$ with constants a^g, d^g, $\bar{\mathrm{u}}_\mathrm{g}^\mathrm{m}$ and a control period $T_u = t_{k+1} - t_k$, $k \in \mathbb{N}_0 \equiv [0, 1, 2, \ldots)$; discrete functions $u_{pk}^g \equiv u_p^g(t_k)$

⋆ The work was supported by Presidium of Russian Academy of Sciences (RAS) (Pr. 22), Division on EMMCP of the RAS (Pr. 15, 18) and the RFBR (Grants 05-08-18175, 07-08-97611, 08-08-99101)

are outputs of digital nonlinear control law (CL), and functions $\mathrm{Sat}(x,a)$ and $\mathrm{Qntr}(x,a)$ are general-usage ones, while the holder model with the period T_u is such: $y(t) = \mathrm{Zh}[x_k, T_u] = x_k \; \forall t \in [t_k, t_{k+1})$.

At given the SC body angular programmed motion $\mathbf{\Lambda}^p(t)$, $\boldsymbol{\omega}^p(t)$, $\boldsymbol{\varepsilon}^p(t) = \dot{\boldsymbol{\omega}}^p(t)$ with respect to the IRF $\mathbf{I}_\oplus$ during time interval $t \in \mathrm{T} \equiv [t_\mathrm{i}, t_\mathrm{f}] \subset \mathrm{T}_{t_0}$, $t_\mathrm{f} \equiv t_\mathrm{i} + T$, and for forming the vector of corresponding continuous control torque $\mathbf{M}^\mathbf{g}(\boldsymbol{\beta}(t), \dot{\boldsymbol{\beta}}(t))$ (2), the vector-columns $\dot{\boldsymbol{\beta}} = \{\dot{\beta}_p\}$ and $\ddot{\boldsymbol{\beta}} = \{\ddot{\beta}_p\}$ must be component-wise module restricted:

$$|\dot{\beta}_p(t)| \le \bar{\mathrm{u}}_\mathrm{g} < \bar{\mathrm{u}}_\mathrm{g}^\mathrm{m}, \;\; |\ddot{\beta}_p(t)| \le \bar{\mathrm{v}}_\mathrm{g}, \; \forall t \in \mathrm{T}, \; p = 1 \div m, \quad (3)$$

where values $\bar{\mathrm{u}}_\mathrm{g}$ and $\bar{\mathrm{v}}_\mathrm{g}$ are constant.

Collinear pair of two stopless GDs was named as *Scissored Pair Ensemble* (*SPE*) in well-known original work *J.W. Crenshaw* (1973). Redundant multiply scheme, based on six gyrodines in the form of three collinear GD's pairs, was named as *3-SPE*. Fig. 1 presents a simplest arrangement of this scheme into a canonical orthogonal gyroscopic basis $\mathrm{O}x_\mathrm{c}^\mathrm{g} y_\mathrm{c}^\mathrm{g} z_\mathrm{c}^\mathrm{g}$. By a slope of the GD pairs suspension axes in this basis it is possible to change essentially a form of the AM variation domain $\mathbf{S}$ at any direction. Based on four gyrodines the minimal redundant scheme *2-SPE* is easily obtained from the *3-SPE* scheme – without third pair (GD #5 and GD #6). In park state of above schemes one can have a vector of summary normed GMC's AM $\mathbf{h}(\boldsymbol{\beta}) \equiv \sum \mathbf{h}_p(\beta_p) = \mathbf{0}$.

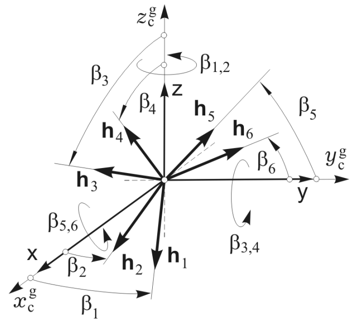

Fig. 1. The scheme *3-SPE*

3. THE PROBLEM STATEMENT

Principle problem gets up on the SC angular guidance at a spatial course motion (SCM) when a space opto-electronic observation is executed at given time interval $t \in \mathrm{T}_n \equiv [t_\mathrm{i}^n, t_\mathrm{f}^n]$. This problem consists in determination of quaternion $\mathbf{\Lambda}(t)$ by the SC BRF $\mathbf{B}$ attitude with respect to the IRF $\mathbf{I}_\oplus$, angular rate vector $\boldsymbol{\omega}(t)$, vectors of angular acceleration $\boldsymbol{\varepsilon}(t)$ and its derivative $\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$ in the form of explicit functions, proceed from principle requirement: optical image of the Earth given part must to move by desired way at focal plane $y^i \mathrm{O}_i z^i$ of the telescope.

Into IRF the SC's spatial rotation maneuver (SRM) is described by kinematic relations

$$\dot{\mathbf{\Lambda}}(t) = \frac{1}{2} \mathbf{\Lambda} \circ \boldsymbol{\omega}(t); \; \dot{\boldsymbol{\omega}}(t) = \boldsymbol{\varepsilon}(t); \; \dot{\boldsymbol{\varepsilon}}(t) = \mathbf{v} \qquad (4)$$

during given time interval $t \in \mathrm{T}_p \equiv [t_\mathrm{i}^p, t_\mathrm{f}^p]$, $t_\mathrm{f}^p \equiv t_\mathrm{i}^p + T_p$. The optimization problem consists in determination of time functions $\mathbf{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ for the boundary conditions on left ($t = t_\mathrm{i}^p$) and right ($t = t_\mathrm{f}^p$) trajectory ends

$$\mathbf{\Lambda}(t_\mathrm{i}^p) = \mathbf{\Lambda}_\mathrm{i}; \; \boldsymbol{\omega}(t_\mathrm{i}^p) = \boldsymbol{\omega}_\mathrm{i}; \; \boldsymbol{\varepsilon}(t_\mathrm{i}^p) = \boldsymbol{\varepsilon}_\mathrm{i}; \qquad (5)$$

$$\mathbf{\Lambda}(t_\mathrm{f}^p) = \mathbf{\Lambda}_\mathrm{f}; \; \boldsymbol{\omega}(t_\mathrm{f}^p) = \boldsymbol{\omega}_\mathrm{f}; \; \boldsymbol{\varepsilon}(t_\mathrm{f}^p) = \boldsymbol{\varepsilon}_\mathrm{f} \qquad (6)$$

with optimization of the integral quadratic index

$$\mathrm{I_o} = \frac{1}{2} \int_{t_\mathrm{i}^p}^{t_\mathrm{f}^p} \langle \mathbf{v}(\tau), \mathbf{v}(\tau) \rangle \, d\tau \Rightarrow \min. \qquad (7)$$

Onboard algorithms are needed for the SC guidance at a SRM taking into account the restrictions (3) to vectors $\dot{\boldsymbol{\beta}}(t)$ and $\ddot{\boldsymbol{\beta}}(t)$. Here for given time interval T_p a problem consists in determination the explicit time functions $\mathbf{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\varepsilon(t)$ and $\dot{\boldsymbol{\varepsilon}}(t)$ for the boundary conditions (5), (6) and also for given condition

$$\dot{\boldsymbol{\varepsilon}}(t_\mathrm{f}^p) = \dot{\boldsymbol{\varepsilon}}_\mathrm{f} \equiv \boldsymbol{\varepsilon}_\mathrm{f}^* + \boldsymbol{\omega}_\mathrm{f} \times \boldsymbol{\varepsilon}_\mathrm{f}, \qquad (8)$$

which presents requirements to a *smooth conjugation* of guidance by a SRM with guidance at next the SC SCM.

Applied onboard measuring subsystem is based on inertial gyro unit corrected by the fine fixed-head star trackers. Contemporary filtering & alignment calibration algorithms give finally a fine discrete estimating the SC angular motion coordinates by the quaternion $\mathbf{\Lambda}_s^\mathrm{m} = \mathbf{\Lambda}_s \circ \mathbf{\Lambda}_s^\mathrm{n}$, $s \in \mathbb{N}_0$, where $\mathbf{\Lambda}_s \equiv \mathbf{\Lambda}(t_s)$, $\mathbf{\Lambda}_s^\mathrm{n}$ is a "noise-drift" digital quaternion and a measurement period $T_q = t_{s+1} - t_s \le T_u$ is multiply with respect to a control period T_u.

At a land-survey SC lifetime up to 5 years its structure inertial and flexible characteristics are slowly changed in wide boundaries, the solar array panels are rotated with respect to the SC body and the communication antennas are pointing for information service. Therefore inertial matrix $\mathbf{A}^\mathrm{o}$ (1) and partial frequencies Ω_j^q of the SC structure are not complete certain. Problems consist in synthesis of the SC guidance laws at its both the SCM and the SRM, and also in dynamical designing the GMC's robust digital control law $\mathbf{u}_k^g = \{u_{pk}^g\}$ on the quaternion values $\mathbf{\Lambda}_s^\mathrm{m}$ when the SC structure characteristics are uncertain and its damping is very weak, decrement of the SC structure oscillations $\delta_j^q \approx 5 \cdot 10^{-3}$ in (1).

4. SYNTHESIS OF FEEDBACK CONTROL

Applied general approach to synthesis of nonlinear control system (NCS) is presented, moreover the method of *vector Lyapunov functions* (VLF) is used in cooperation with the *exact feedback linearization* (EFL) technique. Let there be given a nonlinear controlled object

$$D^+\mathrm{x}(t) = \mathcal{F}(\mathrm{x}(t), \mathrm{u}); \;\; \mathrm{x}(t_0) = \mathrm{x}_0; \; t \in \mathrm{T}_{t_0},$$

where $\mathrm{x}(t) \in \mathcal{H} \subset \mathbb{R}^n$ is a state vector with an initial condition $\mathrm{x}_0 \in \mathcal{H}_0 \subseteq \mathcal{H}$ and vector-column $\mathrm{u} = \{u_j\} \in \mathrm{U} \subset \mathbb{R}^r$ is a control vector. Let some *vector norms* $\rho(\mathrm{x}) \in \overline{\mathbb{R}}_+^l$ and $\rho^0(\mathrm{x}_0) \in \overline{\mathbb{R}}_+^{l_0}$ also be *given*. For any control law (CL) $\mathrm{u} = \mathcal{U}(\mathrm{x})$ the closed-loop system has the form

$$D^+\mathrm{x}(t) = \mathcal{X}(t, \mathrm{x}); \;\; \mathrm{x}(t_0) = \mathrm{x}_0, \qquad (9)$$

where $\mathcal{X}(t, \mathrm{x}) = \mathcal{F}(\mathrm{x}, \mathcal{U}(\mathrm{x}))$, $\mathcal{X} : \mathrm{T}_{t_0} \times \mathcal{H} \rightarrow \mathcal{H}$ is a *discontinuous* operator. Assuming the existence and the non-local continuability of the *right-sided* solution $x(t) \equiv \mathrm{x}(t_0, \mathrm{x}_0; t)$ of the system (9) for its extended definition, there is obtained the property of $\rho\rho^0$-exponential invariance by the solution $x(t) = 0$ under the desired $\gamma \in \overline{\mathbb{R}}_+^l$:

$$(\exists \alpha \in \mathbb{R}_+) \; (\exists \mathcal{B} \in \overline{\mathrm{B}}_+^{l \times l_0}) \; (\exists \delta \in \mathbb{R}_+^{l_0}) \; (\forall \rho^0(\mathrm{x}_0) < \delta)$$

$$\rho(x(t)) \le \gamma + \mathcal{B} \; \rho^0(\mathrm{x}_0) \, \exp(-\alpha(t - t_0)) \quad \forall t \in \mathrm{T}_{t_0}.$$

The basis of this inequality for vector norm $\rho\,(x(t))$ is attained by the *comparison principle*, using the maximum right-sided solution of a comparison system for the VLF, see *Theorem 1* in Somov et al. [1999].

There is such an important problem: by what approach is it possible to create *constructive* techniques for constructing the VLF $v(\mathrm{x})$ and *simultaneous* synthesis of a nonlinear control law $\mathrm{u} = \mathcal{U}(\mathrm{x})$ for the close-loop system (9) with given vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x}_0)$? Recently, a pithy technique have been elaborated, which is based on a *nonlinear transformation* of the NCS model and solving the problem in two stages. *In stage 1*, the right side $\mathcal{F}(\cdot)$ in (9) is transformed as $\mathcal{F}(\cdot)=\mathrm{f}(\mathrm{x})+\mathrm{G}(\mathrm{x})\,\mathrm{u}+\tilde{\mathcal{F}}(t,\mathrm{x}(t),\mathrm{u})$, some principal variables in a state vector $\mathrm{x}\in\tilde{\mathcal{H}}\subset\mathbb{R}^{\tilde{n}}\subseteq\mathbb{R}^n$ with $\tilde{n}\le n$, $\mathrm{x}_0\in\tilde{\mathcal{H}}_0\subseteq\tilde{\mathcal{H}}$ are selected and a *simplified nonlinear model* of the object (9) is presented in the form of an affine quite smooth nonlinear control system

$$\dot{\mathrm{x}}=\mathrm{F}(\mathrm{x},\mathrm{u})\equiv\mathrm{f}(\mathrm{x})+\mathrm{G}(\mathrm{x})\mathrm{u}\equiv\mathrm{f}(\mathrm{x})+\sum \mathrm{g}_j(\mathrm{x})u_j,$$

which is structurally synthesized by the EFL technique. In this aspect, based on the structural analysis of *given* vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x})$, and also vector-functions $\mathrm{f}(\mathrm{x})$ and $\mathrm{g}_j(\mathrm{x})$, the *output vector-function* $\mathrm{h}(\mathrm{x})=\{\mathrm{h}_i(\mathrm{x})\}$ is carefully selected. Furthermore, the nonlinear invertible (one-to-one) coordinate transformation $\mathrm{z} = \Phi(\mathrm{x})\ \forall\mathrm{x}\in\mathcal{S}_\mathrm{h}\subseteq\tilde{\mathcal{H}}$ with $\Phi(0) = 0$ is analytically obtained with *simultaneous* constructing the VLF. Finally, bilateral component-wise inequalities for the vectors $\mathrm{x}, \mathrm{z}, v(\mathrm{x}), \rho(\mathrm{x}), \rho^0(\mathrm{x}_0)$ are derived, it is most desirable to obtain the *explicit* form for the nonlinear transformation $\mathrm{x} = \Psi(\mathrm{z})$, inverse with respect to $\mathrm{z} = \Phi(\mathrm{x})$, and the VLF aggregation procedure is carried out with analysis of proximity for a singular directions in the *Jacobian* $[\partial\mathrm{F}(\mathrm{x},\mathcal{U}(\mathrm{x}))/\partial\mathrm{x}]$. *In stage 2*, the problem of nonlinear CL synthesis for complete model of the NCS (9) is solved by the VLF-method, taking into account rejected coordinates, nonlinearities and restrictions on control. If a forming control is digital, a measurement of the state is discrete and incomplete, then a simplified nonlinear discrete object's model is obtained by *Teylor-Lie* series, a *nonlinear* digital CL is formed and its parametric synthesis is carried out with a simultaneously constructing a *discrete* VLF.

5. GUIDANCE AT A COURSE MOTION

Analytic matching solution have been obtained for problem of the SC angular guidance at the SCM at given time interval $t\in\mathrm{T}_n$. The solution is based on a vector composition of all elemental motions in the GRF $\mathbf{E}_\mathrm{e}$ using next reference frames: the HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, the SRF $\boldsymbol{\mathcal{S}}$ and the FRF $\boldsymbol{\mathcal{F}}$. Vectors $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are presented in the GRF $\mathbf{E}_\mathrm{e}$ as $\mathbf{r}^\mathrm{e}=\mathbf{T}_\mathrm{I}^\mathrm{e}\,\mathbf{r}$ and $\mathbf{v}^\mathrm{e}=\mathbf{T}_\mathrm{I}^\mathrm{e}\,(\mathbf{v}-[\omega_\oplus\mathbf{i}_3\times]\mathbf{r}_\mathrm{o})$, $\mathbf{T}_\mathrm{I}^\mathrm{e}=[\rho_\mathrm{e}(t)]_3$ and $\rho_\mathrm{e}(t)=\rho_\mathrm{e}^\mathrm{i}+\omega_\oplus(t-t_\mathrm{i})$. Vectors $\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}$ and $\mathbf{v}_\mathrm{e}^\mathrm{s}$ are defined as

$$\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}=\{\omega_{\mathrm{e}i}^\mathrm{s}\}=\mathbf{T}_b^\mathrm{s}(\boldsymbol{\omega}-\tilde{\boldsymbol{\Lambda}}\circ\omega_\oplus\mathbf{i}_3\circ\boldsymbol{\Lambda});\ \ \mathbf{v}_\mathrm{e}^\mathrm{s}=\tilde{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s}\circ\mathbf{v}_\mathrm{o}^\mathrm{e}\circ\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s},$$

where $\boldsymbol{\Lambda}=\boldsymbol{\Lambda}_\mathrm{I}^b;\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s}=\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{I}\circ\boldsymbol{\Lambda}_\mathrm{I}^b\circ\boldsymbol{\Lambda}_b^\mathrm{s}$, $\dot{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s}=\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s}\circ\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}/2$, and constant matrix $\mathbf{T}_b^\mathrm{s}$ represents the telescope fixation on the SC body. For any observed point C the oblique range D is analytically calculated as $\mathrm{D}=|\mathbf{r}_\mathrm{c}^\mathrm{e}-\mathbf{r}^\mathrm{e}|$. If orthogonal matrix $\mathbf{C}_\mathrm{h}^\mathrm{s}\equiv\tilde{\mathbf{C}}=\|\tilde{c}_{ij}\|$ defines the SRF $\boldsymbol{\mathcal{S}}$ attitude with respect to the HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, then for any point $\mathrm{M}(\tilde{y}^i,\tilde{z}^i)$ at the telescope focal plane $y^i\mathrm{O}_i z^i$ the components $\tilde{V}_y^i$ and $\tilde{V}_z^i$ of normed vector by an image motion velocity is appeared as

$$\begin{bmatrix}\tilde{V}_y^i\\ \tilde{V}_z^i\end{bmatrix}\equiv\begin{bmatrix}\dot{\tilde{y}}^i\\ \dot{\tilde{z}}^i\end{bmatrix}=\begin{bmatrix}\tilde{y}^i & 1 & 0\\ \tilde{z}^i & 0 & 1\end{bmatrix}\begin{bmatrix}q^i\tilde{\mathrm{v}}_{\mathrm{e}1}^\mathrm{s}-\tilde{y}^i\ \omega_{\mathrm{e}3}^\mathrm{s}+\tilde{z}^i\ \omega_{\mathrm{e}2}^\mathrm{s}\\ q^i\tilde{\mathrm{v}}_{\mathrm{e}2}^\mathrm{s}-\quad\omega_{\mathrm{e}3}^\mathrm{s}-\tilde{z}^i\ \omega_{\mathrm{e}1}^\mathrm{s}\\ q^i\tilde{\mathrm{v}}_{\mathrm{e}3}^\mathrm{s}+\quad\omega_{\mathrm{e}2}^\mathrm{s}+\tilde{y}^i\ \omega_{\mathrm{e}1}^\mathrm{s}\end{bmatrix}. \quad (10)$$

Here normed focal coordinates $\tilde{y}^i = y^i/f_e$ and $\tilde{z}^i = z^i/f_e$, where f_e is the telescope equivalent focal distance; function $q^i\equiv1-(\tilde{c}_{21}\tilde{y}^i+\tilde{c}_{31}\tilde{z}^i)/\tilde{c}_{11}$, and vector of normed SC's mass center velocity have the components $\tilde{\mathrm{v}}_{\mathrm{e}i}^\mathrm{s}=\mathrm{v}_{\mathrm{e}i}^\mathrm{s}/\mathrm{D}$, $i=1\div3$. For given image velocity $\tilde{\mathrm{W}}_y^s=$const and conditions

$$\tilde{V}_y^i(0,0)=\tilde{\mathrm{W}}_y^i=-\tilde{\mathrm{W}}_y^s;\ \tilde{V}_z^i(0,0)=0;\ \partial\tilde{V}_y^i(0,0)/\partial\tilde{z}^i=0$$

calculation of vector $\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}$ is carried out by the relations

$$\omega_{\mathrm{e}1}^\mathrm{s}=-\tilde{\mathrm{v}}_{\mathrm{e}2}^\mathrm{s}\tilde{c}_{31}/\tilde{c}_{11};\ \omega_{\mathrm{e}2}^\mathrm{s}=-\tilde{\mathrm{v}}_{\mathrm{e}3}^\mathrm{s};\ \omega_{\mathrm{e}3}^\mathrm{s}=-\tilde{W}_y^i+\tilde{\mathrm{v}}_{\mathrm{e}2}^\mathrm{s}. \quad (11)$$

By numerical solution of the quaternion differential equation $\dot{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s}=\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s}\circ\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}/2$ with regard to (11) one can obtain values $\boldsymbol{\lambda}_{\mathrm{e}s}^\mathrm{s}\equiv\boldsymbol{\lambda}_\mathrm{e}^\mathrm{s}(t_s)$ for the discrete time moments $t_s\in\mathrm{T}_n$ with period T_q, $s=0\div n_q$, $n_q=T_n/T_q$ when initial value $\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s}(t_\mathrm{i}^n)$ is given. Further solution is based on the elegant extrapolation of values $\boldsymbol{\sigma}_{\mathrm{e}s}^\mathrm{s}=\boldsymbol{\lambda}_{\mathrm{e}s}^\mathrm{s}/(1+\lambda_{0\,\mathrm{e}s}^\mathrm{s})$ by the vector of modified Rodrigues parameters and values $\boldsymbol{\omega}_{\mathrm{e}s}^\mathrm{s}$ by the angular rate vector. The extrapolation is carried out by two sets of n_q coordinated 3-degree vector splines with analytical obtaining a high-precise approximation of the SRF $\boldsymbol{\mathcal{S}}$ guidance motion with respect to the GRF $\mathbf{E}_\mathrm{e}$ both on vector of angular acceleration and on vector of its local derivative. At last stage, required functions $\boldsymbol{\Lambda}(t), \boldsymbol{\omega}(t), \boldsymbol{\varepsilon}(t)$ and $\dot{\boldsymbol{\varepsilon}}(t)=\boldsymbol{\varepsilon}^*(t)+\boldsymbol{\omega}(t)\times\boldsymbol{\varepsilon}(t)$ is calculated by *explicit* formulas. These functions are applied at onboard computer for the time moments $t_s\in\mathrm{T}_n$, and also for calculation (10) of the image velocity at any point $\mathrm{M}(\tilde{y}^i,\tilde{z}^i)$ into the telescope focal plane for any $t\in\mathrm{T}_n$.

6. A ROTATION MANEUVER OPTIMIZATION

Optimal one-axis problem is very simple, the SC optimal motion with respect to any k axis is presented by the analytic function $\varphi_k(t)$ in a class of the five degree polynomials (splines) by normed time $\tau=(t-t_\mathrm{i}^p)/T_p\subset[0,1]$.

Developed analytical approach to the problem is based on necessary and sufficient condition for solvability of Darboux problem. At general case the solution is presented as result of composition by three ($k=1\div3$) simultaneously derived elementary rotations of embedded bases $\mathbf{E}_k$ about units $\mathbf{e}_k$ of Euler axes, which positions are defined from the boundary conditions (6) and (7) for initial spatial problem. For all 3 elementary rotations with respect to units $\mathbf{e}_k$ the boundary conditions are analytically assigned. Into the IRF $\mathbf{I}_\oplus$ the quaternion $\boldsymbol{\Lambda}(t)$ is defined by the production

$$\boldsymbol{\Lambda}(t)=\boldsymbol{\Lambda}_\mathrm{i}\circ\boldsymbol{\Lambda}_1(t)\circ\boldsymbol{\Lambda}_2(t)\circ\boldsymbol{\Lambda}_3(t), \quad (12)$$

where $\boldsymbol{\Lambda}_k(t)=(\cos(\varphi_k(t)/2),\sin(\varphi_k(t)/2)\mathbf{e}_k)$, and functions $\varphi_k(t)$ analytically present the elementary rotation angles. Let the quaternion $\boldsymbol{\Lambda}^*\equiv(\lambda_0^*,\boldsymbol{\lambda}^*)=\tilde{\boldsymbol{\Lambda}}_\mathrm{i}\circ\boldsymbol{\Lambda}_\mathrm{f}\ne\mathbf{1}$ have the Euler axis unit $\mathbf{e}_3=\boldsymbol{\lambda}^*/\sin(\varphi^*/2)$ by 3-rd elementary rotation, where angle $\varphi^*=2\arccos(\lambda_0^*)$. For elementary rotations there are applied the boundary quaternions:

$$\begin{aligned}&\boldsymbol{\Lambda}_1(t_\mathrm{i}^p)=\boldsymbol{\Lambda}_1(t_\mathrm{f}^p)=\boldsymbol{\Lambda}_2(t_\mathrm{i}^p)=\boldsymbol{\Lambda}_2(t_\mathrm{f}^p)=\boldsymbol{\Lambda}_3(t_\mathrm{i}^p)=\mathbf{1};\\ &\boldsymbol{\Lambda}_3(t_\mathrm{f}^p)=(\cos(\varphi_3^\mathrm{f}/2),\mathbf{e}_3\sin(\varphi_3^\mathrm{f}/2)),\end{aligned} \quad (13)$$

where $\varphi_3^\mathrm{f}=\varphi^*$ and $\mathbf{1}$ is a single quaternion. Unit $\mathbf{e}_1$ of 1-st elementary rotation's Euler axis is selected by simple algorithm [Somov, 2007] and then unit $\mathbf{e}_2$ is defined as $\mathbf{e}_2=\mathbf{e}_3\times\mathbf{e}_1$. All vectors $\boldsymbol{\omega}_k(t)$, $\boldsymbol{\varepsilon}_k(t)$ and $\dot{\boldsymbol{\varepsilon}}_k(t)$ have analytic form which is optimal on index (7) for each elementary rotation. Vectors $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\dot{\boldsymbol{\varepsilon}}(t)\equiv\mathbf{v}(t)$ are analytically defined by recurrent algorithm [Somov et al., 2007] using functions $\varphi_k(t)$ and their derivatives.

For nonlinear problem (4) – (7) Hamilton function

$$H = -\frac{1}{2}\langle \mathbf{v}, \mathbf{v}\rangle + \frac{1}{2}\langle \mathrm{vect}(\tilde{\mathbf{\Lambda}} \circ \mathbf{\Psi}), \boldsymbol{\omega}\rangle + \langle \boldsymbol{\mu}, \boldsymbol{\varepsilon}\rangle + \langle \boldsymbol{\nu}, \mathbf{v}\rangle$$

have associated variables – vectors $\boldsymbol{\mu}, \boldsymbol{\nu}$ and quaternion $\mathbf{\Psi} = \mathbf{C}_\varphi \circ \mathbf{\Lambda}$, where $\mathbf{C}_\varphi = (\mathrm{c}_{\varphi 0}, \mathbf{c}_\varphi)$ is the normed quaternion [Branetz and Shmyglevsky, 1973] with a vector part $\mathbf{c}_\varphi = \{\mathrm{c}_{\varphi k}\}$. The associated differential system

$$\dot{\mathbf{\Psi}} = \frac{1}{2}\mathbf{\Psi} \circ \boldsymbol{\omega};\ \dot{\boldsymbol{\mu}} = -\frac{1}{2}\tilde{\mathbf{\Lambda}} \circ \mathbf{c}_\varphi \circ \mathbf{\Lambda};\ \dot{\boldsymbol{\nu}} = -\boldsymbol{\mu} \qquad (14)$$

and the optimality condition $\partial H/\partial \mathbf{v} = -\mathbf{v} + \boldsymbol{\nu} = \mathbf{0}$ give the optimal " control "

$$\mathbf{v}(t) = \boldsymbol{\nu}(t) = \mathbf{c}_\varepsilon - \mathbf{c}_\omega(t - t_{\mathrm{i}}^p) + \frac{1}{2}\int_{t_{\mathrm{i}}^p}^{t}\Big(\int_{t_{\mathrm{i}}^p}^{\tau} \tilde{\mathbf{\Lambda}}(s) \circ \mathbf{c}_\varphi \circ \mathbf{\Lambda}(s) ds\Big)\, d\tau,$$

where vectors $\mathbf{c}_\varphi$, $\mathbf{c}_\omega = \{\mathrm{c}_{\omega k}\}$ and $\mathbf{c}_\varepsilon = \{\mathrm{c}_{\varepsilon k}\}$ must be numerically defined using known analytical structure of solution for direct system (4) and taking into account the boundary conditions (5) and (6). Standard Newton iteration method was applied for numerical obtaining a strict optimal "control" $\mathbf{v}(t)$, moreover analytical solution (initial point) was applied in the form of approximate optimal motion (12) and (13). Difference between approximate optimal motion and strict optimal motion is very light for the SC practical rotational maneuvers.

7. GUIDANCE AT A ROTATION MANEUVER

Fast onboard algorithms for the SC guidance at a SRM with restrictions to $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$, $\dot{\boldsymbol{\varepsilon}}(t)$, corresponding restrictions to $\mathbf{h}(\boldsymbol{\beta}(t)), \dot{\boldsymbol{\beta}}(t)$ and $\ddot{\boldsymbol{\beta}}(t)$ in a class of the SC angular motions, were elaborated. Developed analytical approach to the problem is based on approximate optimal motion (12), (12) with boundary conditions (5), (6) and (8). Here functions $\varphi_k(t)$ are selected in a class of splines by five and six degree, moreover a module of a angular rate $\dot{\varphi}_3(t)$ in a position transfer $(k = 3)$ may be limited when functions $\dot{\varphi}_1(t) = \dot{\varphi}_2(t) \equiv 0$. The technique is based on the generalized integral's properties for the AM of the mechanical system "SC+GMC" and allows to evaluate vectors $\boldsymbol{\beta}(t)$, $\dot{\boldsymbol{\beta}}(t)$, $\ddot{\boldsymbol{\beta}}(t)$ in analytical form for a preassigned SC motion $\mathbf{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$, $\dot{\boldsymbol{\varepsilon}}(t)\ \forall t \in \mathrm{T}_p$.

Into orthogonal canonical basis Oxyz, see fig. 1, the GD's AM units have next projections:

$$x_1 = C_1;\ x_2 = C_2;\ y_1 = S_1;\ y_2 = S_2;\ x_3 = S_3;\ x_4 = S_4;$$
$$z_3 = C_3;\ z_4 = C_4;\ y_5 = C_5;\ y_6 = C_6;\ z_5 = S_5;\ z_6 = S_6,$$

where $S_p \equiv \sin\beta_p$ and $C_p \equiv \cos\beta_p$. Then vector-column $\mathbf{h}(\boldsymbol{\beta}) = \{\mathsf{x}, \mathsf{y}, \mathsf{z}\}$ of normed GMC's summary AM vector and matrix $\mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta}) = \partial \mathbf{h}/\partial \boldsymbol{\beta}$ have the form

$$\mathbf{h}(\boldsymbol{\beta}) = \begin{bmatrix} \Sigma x_p \\ \Sigma y_p \\ \Sigma z_p \end{bmatrix};\ \mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta}) = \begin{bmatrix} -y_1 & -y_2 & z_3 & z_4 & 0 & 0 \\ x_1 & x_2 & 0 & 0 & -z_5 & -z_6 \\ 0 & 0 & -x_3 & -x_4 & y_5 & y_6 \end{bmatrix}.$$

For *3-SPE* scheme singular state is appeared when the matrix Gramme $\mathbf{G}(\boldsymbol{\beta}) = \mathbf{A}_h(\boldsymbol{\beta})\mathbf{A}_h^{\mathrm{t}}(\boldsymbol{\beta})$ loses its full rang, e.g. when $\mathsf{G} \equiv \det \mathbf{G}(\boldsymbol{\beta}) = 0$. At introducing the denotations

$$x_{12} = x_1 + x_2;\quad x_{34} = x_3 + x_4;\quad y_{12} = y_1 + y_2;$$
$$y_{56} = y_5 + y_6;\quad z_{34} = z_3 + z_4;\quad z_{56} = z_5 + z_6;$$
$$\tilde{x}_{12} = x_{12}/\sqrt{4 - y_{12}^2}\ ;\ \tilde{x}_{34} = x_{34}/\sqrt{4 - z_{34}^2};$$
$$\tilde{y}_{12} = y_{12}/\sqrt{4 - x_{12}^2}\ ;\quad \tilde{y}_{56} = y_{56}/\sqrt{4 - z_{56}^2};$$
$$\tilde{z}_{34} = z_{34}/\sqrt{4 - x_{34}^2}\ ;\quad \tilde{z}_{56} = z_{56}/\sqrt{4 - y_{56}^2}$$

components of the GMC explicit vector tuning law

$$\mathbf{f}_\rho(\boldsymbol{\beta}) \equiv \{f_{\rho 1}(\boldsymbol{\beta}), f_{\rho 2}(\boldsymbol{\beta}), f_{\rho 3}(\boldsymbol{\beta})\} = \mathbf{0} \qquad (15)$$

are applied in the form

$$f_{\rho 1}(\boldsymbol{\beta}) \equiv \tilde{x}_{12} - \tilde{x}_{34} + \rho\ (\tilde{x}_{12}\ \tilde{x}_{34} - 1);$$
$$f_{\rho 2}(\boldsymbol{\beta}) \equiv \tilde{y}_{56} - \tilde{y}_{12} + \rho\ (\tilde{y}_{56}\ \tilde{y}_{12} - 1);$$
$$f_{\rho 3}(\boldsymbol{\beta}) \equiv \tilde{z}_{34} - \tilde{z}_{56} + \rho\ (\tilde{z}_{34}\ \tilde{z}_{56} - 1).$$

The analytical proof have been elaborated that vector tuning law (15) ensures absent of singular states by this GMC scheme for all values of the GMC AM vector $\mathbf{h}(t) \in \mathbf{S} \setminus \mathbf{S}^*$, i.e. inside all its variation domain. For the representation

$$x_{12} = (\mathsf{x} + \Delta_x)/2;\quad x_{34} = (\mathsf{x} - \Delta_x)/2;$$
$$y_{56} = (\mathsf{y} + \Delta_y)/2;\quad y_{12} = (\mathsf{y} - \Delta_y)/2;$$
$$z_{34} = (\mathsf{z} + \Delta_z)/2;\quad z_{56} = (\mathsf{z} - \Delta_z)/2$$

and the denotation $\mathbf{\Delta} = \{\Delta_x, \Delta_y, \Delta_z\}$ one can obtain the nonlinear vector equation $\mathbf{\Delta}(t) = \mathbf{\Phi}(\mathbf{h}(t), \mathbf{\Delta}(t))$. At a known vector $\mathbf{h}(t)$ this equation have single solution $\mathbf{\Delta}(t)$, which is readily computed by method of a simple iteration. Further the units $\mathbf{h}_p(\beta_p(t))$ and vector-columns $\boldsymbol{\beta}(t)$, $\dot{\boldsymbol{\beta}}(t)$, $\ddot{\boldsymbol{\beta}}(t)$ are calculated by the explicit analytical relations $\forall t \in \mathrm{T}_p$. For the *2-SPE* scheme such evaluation is carried out by the explicit analytical formulas only.

8. FILTERING AND ROBUST DIGITAL CONTROL

In stage 1, for continuous forming the control torque $\mathbf{M}^{\mathbf{g}}(\boldsymbol{\beta}(t), \dot{\boldsymbol{\beta}}(t))$ (2) and the SC model as a free rigid body the simplified controlled object is such:

$$\dot{\mathbf{\Lambda}} = \mathbf{\Lambda} \circ \boldsymbol{\omega}/2;\quad \mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}\times]\mathbf{G}^o = \mathbf{M}^{\mathbf{g}};\quad \dot{\boldsymbol{\beta}} = \mathbf{u}^{\mathrm{g}}(t). \qquad (16)$$

The error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \tilde{\mathbf{\Lambda}}^p(t) \circ \mathbf{\Lambda}$, Euler parameters' vector is $\boldsymbol{\mathcal{E}} = \{e_0, \mathbf{e}\}$, and the attitude error's matrix is $\mathbf{C}_e \equiv \mathbf{C}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 - 2[\mathbf{e}\times]\mathbf{Q}_e$, where $\mathbf{Q}_e \equiv \mathbf{Q}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 e_0 + [\mathbf{e}\times]$ with $\det(\mathbf{Q}_e) = e_0$. If error $\delta\boldsymbol{\omega} \equiv \tilde{\boldsymbol{\omega}}$ in the rate vector $\boldsymbol{\omega}$ is defined as $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \mathbf{C}_e \boldsymbol{\omega}^p(t)$, and the GMC's required control torque vector $\mathbf{M}^{\mathbf{g}}$ is formed as

$$\mathbf{M}^{\mathbf{g}} = \boldsymbol{\omega} \times \mathbf{G}^o + \mathbf{J}(\mathbf{C}_e \dot{\boldsymbol{\omega}}^p(t) - [\boldsymbol{\omega}\times]\mathbf{C}_e \boldsymbol{\omega}^p(t) + \tilde{\mathbf{m}}),$$

then the simplest nonlinear model of the SC's attitude error is as follows:

$$\dot{e}_0 = -\langle \mathbf{e}, \tilde{\boldsymbol{\omega}}\rangle/2;\quad \dot{\mathbf{e}} = \mathbf{Q}_e \tilde{\boldsymbol{\omega}}/2;\quad \dot{\tilde{\boldsymbol{\omega}}} = \tilde{\mathbf{m}}. \qquad (17)$$

For model (17) a *non-local nonlinear* coordinate transformation is defined and applied at analytical synthesis by the EFL technique. This results in the nonlinear CL

$$\tilde{\mathbf{m}}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}) = -\mathbf{A}_0\ \mathbf{e}\ \mathrm{sgn}(e_0) - \mathbf{A}_1\ \tilde{\boldsymbol{\omega}}, \qquad (18)$$

where $\mathbf{A}_0 = ((2a_0^* - \tilde{\omega}^2/2)/e_0)\mathbf{I}_3$; $\mathbf{A}_1 = a_1^*\mathbf{I}_3 - \mathbf{R}_{e\omega}$, $\mathrm{sgn}(e_0) = (1,\ \text{if}\ e_0 \geq 0) \vee (-1,\ \text{if}\ e_0 < 0)$, matrix $\mathbf{R}_{e\omega} = \langle \mathbf{e}, \tilde{\boldsymbol{\omega}}\rangle \mathbf{Q}_e^{\mathrm{t}}[\mathbf{e}\times]/(2e_0)$, and parameters a_0^*, a_1^* are analytically calculated on spectrum $\mathrm{S}_{ci}^* = -\alpha_c \pm j\omega_c$. Simultaneously the VFL $\boldsymbol{\upsilon}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}})$ is analytically constructed for close-loop system (17) and (18).

Discrete measured error quaternion and Euler parameters' vector are $\mathbf{E}_s = (e_{0s}, \mathbf{e}_s) = \tilde{\mathbf{\Lambda}}^p(t_s) \circ \mathbf{\Lambda}_s^{\mathrm{m}}$ and $\boldsymbol{\mathcal{E}}_s = \{e_{0s}, \mathbf{e}_s\}$, and the attitude error filtering is executed by the relations

$$\tilde{\mathbf{x}}_{s+1} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{B}}\mathbf{e}_s; \mathbf{e}_s^{\mathrm{f}} = \tilde{\mathbf{C}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{D}}\mathbf{e}_s, \qquad (19)$$

where matrices $\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ have conforming dimensions and some general turning parameters. Attitude filtered error vector $\mathbf{e}_k^{\mathrm{f}}$ is applied for forming the digital control $\tilde{\mathbf{m}}_k = \mathbf{u}_k$ taking into account a time delay at incomplete measurement of state and onboard signal processing:

$$\mathbf{v}_k = -(\mathbf{K}_{\mathrm{d}}^{\mathrm{x}}\,\hat{\mathbf{x}}_k + \mathbf{K}_{\mathrm{d}}^{\mathrm{u}}\,\mathbf{u}_k);\quad \mathbf{u}_{k+1} = \mathbf{v}_k,\quad k \in \mathbb{N}_0; \qquad (20)$$

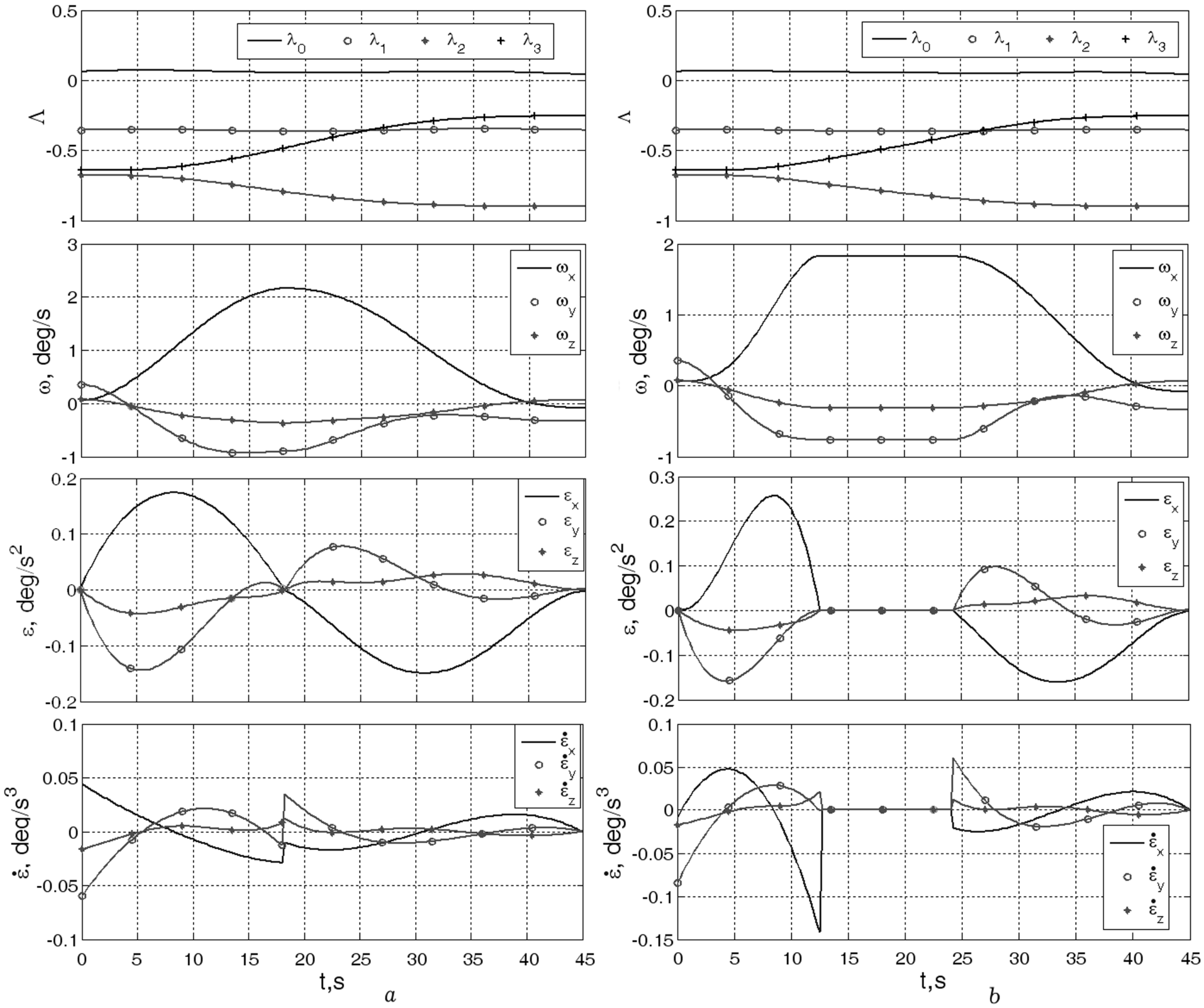

Fig. 2. The SC's SPM: *a* — without a limit on module of the SC angular rate vector; *b* — with such limit.

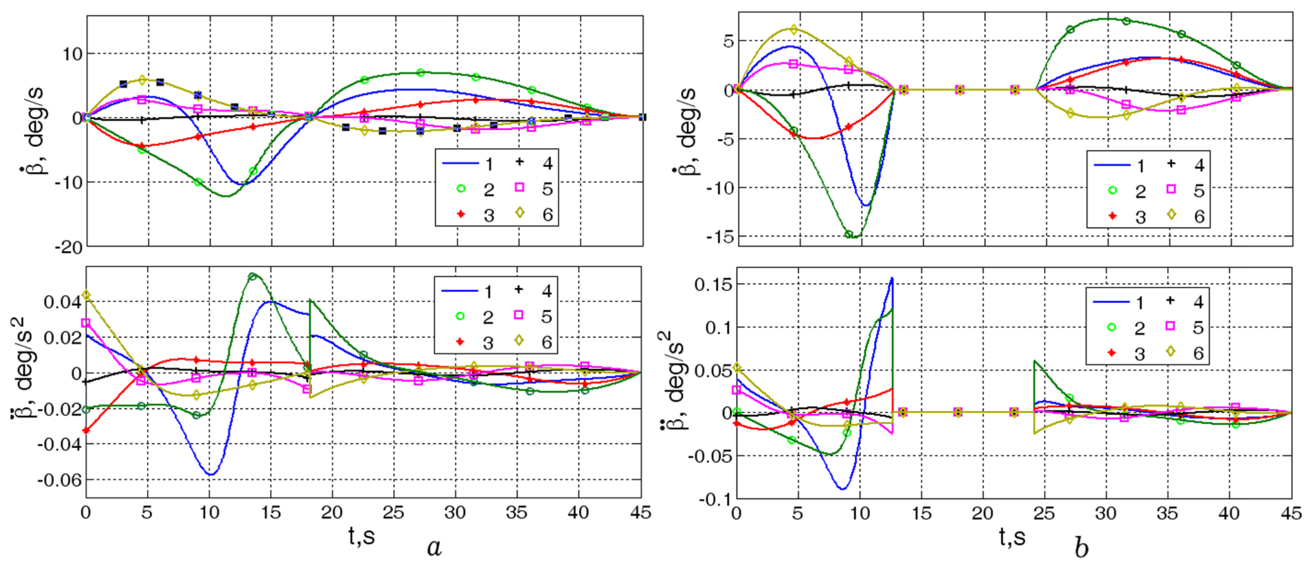

Fig. 3. The GMC *3-SPE* scheme coordinates the same SC's SPM: *a* — without a limit; *b* — with such limit.

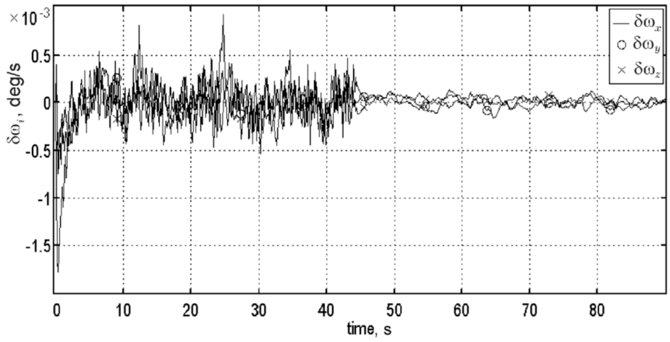

Fig. 4. Rate errors for consequence of the SRM and SCM

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}_{\text{od}}\,\hat{\mathbf{x}}_k + \mathbf{B}^{\text{u}}_{\text{od}}\,\mathbf{u}_k + \mathbf{B}^{\text{v}}_{\text{od}}\,\mathbf{v}_k$$
$$+\mathbf{G}_{\text{d}}(\mathbf{e}^{\text{f}}_k - (\mathbf{C}_{\text{od}}\,\hat{\mathbf{x}}_k + \mathbf{D}^{\text{u}}_{\text{od}}\,\mathbf{u}_k + \mathbf{D}^{\text{v}}_{\text{od}}\,\mathbf{v}_k)),$$

where $\hat{\mathbf{x}}_k = \{\hat{\mathbf{e}}_k, \hat{\tilde{\boldsymbol{\omega}}}_k\}$, matrices have conforming dimensions and also general turning parameters.

In stage 2, the problems of synthesising digital nonlinear CL were solved for model of the flexible spacecraft (1) with incomplete discrete measurement of state. Furthermore, the selection of parameters in the structure of the GMC nonlinear robust CL (which optimizes the main quality criterion for given restrictions, including coupling and damping the SC structure oscillations, see Somov et al. [2005a]) is fulfilled by a parametric optimization of the comparison system for the VLF and multistage numerical simulation. Thereto, the VLF has the structure derived above for the error coordinates $\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}$ and the structure of other VLF components in the form of *sublinear norms* for vector variables $\mathbf{q}(t)$, $\dot{\mathbf{q}}(t)$, $\dot{\boldsymbol{\beta}}(t)$ using the vector $\boldsymbol{\beta}(t)$.

9. COMPUTER SIMULATION

Fig. 2 and fig. 3 present dynamic characteristics of the SC's SRM and the GMC by *3-SPE* scheme during time $t \in \mathrm{T}_p = [0, T_p]$ with $T_p = 45$ sec and next boundary conditions:

$$\boldsymbol{\Lambda}_{\text{i}} = (0.06255029449, -0.35479160599,$$
$$-0.67663869314, -0.64216077108);$$
$$\boldsymbol{\Lambda}_{\text{f}} = (0.04168181290, -0.35479620846,$$
$$-0.89901121936, -0.25330042320);$$
$$\boldsymbol{\omega}_{\text{i}} = \{0.060345, 0.355995, 0.071572\}^\circ/s;$$
$$\boldsymbol{\omega}_{\text{f}} = \{-0.084455, -0.333483, 0.060107\}^\circ/s;$$
$$\boldsymbol{\varepsilon}_{\text{i}} = 10^{-2}\cdot\{0.2960, -0.0643, 0.0303\}^\circ/s^2;$$
$$\boldsymbol{\varepsilon}_{\text{f}} = 10^{-2}\cdot\{-0.2784; 0.1417; -0.0074\}^\circ/s^2;$$
$$\dot{\boldsymbol{\varepsilon}}_{\text{f}} = 10^{-5}\cdot\{0.05, 0.38, 0.01\}^\circ/s^3.$$

Fig. 4 and Fig. 5 present some results on computer simulation of a gyromoment ACS for Russian remote sensing SC by the *Resource-DK* type. Here the rate errors are represented at consequence of the SC spatial rotational maneuver for time $t \in [0, 45)$ sec and the SC spatial course motion for time $t \in [45, 90]$ sec. Applied digital robust nonlinear control law is flexible switched at the time moment $t = 45$ sec on astatic ones with respect to the acceleration.

10. CONCLUSIONS

In progress of Somov et al. [2005b] and Somov et al. [2007] new results were presented on optimizing the SC spatial rotational maneuvers, on precise onboard computing the spatial course motion at a space opto-electronic observation. Principle results were connected with onboard signal processing by multiple discrete filtering and nonlinear digital robust gyromoment control by the *3-SPE* scheme, applied for the agile observation SC.

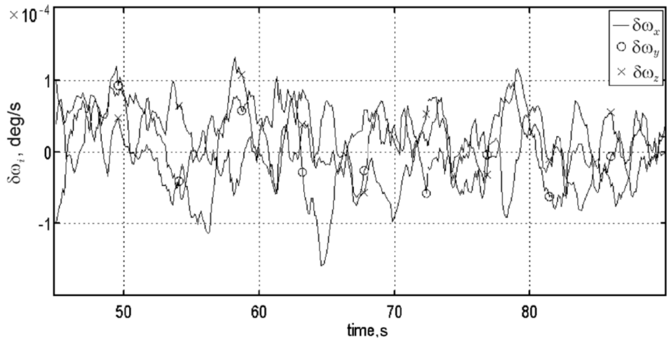

Fig. 5. The rate errors at the spatial course motion

These results were also successfully applied for a space free-flying robot at transportation the flexible large-scale mechanical payload, see Somov [2006, 2007].

REFERENCES

B.N. Branetz and I.P. Shmyglevsky. *Application of Quaternions for Problems of Rigid Body Attitude.* Nauka, Moscow, 1973.

B.R. Hoelscher and S.R. Vadali. Optimal open-loop and feedback control using single gimbal control moment gyroscopes. *Journal of the Astronautical Sciences*, 42 (2):189–206, 1994.

J.L. Junkins and J.D. Turner. *Optimal Spacecraft Rotational Maneuvers.* Elsevier Science, 1986.

Ye.I. Somov. Analytic synthesis of a programme gyromoment conrol by free-flying space robot. *Conrol Problems*, (6):72–78, 2006.

Ye.I. Somov. Optimization of a rotation maneuver and synthesis of gyromoment guidance laws for spacecraft and free-flying robots. *Izvestiya of Samara Scientific Center RAS*, 9(3):824–834, 2007.

Ye.I. Somov, S.A. Butyrin, V.M. Matrosov, and G.P. Anshakov et al. Ultra-precision attitude control of a large low-orbital space telescope. *Control Engineering Practice*, 7(7):1127–1142, 1999.

Ye.I. Somov, S.A. Butyrin, and S.Ye Somov. Coupling and damping the structure oscillations at spacecraft gyromoment digital control. In *Proceedings of the IEEE/EPS International conference "Physics and Control"*, pages 497–502, Saint Petersburg, 2005a.

Ye.I. Somov, S.A. Butyrin, S.Ye Somov, and G.P. Anshakov. Guidance and robust gyromoment attitude control of agile flexible remote sensing spacecraft. In *Preprints of 17th IFAC Symposium on Automatic control in Aerospace, Paper We-04-1/4*, pages 1–6, Toulouse, 2007. ONERA.

Ye.I. Somov, S.A. Butyrin, S.Ye Somov, V.M. Matrosov, and G.P. Anshakov. Robust nonlinear gyromoment control of agile remote sensing spacecraft. In *Preprints of 16th World Congress, Paper Mo-E02-TO/4*, pages 1–6, Praque, 2005b.