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Observability of Timed Continuous Petri Nets: A Class of Hybrid Systems ⋆

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Abstract: Timed continuous Petri net systems with infinite server semantics are piecewise linear systems. This paper addresses several problems regarding the state observability of these systems. We assume that the initial marking/state is not known and measuring some places we want to estimate all the others. First, a study of the different linear systems corresponding to a continuous Petri net system is performed. It is shown that in some cases, some of them are redundant, and so can be disregarded. The notion of *distinguishable configurations* is introduced. It helps to give a necessary and sufficient criterion for the observability in infinitesimal time. Using results from *linear structured systems* (Commault et al. (2005)), the concept of *generic observability* is introduced and it is studied in the case of join free nets.

1. INTRODUCTION

State estimation is a very important part of the control problem when not all the states are directly measurable. In last years, state estimation has been studied by many researchers (Babaali and Egerstedt (2004); Collins and van Schuppen (2004)) but many aspects are still open, specially in the case of hybrid systems. In this paper we study observability in infinitesimal time and generic observability of continuous Petri nets (contPN) with infinite server semantics. Under this semantics, a contPN system with joins corresponds to a set of switching linear systems, where the switches depend on the marking and the rates of the transitions. A brief introduction to timed contPN is given in Section 2 recalling basic concepts and results; moreover, an assembly system modeled by a contPN is used to illustrate the behavior of such systems.

Being contPN of piecewise linear systems, the results regarding its observability are similar to those obtained in hybrid systems, but they differ for two main reasons: (1) the existence of linear systems that are redundant, i.e., systems that it is not necessary to consider; (2) when the marking is at the border of two regions, more than one linear system can be used indistinctively (thus it is not important which one is taken), which makes harder to distinguish between them.

In Section 3 the concept of *redundant configuration* is introduced. An exponential number of linear systems can be embedded in a contPN but not all of them are always fundamental for the evolution. A sufficient and necessary condition for a configuration to be redundant is presented.

In Section 4 the notion of *distinguishable* configurations is initiated, concept similar to the one of hybrid systems: distinguishable discrete states. A condition for two configurations to be distinguishable is given and, finally, a necessary and sufficient criterium for the observability in infinitesimal time of general contPN is proved.

In Mahulea et al. (2005) an interpretation of the loss of observability in the case of join free (JF) contPN systems is given, when the system contains attributions. It is pointed out that observability cannot be checked locally and can be lost for some specific values of the firing rates of the transitions. Moreover, using the results on structured systems (Commault et al. (2005)), the problem of generic observability (here observability for almost all timing interpretation $\boldsymbol{\lambda}$) is solved in Section 5. In this problem, the firing rates of the transitions become parameters and the system is called generically observable if it is observable for almost all values of its firing rates. Hence, the attributions will not create problems anymore for generic observability.

2. TIMED CONTINUOUS PETRI NETS

ContPN have been introduced as a relaxation of discrete models (see David and Alla (2005); Silva and Recalde (2002) for two broad perspectives). In continuous net systems, the firing of transitions is not limited to a natural quantity but to a real positive quantity. In this way, the states/markings of the continuous systems can take real positive values.

Definition 1. A contPN system is a pair $\langle \mathcal{N}, \boldsymbol{m_0} \rangle$, where $\mathcal{N} = \langle P, T, \boldsymbol{Pre}, \boldsymbol{Post} \rangle$ is a net structure, with two disjoint sets of places P and of transitions T, for a place $p \in P$ and a transition $t \in T$, $\boldsymbol{Pre}[p,t]$ ($\boldsymbol{Post}[p,t]$) represents the weight of the arc from p to t (and from t to p, respectively), and $\boldsymbol{m}_0 \in \mathbb{R}^{|P|}_{\geq 0}$ is the initial marking.

A transition t is *enabled* at a marking $\boldsymbol{m}$ if all its input places are marked, and its *enabling degree* is given by:

$$enab(t, \boldsymbol{m}) = \min_{p \in {}^\bullet t} \left\{ \frac{\boldsymbol{m}[p]}{\boldsymbol{Pre}[p,t]} \right\} \quad (1)$$

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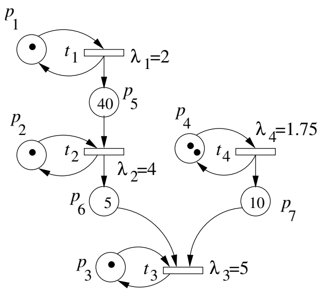


Fig. 1. ContPN taken from David and Alla (2005) (Figure 7.13).

where, for a node $v \in P \cup T$, ${}^\bullet v (v^\bullet)$ denotes the set of input (output) nodes.

A transition t enabled at $\boldsymbol{m}$ can fire in any real amount between 0 and $enab(t, \boldsymbol{m})$ leading to a new marking $\boldsymbol{m'}$. This firing can be written as: $\boldsymbol{m'} = \boldsymbol{m} + \alpha \cdot \boldsymbol{C}[\cdot, t]$ with $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$ called *token flow matrix*. In general, if a sequence $\sigma = \alpha_1 t_1 \ldots \alpha_k t_k$ of transitions are fired from $\boldsymbol{m}_0$ yielding a marking $\boldsymbol{m}$, it is said that $\boldsymbol{m}$ is *reachable* from $\boldsymbol{m}_0$ and the evolution can be described as:

$$\boldsymbol{m} = \boldsymbol{m}_0 + \boldsymbol{C} \cdot \boldsymbol{\sigma} \tag{2}$$

with $\boldsymbol{\sigma} : T \to \mathbb{R}_{\geq 0}$ the firing count vector of σ.

Equation (2) is called the *state* or *fundamental equation* of a contPN. The set of all reachable markings of an untimed net system is denoted by $RS^{ut}(\mathcal{N}, \boldsymbol{m}_0)$. In the case of contPN, reachability space can be extended to lim-reachability space considering infinitely long sequences, and the set of all lim-reachable markings is denoted by $lim - RS^{ut}(\mathcal{N}, \boldsymbol{m}_0)$ (Silva and Recalde (2002)).

Definition 2. A *configuration* $\mathcal{C}_k$ of $\mathcal{N}$ is a set of $|T|$ arcs (p_i, t_j) such that $\bigcup_{(p_i,t_j)\in\mathcal{C}_k} t_j = T$.

Therefore, a configuration is a set of (p_i, t_j) arcs, one per transition. A configuration $\mathcal{C}_k$ can be represented with a *constraint matrix* $\boldsymbol{\Pi}_k : T \times P \to \mathbb{R}_{\geq 0}$:

$$\boldsymbol{\Pi}_k[t_j, p_i] = \begin{cases} \dfrac{1}{\boldsymbol{Pre}[p_i, t_j]}, \text{ if } (p_i, t_j) \in \mathcal{C}_k \\ \\ 0, \text{ otherwise.} \end{cases} \tag{3}$$

The reachability space of a contPN can be partitioned, with some overlapping at the borders, associating to each configuration $\mathcal{C}_k$ a *region* $\mathcal{R}_k$, i.e., $lim - RS^{ut} = \mathcal{R}_1 \cup \cdots \cup \mathcal{R}_\gamma$.

Definition 3. A region $\mathcal{R}_k$ associated to the configuration $\mathcal{C}_k$ is the convex set of markings $\boldsymbol{m} \in lim - RS^{ut}(\mathcal{N}, \boldsymbol{m}_0)$ such that

$$\forall \boldsymbol{m} \in \mathcal{R}_k, \forall (p_i, t_j) \in \mathcal{C}_k, \frac{\boldsymbol{m}[p_i]}{\boldsymbol{Pre}[p_i, t_j]} = \min_{p_l \in {}^\bullet t_j} \frac{\boldsymbol{m}[p_l]}{\boldsymbol{Pre}[p_l, t_j]}.$$

Example 4. Let us consider the contPN system in Fig. 1 that models a simple assembly system. It has 6 configurations because the enabling degree of t_2 can be given by the marking of p_2 or p_5 and the enabling degree of t_3 can be given by the marking of p_3, p_6 or p_7. These configurations are: $\mathcal{C}_1 = \{(p_1, t_1), (p_2, t_2), (p_3, t_3), (p_4, t_4)\}$, $\mathcal{C}_2 = \{(p_1, t_1), (p_5, t_2), (p_3, t_3), (p_4, t_4)\}$, $\mathcal{C}_3 = \{(p_1, t_1), (p_2, t_2), (p_6, t_3), (p_4, t_4)\}$, $\mathcal{C}_4 = \{(p_1, t_1), (p_5, t_2), (p_6, t_3), (p_4, t_4)\}$, $\mathcal{C}_5 = \{(p_1, t_1), (p_2, t_2), (p_7, t_3), (p_4, t_4)\}$ and $\mathcal{C}_6 = \{(p_1, t_1), (p_5, t_2), (p_7, t_3), (p_4, t_4)\}$. Each configuration corresponds to different constraint matrices, for example, in the case of $\mathcal{C}_3$ it is:

$$\boldsymbol{\Pi}_3 = \begin{bmatrix} 1\,0\,0\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,1\,0\,0\,0 \end{bmatrix} \tag{4}$$

When time is introduced, the fundamental equation depends on it, and deriving with respect to time it becomes:

$$\begin{cases} \dot{\boldsymbol{m}}(\tau) = \boldsymbol{C} \cdot \dot{\boldsymbol{\sigma}} = \boldsymbol{C} \cdot \boldsymbol{f} \\ \boldsymbol{m}(\tau_0 = 0) = \boldsymbol{m}_0 \end{cases} \tag{5}$$

Depending on how $\boldsymbol{f}$ is defined different firing semantics are obtained. The most used in literature are *finite* and *infinite server semantics* Silva and Recalde (2002), also called *constant* and *variable speed* (David and Alla (2005)). These two semantics provide in general different approximations of the underlying discrete net systems. Nevertheless, for a broad class of net systems and under some general conditions it is proved that infinite server semantics provides always a better approximation of the steady state throughput (Mahulea et al. (2006)). In this paper, timed contPN systems are considered under infinite server semantics, for which the firing of a transition $t_j \in T$ at a marking $\boldsymbol{m}$ is defined as:

$$f_j = \boldsymbol{f}[t_j] = \lambda_j \cdot enab(t_j, \boldsymbol{m}) = \lambda_j \cdot \min_{p \in {}^\bullet t_j} \left\{ \frac{\boldsymbol{m}[p]}{\boldsymbol{Pre}[p, t_j]} \right\} \tag{6}$$

where λ_j is the firing rate of the transition t_j. Since the flow of a transition is based on a minimum function, timed contPN systems with infinite server semantics are a subclass of piecewise linear systems, where the switch appears when the place that constraints the enabling degree of a transition changes.

Defining $\boldsymbol{\Lambda} : T \times T \to \mathbb{R}_{\geq 0}$, $\boldsymbol{\Lambda}[i, j] = \lambda_j$ if $i = j$ and 0 otherwise, the evolution of a contPN with infinite server semantics can be described by:

$$\dot{\boldsymbol{m}}(\tau) = \boldsymbol{C} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\Pi}(\boldsymbol{m}(\tau)) \cdot \boldsymbol{m}(\tau) \tag{7}$$

where $\boldsymbol{\Pi}(\boldsymbol{m}(\tau))$ is a configuration such that $\boldsymbol{m}(\tau)$ belongs to the associated region.

Since a region is associated to each configuration and the constraint matrix $\boldsymbol{\Pi}_k$ is constant in each configuration, the behavior is linear while the marking keeps in a region.

Example 5. Let us go back to Ex. 4. The evolution while the marking is inside $\mathcal{R}_3$ (the corresponding region of $\mathcal{C}_3$) is:

$$\dot{\boldsymbol{m}}(\tau) = \boldsymbol{C}\boldsymbol{\Lambda}\boldsymbol{\Pi}_3\boldsymbol{m}(\tau) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1.75 & 0 & -5 & 0 \end{bmatrix} \boldsymbol{m}(\tau) \tag{8}$$

where $\boldsymbol{\Lambda} = diag([2, 4, 5, 1.75]^T)$.

In Fig. 2 the time evolution of the contPN system is presented. It can be seen that 5 linear systems govern the evolution until 25 t.u. At the initial marking, $enab(t_2, \boldsymbol{m}_0) = m_2 = 1$ and $enab(t_3, \boldsymbol{m}_0) = m_3 = 1$,

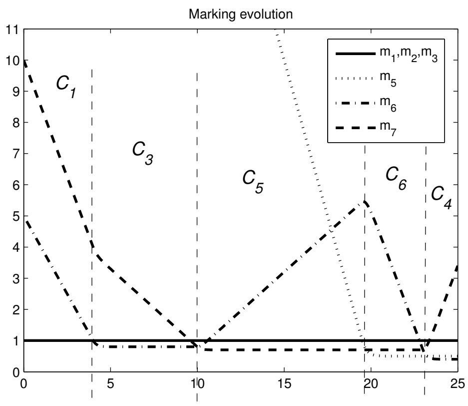
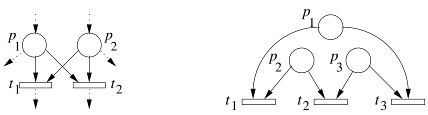


Fig. 2. Evolution of contPN system in Fig. 1.



(a) (b)

Fig. 3. ContPNs with redundant regions.

hence the system is inside the region $\mathcal{R}_1$ associated to $\mathcal{C}_1$. Arriving at $m_6 = m_3 = 1$ ($\tau = 4$ t.u.) a switch will occur since $enab(t_3, \cdot) = m_6$ and the new configuration will be $\mathcal{C}_3$. Then, around 10 t.u., there will be another switch because $m_6 = m_7$ and the new configuration will be $\mathcal{C}_5$. Later, when $m_5 = m_2$ ($\tau = 20$ t.u.) the system will evolve in configuration $\mathcal{C}_6$; and finally a new switch around 23 t.u. will bring the system to $\mathcal{C}_4$.

Continuous nets can be classified according to their structure:

- $\mathcal{N}$ is *Join-Free* (JF) if $\forall t \in T : |^\bullet t| \leq 1$.
- $\mathcal{N}$ is *Continuous Equal Conflict* (CEQ) if $^\bullet t \cap {}^\bullet t' \neq \emptyset \Rightarrow Pre[P,t] = k \cdot Pre[P,t']$.
- $\mathcal{N}$ is *Attribution Free* (AF) if $\forall p \in P : |^\bullet p| \leq 1$.
- $\mathcal{N}$ is pure if $\forall t \in T$, $^\bullet t \cap t^\bullet = \emptyset$.

3. REDUNDANT CONFIGURATIONS

It may happen that for every initial marking all the reachable markings of a region are on the border (one example is illustrated later in Ex. 7). Hence, these markings belong always to other regions, so it is not necessary to consider this configuration and obviously neither to check the observability of the corresponding linear system. Remember that the number of configurations/regions can be exponential so if this number is reduced, the complexity of the analysis is reduced. Therefore, our first step is to characterize these *redundant configurations* and to remove them.

Definition 6. Let $\mathcal{C}_i$ be a configuration with associated region $\mathcal{R}_i$. If for all $\boldsymbol{m}_0$, $\mathcal{R}_i \subseteq \bigcup_{j \neq i} \mathcal{R}_j$ then $\mathcal{C}_i$ is a *redundant configuration* and the corresponding linear system is a *redundant linear system*.

Example 7. Let us consider the subnet in Fig. 3(a). Assume the arcs $(p_1, t_1), (p_2, t_2) \in \mathcal{C}_1$ (with associated region $\mathcal{R}_1$) and the arcs $(p_1, t_1), (p_1, t_2) \in \mathcal{C}_2$ (with associated region $\mathcal{R}_2$). Assume also that the other arcs of the configurations are the same, i.e., $\mathcal{C}_1 \setminus \mathcal{C}_2 = \{(p_2, t_2)\}$ and $\mathcal{C}_2 \setminus \mathcal{C}_1 = \{(p_1, t_2)\}$.

Let $\boldsymbol{m}_0$ be an arbitrary initial marking. All reachable markings $\boldsymbol{m} \in \mathcal{R}_1$ satisfy:

(1) $\boldsymbol{m}[p_1] \leq \boldsymbol{m}[p_2]$ since for the join t_1, the flow is given by the marking of p_1, and
(2) $\boldsymbol{m}[p_2] \leq \boldsymbol{m}[p_1]$ since the flow of the join t_2 is given by the marking of p_2.

But, (1) & (2) implies $\boldsymbol{m}[p_1] = \boldsymbol{m}[p_2]$, $\forall \boldsymbol{m} \in \mathcal{R}_1$, so $\mathcal{R}_1$ is reduced to its border.

Since the flow of the other transitions is given by the same places by assumption, it is obvious that $\mathcal{R}_1 \subset \mathcal{R}_2$ and the linear system associated to $\mathcal{C}_2$ provides the same time-evolution for the markings $\boldsymbol{m} \in \mathcal{R}_1$. Hence, $\mathcal{C}_1$ can be ignored.

To see if a configuration $\mathcal{C}_i$ is non-redundant, check if there exists a marking such that the enabling degree of all the join transitions can be satisfied *only* according to the arcs in the configuration. In other words, if t is a join and (p_i, t) belongs to $\mathcal{C}_i$ then check if there exists a marking $\boldsymbol{m} \geq 0$ such that for all $p_j \in {}^\bullet t$, $p_j \neq p_i$, $\frac{\boldsymbol{m}[p_j]}{\boldsymbol{Pre}[p_j,t]} > \frac{\boldsymbol{m}[p_i]}{\boldsymbol{Pre}[p_i,t]}$. If such a marking does not exist, it means that the region is included into another one.

Proposition 8. Let $\mathcal{N}$ be a timed contPN system. The configuration $\mathcal{C}_i$ is redundant iff $\nexists \boldsymbol{m} \geq 0$ solution of the following inequations written for all $(p_k, t_j) \in \mathcal{C}_i$ and for all $p_u \in \{^\bullet t_j\} \setminus p_k$:

$$\frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k, t_j]} < \frac{\boldsymbol{m}[p_u]}{\boldsymbol{Pre}[p_u, t_j]}, \tag{9}$$

Proof. Obviously, if (9) has a solution this is an interior point of $\mathcal{R}_i$, the region corresponding to $\mathcal{C}_i$; thus it does not belong to another region.

For the reverse sense, let us assume that (9) has no solution. This means that for all $\boldsymbol{m}_0 \geq 0$ there exists at least one join transition t_j such that $\frac{\boldsymbol{m}[p_k]}{\boldsymbol{Pre}[p_k,t_j]} \geq \frac{\boldsymbol{m}[p_u]}{\boldsymbol{Pre}[p_u,t_j]}$ with $(p_k, t_j) \in \mathcal{C}_i$. If for all $\boldsymbol{m}$ this inequality is satisfied strictly the region is empty and can be eliminated without problems together with the corresponding configuration. Otherwise, if it is an equality, considering that the flow of t_j is given by $\boldsymbol{m}[p_u]$ not by $\boldsymbol{m}[p_k]$ it is clear that the corresponding regions include the region corresponding to $\mathcal{C}_i$. Hence, $\mathcal{C}_i$ is a redundant configuration. □

In Recalde et al. (2006) is introduced the notion of *time implicit arc* as those arcs that never constraint the enabling degree of the output transition for a given initial marking. It may seem that if a configuration is redundant, a set of arcs has to be implicit, since they cannot define the enabling. However, it is not true, since: (1) it is not that an arc never defines the enabling, but that a combination of arcs may never define the enabling, and (2) a redundant configuration is purely structural, i.e., it does not depend on the initial marking. For example, in the net in Fig. 3(a), none of the arcs is implicit, although a configuration is redundant. In this example, the redundant configuration could also have been avoided by fusing transitions t_1 and t_2 into a single one (see Recalde et al. (2006)). However,

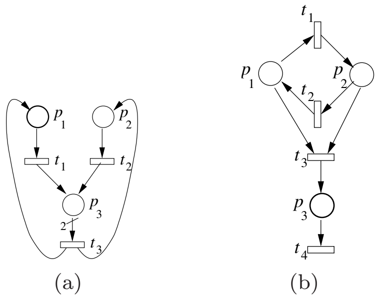


Fig. 4. Two timed contPNs

this kind of transformation cannot always be applied, as shown in the following example.

Example 9. Let us consider the contPN in Fig. 3(b) and let us write the inequalities (9) corresponding to $\mathcal{C}_1 = \{(p_2, t_1), (p_3, t_2), (p_1, t_3)\}$. These are:

$$\begin{cases} m_2 < m_1 \;\; (p_2, t_1) \in \mathcal{C}_1, p_1 \in {}^\bullet t_1 \;\; (1) \\ m_3 < m_2 \;\; (p_3, t_2) \in \mathcal{C}_1, p_2 \in {}^\bullet t_2 \;\; (2) \\ m_1 < m_3 \;\; (p_1, t_3) \in \mathcal{C}_1, p_3 \in {}^\bullet t_3 \;\; (3) \end{cases} \tag{10}$$

Combining (10.2) and (10.3) we obtain $m_1 < m_2$ that is in contradiction with (10.1). Therefore, $\mathcal{C}_1$ is redundant.

4. OBSERVABILITY OF TIMED CONTPN

Let us assume that the marking of some places can be measured, i.e., the token charge at every time instant is known, due to some sensors. The problem is to estimate the other marking variables using these measurements. Going back to (7), the system considered here is given by:

$$\begin{cases} \dot{\boldsymbol{m}}(\tau) = \boldsymbol{C} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\Pi}(\boldsymbol{m}(\tau)) \cdot \boldsymbol{m}(\tau) \\ \boldsymbol{y}(\tau) = \boldsymbol{S} \cdot \boldsymbol{m}(\tau) \end{cases} \tag{11}$$

where $\boldsymbol{S}$ is a $|P_o| \times |P|$ matrix, with P_o the set of observable places, each row of $\boldsymbol{S}$ has all components zero except the one corresponding to the i^{th} measurable place that is 1. Obviously, this is a piecewise linear system since the matrix $\boldsymbol{\Pi}$ is dynamically changing with the marking but the matrix $\boldsymbol{S}$ is the same for all linear systems. We consider that all linear systems are deterministic, i.e., noise-free. Therefore:

Definition 10. A timed contPN system $\langle \mathcal{N}, \boldsymbol{\lambda} \rangle$ under infinite server semantics is *observable in infinitesimal time* if it is always possible to compute its initial state $\boldsymbol{m}_0$ in any time interval $[0, \epsilon)$, observing a set of $P_o \subseteq P$ places.

In the rest of the paper, our attention is bound to observability in infinitesimal time. Thus, when we say that a system is observable we understand that it is observable in infinitesimal time. To study this kind of observability, the following assumptions are considered:

(A1) $\langle \mathcal{N}, \boldsymbol{\lambda} \rangle$: the net structure and timing are known;
(A2) The redundant configurations are removed.

The marking (state) estimation procedure has an interesting interpretation in contPN: going backward on path (Júlvez et al. (2004)).

Example 11. Let us consider the contPN in Fig. 4(a) and assume that p_1 is measured. So, $m_1(\tau)$ is known at every time instant. Then, the derivative of the marking, i.e., $\dot{m}_1(\tau)$, can be estimated, and also the flow of the transition t_1 because $f_1(\tau) = \lambda_1 \cdot m_1(\tau)$. Evidently, the flow of t_3 is deduced immediately using that $f_3(\tau) = \dot{m}_1(\tau) + f_1(\tau)$ and then the marking of p_3 can be computed because, on the other hand, $f_3(\tau) = \lambda_3 \cdot \frac{m_3(\tau)}{2}$. Knowing $m_3(\tau)$ we can estimate $\dot{m}_3(\tau)$, but $f_1(\tau)$ and $f_3(\tau)$ are also known, hence $f_2(\tau)$ can be estimated as: $f_2(\tau) = \dot{m}_3(\tau) + f_3(\tau) - f_1(\tau)$ that permits to estimate $m_2(\tau)$ since $f_2(\tau) = \lambda_2 \cdot m_2(\tau)$.

A very important problem in the observability of hybrid systems is the determination of the discrete state. Hence, the problem consists not only in estimating the continuous state but also the discrete one. In contPN systems, the discrete state (the configuration) can be deduced if the continuous state is known. However, if not all places are observed, it may happen that the observation fits with different discrete states, i.e., observing some places, it may happen that more than one system satisfy the observation. If the continuous states are on the border of some regions, it is not important which linear system is assigned, but it may happen that the solution corresponds to interior points of some regions and it is necessary to distinguish between them.

Example 12. Let us consider the timed contPN in Fig. 4(b) and assume $\boldsymbol{\lambda} = \boldsymbol{1}$ and p_3 is the measured place, i.e., $\boldsymbol{S} = [0, 0, 1]^T$. This system has two configurations: $\mathcal{C}_1 = \{(p_1, t_1);\ (p_2, t_2);\ (p_1, t_3);\ (p_3, t_4)\}$ and $\mathcal{C}_2 = \{(p_1, t_1);\ (p_2, t_2);\ (p_2, t_3);\ (p_3, t_4)\}$, corresponding to the following linear systems:

$$\Sigma_i = \begin{cases} \dot{\boldsymbol{m}}(\tau) = \boldsymbol{C} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\Pi}_i \cdot \boldsymbol{m}(\tau) \\ \boldsymbol{y}(\tau) = [0, 0, 1] \cdot \boldsymbol{m}(\tau) \end{cases} \tag{12}$$

where,

$$\boldsymbol{C} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\Pi}_1 = \begin{bmatrix} -1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \cdot \boldsymbol{I}_4 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

and
$$\boldsymbol{C} \cdot \boldsymbol{\Lambda} \cdot \boldsymbol{\Pi}_2 = \begin{bmatrix} -1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \cdot \boldsymbol{I}_4 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

The observability matrices of these two linear systems, i.e., $\vartheta_i = \left[\boldsymbol{S}, (\boldsymbol{S}\,(\boldsymbol{C\Lambda\Pi}_i))^T, \left(\boldsymbol{S}\,(\boldsymbol{C\Lambda\Pi}_i)^2\right)^T, \ldots, \left(\boldsymbol{S}\,(\boldsymbol{C\Lambda\Pi}_i)^{|P|}\right)^T\right]^T$, are:

$$\vartheta_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ -3 & 1 & 1 \end{bmatrix}; \quad \vartheta_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -3 & 1 \end{bmatrix}$$

which have both full rank, meaning that both linear systems are observable. Let us take $\boldsymbol{m}_1 = [1, 2, 0]^T \in \mathcal{R}_1 \setminus \mathcal{R}_2$ and $\boldsymbol{m}_2 = [2, 1, 0]^T \in \mathcal{R}_2 \setminus \mathcal{R}_1$. The corresponding observations are $\vartheta_i \boldsymbol{m}_i(\tau) = [\boldsymbol{y}(\tau), \dot{\boldsymbol{y}}(\tau), \ldots]^T$ and for the selected markings we have that $\vartheta_1 \cdot \boldsymbol{m}_1 = \vartheta_2 \cdot \boldsymbol{m}_2 = [0, 1, -1]^T$. Therefore, being equal, it is impossible to distinguish between $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$.

Definition 13. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two configurations with $\mathcal{R}_1$, $\mathcal{R}_2$ the associated regions. $\mathcal{C}_1$ and $\mathcal{C}_2$ are *distinguishable* if for any $\boldsymbol{m}_1 \in \mathcal{R}_1 \setminus \mathcal{R}_2$ and any $\boldsymbol{m}_2 \in \mathcal{R}_2 \setminus \mathcal{R}_1$ the observation $\boldsymbol{y_1}(\tau)$ for the trajectory through $\boldsymbol{m}_1$ and the observation $\boldsymbol{y_2}(\tau)$ for the trajectory through $\boldsymbol{m}_2$ are *different* on an interval $[0, \epsilon)$.

Remark that we remove the solutions at the border $\mathcal{R}_1 \cap \mathcal{R}_2$ since for those points both linear systems lead to identical behavior, therefore it is not important which one is chosen. This is an important difference between piecewise affine hybrid systems and contPNs: in general, for piecewise affine systems, given two states $(q, \boldsymbol{x}) \neq (q', \boldsymbol{x})$, the evolution may be different; in contPN, the same continuous state can be associated to two discrete states (configurations) only if the marking is on the border of the corresponding regions, and the evolution is identical.

An immediate sufficient condition for being distinguishable is:

Proposition 14. Let $\mathcal{C}_i$ $\{i = 1, 2\}$ be a configuration, ϑ_i and $\mathcal{R}_i$ the corresponding observability matrix and region. If the linear system

$$\vartheta_1 \cdot \boldsymbol{m}_1 = \vartheta_2 \cdot \boldsymbol{m}_2 \tag{13}$$

has no solution $(\boldsymbol{m}_1, \boldsymbol{m}_2) \in (\mathcal{R}_1 \setminus \mathcal{R}_2) \times (\mathcal{R}_2 \setminus \mathcal{R}_1)$, then the configurations $\mathcal{C}_1$ and $\mathcal{C}_2$ are distinguishable.

Proof. If (13) has no solution then the outputs for any two markings belonging to those regions are distinct. So, given a marking in any of these regions we can determine which is the configuration that governs the evolution of the contPN system. □

Example 15. In Ex. 12, for the timed contPN in Fig. 4(b) it is shown that $\vartheta_1 \cdot \boldsymbol{m}_1 = \vartheta_2 \cdot \boldsymbol{m}_2 = [0, 1, -1]^T$. Hence Prop. 14 does not allow to conclude that $\mathcal{C}_1$ and $\mathcal{C}_2$ are distinguishable. For the interpretation of this result, let us consider the equations that govern the evolution of the system:

$$f_3 = \lambda_3 \cdot \min\{m_1, m_2\} \tag{14}$$

$$\dot{m}_1 = \lambda_2 \cdot m_2 - \lambda_1 \cdot m_1 - f_3 \tag{15}$$

$$\dot{m}_2 = \lambda_1 \cdot m_1 - \lambda_2 \cdot m_2 - f_3 \tag{16}$$

Summing and integrating (15) and (16), we obtain

$$(m_1 + m_2)(\tau) = (m_1 + m_2)(0) - 2\int_0^{\tau} f_3(\theta) \cdot d\theta \tag{17}$$

Obviously, if p_3 is measured, f_3 can be estimated since $f_3(\tau) = \dot{m}_3(\tau) + \lambda_4 \cdot \dot{m}_3(\tau)$. Therefore, according to (14), the minimum between m_1 and m_2 is estimated and according to (17) their sum is also known. Nevertheless, these two equations are not enough to compute the markings, i.e., we have the values but it is impossible to distinguish which one corresponds to which place.

Using the notion of distinguishable configurations, an immediate criterium for observability in infinitesimal time is:

Theorem 16. A timed continuous Petri net system $\langle \mathcal{N}, \boldsymbol{\lambda} \rangle$ under infinite server semantics is observable in infinitesimal time iff:

(1) All configurations are distinguishable,
(2) For each configuration, the associated linear system is observable.

Proof. Assume that given an observation $\bar{\boldsymbol{m}}$, there are two different markings $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ coherent with $\bar{\boldsymbol{m}}$. Since the linear systems are observable, $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ belong to different configurations. But the configurations are all distinguishable, contradiction.

If the contPN is observable, for any initial marking in any configuration it must be possible to reconstruct it from observation, hence all the linear systems associated to the configurations have to be observable. Moreover, the configurations have to be distinguishable, since otherwise it would be possible to have two different markings that fit with the observation. □

5. GENERIC OBSERVABILITY

In Mahulea et al. (2005) it is shown that if the net has attributions, i.e., $p \in P$ with $|^\bullet p| \geq 2$, observability can be lost. In this case, a pole-zero cancelation can appear, which happens for very specific values of $\boldsymbol{\lambda}$, i.e., the denominator and the numerator of the transfer function vector between the input flow in places and the outputs: $\mathcal{Y}(s) = \boldsymbol{S}\left(s\boldsymbol{I} - \boldsymbol{C}\boldsymbol{\Lambda}\boldsymbol{\Pi}_i\right)^{-1}$ have a common factor. If the firing rates of the transitions are chosen randomly in $\mathbb{R}^+$, the probability to obtain this cancelation is null.

In previous section, observability for a fixed vector of firing rates is studied, while in Júlvez et al. (2004) *structural observability*, i.e., observability for all possible values of the firing rates, is introduced. Here, we define an intermediate concept, observability for almost all firing rate vectors, called *generic observability*.

Remark 17. Generic observability for $\mathcal{N}$ does not imply observability for a particular $\boldsymbol{\lambda}$, but for almost all values.

Generic observability is defined following the ideas presented in Commault et al. (2005) for linear structured systems. Hence we consider JF nets for which the behavior is linear and not piecewise linear as in general case. In this section, we try to interpret in net system terms generic observability of linear structured systems. We consider:

(A1) The net structure $\mathcal{N}$ is known and $\boldsymbol{\lambda}$ is a parameter;
(A2) $\mathcal{N}$ is JF.

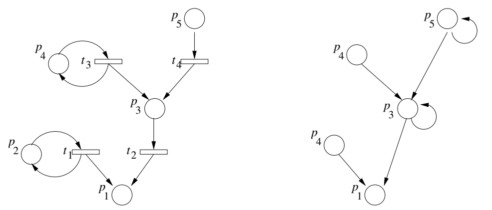
Generic observability can be studied also for AF and CEQ nets. In both cases, joins can be removed and the obtained net is JF, being observable iff the original one is. In the case of CEQ nets, all the input places of the join transitions should be measured, while in the case of AF the flow of the join transitions provide only minimum functions of some weighted markings that are not enough to estimate others markings, see Mahulea (2007).

Definition 18. Let $\langle \mathcal{N}, \boldsymbol{\lambda}, \boldsymbol{m_0} \rangle$ be a JF contPN system and P_o a set of measured places. $\mathcal{N}$ is *generically observable* from P_o if $\langle \mathcal{N}, \boldsymbol{\lambda}, \boldsymbol{m_0} \rangle$ is observable for all values of $\boldsymbol{\lambda}$ outside a proper algebraic variety of the parameter space.

Connections between observability for a given $\boldsymbol{\lambda}$ and generic observability are immediate.

In Commault et al. (2005), generic observability is studied for structured linear systems using an *associated graph*; observability is guaranteed when there exists a state-output connection for every state variable (the system is said to be *output connected*) and no *contraction* (defined after) exists.

The associated graph of an unforced linear system, $G = (Z, W)$ is defined by a vertex set Z and an edge set W (Commault et al. (2005)). The vertex set $Z = X \cup Y$ with X the set of state vertices and Y the set of output



(a) A JF ContPN. (b) Associated graph.

Fig. 5. Obtaining the associated graph of a JF contPN.

vertices. Denoting (v, v') for a direct edge from the vertex $v \in Z$ to a vertex $v' \in Z$, the edge set W is described by $W_A \cup W_S$ with $W_A = \{(x_j, x_i) | A[i,j] \neq 0\}$ and $W_S = \{(x_j, y_i) | S[i,j] \neq 0\}$.

The transformation of a JF net into its corresponding *associated directed graph* can be computed as follows (see Fig. 5). The vertex set Z is given by the set P of places (i.e. $Z = P$). The edge set W is computed as: $W = \{(p_i, p_j) | p_j \in (p_i{}^\bullet)^\bullet \bigwedge p_i \neq p_j\} \cup \{(p_i, p_i) | \exists t \in p_i{}^\bullet, \boldsymbol{Pre}[p_i, t] \neq \boldsymbol{Post}[p_i, t]\}$. The first set adds an edge from a place p_i to all places in $(p_i{}^\bullet)^\bullet$ since the dynamic matrix has a non null entry and prevents adding an edge in the case of a self-loop. The second subset will add a self-loop in the associated graph for any place with $\boldsymbol{Pre}[p_i, t] \neq \boldsymbol{Post}[p_i, t]$, i.e., the marking of p_i will change firing t, implying that the dynamical matrix has an non zero entry.

Definition 19. Let $\mathcal{N}$ be a contPN system and $G(\mathcal{N})$ its associated graph with vertex set Z and edge set W. Consider a set S made of k_S state vertices. Denote $E(S)$ the set of vertices w_i for $i = 1, \ldots, l_S$ of Z, such that there exists an edge $(x_j, w_i) \in W$ with $x_j \in S$. S is said to be a *contraction* if $k_S - l_S > 0$.

Based on the procedure to generate the associated graph (Fig. 5), and using Prop. 1 in Commault et al. (2005), the following is true:

Proposition 20. Let $\mathcal{N}$ be a contPN and $G(\mathcal{N})$ its associated graph. $\mathcal{N}$ is generically observable iff:

(1) $\mathcal{N}$ is output connected
(2) $G(\mathcal{N})$ contains no contraction.

Example 21. Let us consider the contPN in Fig. 5(a) whose associated graph is sketched in Fig. 5(b). Taking $S = \{p_2, p_3, p_4, p_5\}$ $(k_S = 4)$, $E(S) = \{p_1, p_3, p_5\}$ $(l_S = 3)$. Thus, the net has a contraction $(k_S - l_S = 4 - 3 = 1)$, so it is not generically observable. This happens because the flows of the transitions t_1 and t_3 are constant and measuring p_1 it is impossible to distinguish between these two constant incoming flows.

In the case of pure contPN systems, the necessary and sufficient condition of generic observability can be simplified. Since the associated graph of a pure PN has in every node a self-loop (under infinite server semantics, if p_i has at least one output transition t_j the derivative of the marking is: $\dot{m}_i = \ldots - \lambda_j \cdot m_i + \ldots$). Therefore, no contraction can exist and the only remaining condition in Prop. 20 is the output connectedness.

Corollary 22. Let $\mathcal{N}$ be a pure JF contPN. $\mathcal{N}$ is generically observable iff at least one place from each terminal strongly connected component is measured.

Therefore, for AF, CEQ and JF nets, if one place from each terminal strongly connected component is measured the net is generic observable. It may be not observable since if there exists attributions a pole zero cancelation can occur and thus the system will not be observable.

6. CONCLUSIONS AND FUTURE WORK

In this paper we provide a necessary and sufficient condition for the observability in infinitesimal time of general contPN systems and we give a necessary and sufficient condition for the generic observability of pure JF net systems. For these purposes we introduce and study the notions of redundant and distinguishable configurations. Our future work includes observability in finite and infinite time and also generic observability of general contPNs.

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