Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

# Fast and Accurate Modeling of Distributed RLC Interconnect and Transmission Line in Time and Frequency Domains

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**Abstract:** This paper presents the closed forms of the state space models and the recursive algorithms of the transfer function models for fast and accurate modeling of large scale complex systems of the evenly or unevenly distributed RLC interconnect and transmission lines. Considered models include the distributed RLC interconnect lines with or without external source and load connection. The effective closed forms do not involve any matrix inverse, factorization, or multiplication, thus dramatically reduce the computation complexity. Especially, the computation complexity of the closed forms for any evenly or unevenly distributed RLC interconnect line circuits is only O(1) or O($m$) respectively, where $m<<N$, $N$ is the system order, and $m$ represents the number of even sections which compose the uneven interconnect line. The features of new recursive algorithms are two recursive s-polynomials and their low computation complexity too. Illustration examples are provided to demonstrate the results in both time and frequency domains. The results can be applied to the RLC interconnect analysis and model reduction as a key to new approach, and to control systems with transmission lines, internet or delay lines.

## 1. INTRODUCTION

This paper presents the fast and accurate modelling approaches for large scale complex systems of distributed RLC interconnect and transmission lines circuits in both time and frequency domains. It is important to point out that the topic was among the interesting topics which Prof. Kalman (2005) addressed in the plenary speech at the 2005 IFAC.

Advancement of high-speed deep-submicron VLSI technology makes the interconnect to be a main factor of signal propagation delay and a key factor of modeling difficulty as a complex system of distributed RLC circuits (Reed and Rohrer 1999; Zhou *et al.* 1991). In some standard ASIC with 90nm nanotechnology, the ratio of the interconnect delay to the gate delay may approach to 4:1. Furthermore, its distribution feature and complex structure make the system order in millions. Therefore, the modelling and model reduction of interconnects have been a major challenge and a necessity to the analysis and design in the areas.

It is well known that the original model is not only a starting point for model reduction, but also a basis for performance evaluation of the reduced models and model reduction methods. It is important to reveal more fundamental characteristics of the distributed interconnect from its original models. As we know, the state space model in the time domain and the transfer function model in the frequency domain (s-domain) are two useful models, in addition to the maturated MNA which is directly from the KCL or KVL. The later is useful for model reduction approaches of the AWE (Asymptotic Waveform Evaluation, Pillage and Rohrer 1990) and the PVL (Padé via Lanczos process in the Krylov space, Feldmann and Freund 1995). Furthermore, the Balanced Truncation Method (BTM) (Glover 1984) is based on the state space model as a useful model reduction method (Zhou 1998, Yuan *et al.* 2004, 2005, Heydari and Pedram 2006), especially to provide an upper-bound of the approximation error. Wang *et al*. (2002) presented an insight review on the projection-based algorithms for model order reduction. They further presented algorithms based on generalized orthonormal basis functions in Hilbert and Hardy space. Antoulas and Sorensen (2003) provided an excellent overview on approximation and partial realization of systems. Recently, Wang *et al*. (2005, 2007) presented ELO (even-length-order) model simplification methods for RC interconnects. Clearly the BTM, ELO, other state space equation-based methods (Li *et al.* 1999), and some projection-based algorithms need to start from the state space model in the time domain. On the other hand, the transfer function in s-domain has been widely used for analysis and model reduction, such as DTT method (Ismail and Friedman 2003) that directly truncates the transfer function for model reduction with a nice approach to the tree-structure.

Even though the progress has been made, there are still many challenging problems to us (Wang 2005), e.g., (1) the lack of the state space model for the distributed RLC interconnect; and (2) the current transfer function model of the RLC interconnect lines involve computation of s-rational functions in recursive algorithms, or *sinh* and *cosh* functions. The traditional time domain model of interconnects is the MNA. Then, in order to get the state space model from the MNA, it needs matrix inverse (or decomposition) and multiplication, making a computation complexity of $O(n^2)$~$O(n^3)$. Here, the computation complexity is defined in the number of scalar multiplications, in a more detailed level than the conventional

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10.3182/20080706-5-KR-1001.0099

definition of the number that the method traverses nodes or components.

Thus, we ask: (1) can we find a closed-form of the state space model of the distributed RLC interconnect to avoid calculation of large dimension matrix inverse, decomposition, or multiplication, and (2) can we develop an even simpler recursive algorithm to establish the original transfer function model for model reduction in view of such high system order (size). We notice that the common structure of interconnects is a tree-type. At the same time, in order to investigate a tree or net type interconnect, we must study their basic components of lines first. Thus, this paper is to address these questions first for the distributed RLC interconnect lines.

Our resulted computation complexity of the state space closed forms is only $O(1)$ or $O(m)$ for any evenly or unevenly distributed RLC interconnect line, respectively, where the system order is $2n$, and $m<n$ (or $m<<n$). In cases of unevenly distributed interconnect, we can divide this uneven interconnect line into several, say $m$, sections which are evenly distributed sub-interconnect lines. The characteristics of our recursive algorithms are two recursive s-*polynomials*, and the low computation complexity. It is learnt that for a low order model approximation, the recursive algorithm may be modified by computing the first few (relevant) moments, i.e., limiting the polynomial multiplication order, with a lower complexity than $O(n^2)$.

Due to the page limit, the derivations of the results are omitted here, and the tree-type interconnect modelling based on this paper will be presented separately. However, we have included examples here to demonstrate the results.

## 2. PROBLEM FORMULATION

The considered RLC interconnect line circuits include unevenly or evenly distributed ones, and with or without external source and load. Thus, we may have four different models. The system order is assumed as $2n$ as general. The input port is with a voltage $v_{in}(t)$ and the output port then has a voltage $v_o(t)$. The distribution parameters are resistors $R_i$, inductors $L_i$ and capacitors $C_i$, $i=1,\cdots,n$. The index is ordered from the output/sink terminal to the input/source terminal. Denote the circuit node voltages as $v_i$, $i=1,\cdots,n$, respectively. The output may be any node voltage, e.g.,

$$v_o(t)=v_{out}(t)=v_1(t) \tag{1}$$

or any internal node voltage $v_i$. Circuit Model 1 is a general distributed RLC line circuit with a consideration of its external connection as shown in Fig. 1, where the external source resistor is $R_s$, the load resistor is $R_0$, and the load capacitor is $C_0$. Model 2 is an evenly distributed RLC interconnect line circuit with its external connection. Model 3 is a pure general distributed RLC line itself. Finally, Model 4 is an evenly distributed RLC interconnect line itself as shown in Fig. 2. Models 3 and 4 may be considered as a special case of Models 1 and 2, respectively, when the external parameters are much less significant or omitted as

$$R_s=0,\ R_0=\infty,\ C_0=0. \tag{2}$$

Notice that the pure interconnect really reflects itself without any distortion, thus it is important. Model 2 and Model 4 may be considered as a special case of Model 1 and 3, respectively, i.e.,

$$R_i=r,\ L_i=l,\ \text{and}\ C_i=c,\ i=1,\cdots,n. \tag{3}$$

Thus, the paper mainly discusses a general Model 1 and a special Model 4.

A linear state-space model {A, B, C, D}, which presents the dynamics of the system, has the form:

$$\dot{x}(t)=Ax(t)+Bu(t),\quad y(t)=Cx(t)+Du(t). \tag{4}$$

It is well known that the system matrix $A$ and its eigenvalues reflect the system characteristics. The transfer function specifies the relationship from the input signal $V_{in}(s)$ to the output signal $V_o(s)$ in s-domain, and has the following form:

$$T(s)=T_n(s)=V_o(s)/V_{in}(s)=N_n(s)/D_n(s) \tag{5}$$

where $N_n(s)$ and $D_n(s)$ are respectively the numerator and denominator polynomials of the 2n-th order transfer function $T_n(s)$. Its poles and zeros reflect the system characteristics.

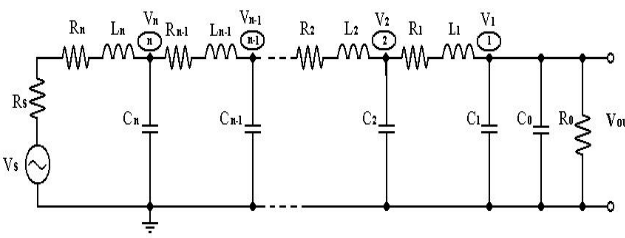


Fig. 1. General distributed RLC interconnect line circuit with external connection parameters

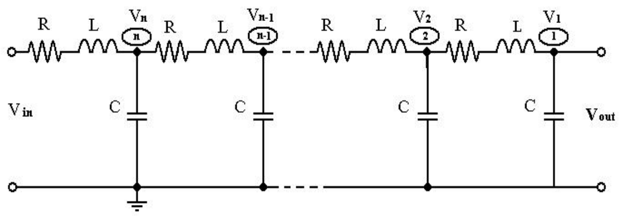


Fig. 2. Evenly distributed RLC interconnect line circuit

The goal of the paper is to present the close-form of the state space model (4), and the new recursive algorithm of the transfer function (5) leading to the reduction of computation complexity.

## 3. STATE SPACE MODEL AND ITS CLOSED FORMS

The closed forms of the state space model of distributed RLC interconnect circuits are presented below.

**Theorem 3.1.** For a general distributed RLC interconnect Model 1 in Fig. 1, choose the state variable vector $x(t)$, the input variable $u(t)$ and the output variable $y(t)$ as

$$x(t)=[v^T(t)\ \ \dot{v}^T(t)]^T,\ v(t)=[v_n(t)\cdots\ v_2(t)\ \ v_1(t)]^T$$
$$u(t)=v_{in}(t),\ y(t)=v_o(t)=v_1(t). \tag{6}$$

Then, the state space model (4) of Model 1 has its closed form as

$$A = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix} \tag{7}$$

$$A_{21} =$$

$$\begin{bmatrix} -\frac{1}{C_n}(\frac{1}{L_n}+\frac{1}{L_{n-1}}) & \frac{1}{C_n L_{n-1}} & 0 & \cdots & 0 & \frac{1}{R_0 C_n}(\frac{R_{n-1}}{L_{n-1}}-\frac{R_n+R_S}{L_n}) \\ \frac{1}{C_{n-1}L_{n-1}} & \ddots & \ddots & \ddots & \vdots & \frac{1}{R_0 C_{n-1}}(\frac{R_{n-2}}{L_{n-2}}-\frac{R_{n-1}}{L_{n-1}}) \\ 0 & \ddots & \ddots & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & \frac{1}{C_3 L_2} & \frac{1}{R_0 C_3}(\frac{R_2}{L_2}-\frac{R_3}{L_3}) \\ \vdots & & \ddots & \ddots \; \frac{1}{C_2 L_2} & -\frac{1}{C_2}(\frac{1}{L_2}+\frac{1}{L_1}) & \frac{1}{C_2 L_1}+\frac{1}{R_0 C_2}(\frac{R_1}{L_1}-\frac{R_2}{L_2}) \\ 0 & \cdots & \cdots & 0 & \frac{1}{(C_1+C_0)L_1} & -\frac{1+R_1/R_0}{(C_1+C_0)L_1} \end{bmatrix} \tag{8}$$

$$A_{22} = \begin{bmatrix} -\frac{R_n+R_s}{L_n} & \frac{C_{n-1}}{C_n}(\frac{R_{n-1}}{L_{n-1}}-\frac{R_n+R_s}{L_n}) & \cdots & \frac{C_2}{C_n}(\frac{R_{n-1}}{L_{n-1}}-\frac{R_n+R_s}{L_n}) & \frac{C_1+C_0}{C_n}(\frac{R_{n-1}}{L_{n-1}}-\frac{R_n+R_s}{L_n}) \\ 0 & -\frac{R_{n-1}}{L_{n-1}} & \ddots & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{R_2}{L_2} & \frac{C_1+C_0}{C_2}(\frac{R_1}{L_1}-\frac{R_2}{L_2}) \\ 0 & \cdots & \cdots & 0 & -\frac{R_1}{L_1}-\frac{1}{R_0(C_1+C_0)} \end{bmatrix} \tag{9}$$

$$B = [0 \cdots 0 \vdots 1/(C_n L_n) \; 0 \cdots 0]^T, \; C = [0 \cdots 0 \; 1 \vdots 0 \cdots 0], \; D = 0 \tag{10}$$

where matrices *A, B* and *C* have appropriate dimensions as

$$A \in R^{2n\times 2n}, \; A_{21} \in R^{n\times n}, \; A_{22} \in R^{n\times n}, \; B \in R^{2n\times 1}, \; C \in R^{1\times 2n}. \tag{11}$$

**Remark 3.1:** In general, for any intermediate point output $y(t) = v_o(t) = v_i(t)$, $i = 1,\cdots,n$, their state space models share the same matrices *A*, *B* and *D*, but matrix $C = [e_{n-i+1}^T \vdots 0 \cdots 0]$, where $e_i \in R^n$ is a unit vector with all entries 0 but the i-th entry 1.

**Remark 3.2:** The system sub-matrix $A_{21}$ has elements in the tri-diagonals and last column, but its all other entries are 0. Its tri-diagonals in the i-th row have elements $C_{n-i+1}$, $L_{n-i+1}$ and $L_{n-i}$, and their sum equals to 0 for $i = 2,\cdots,n-2$. Its last column has elements all with the load resistor $R_0$. The first row has elements of $C_n$, $L_n$, $L_{n-1}$, $R_n$, $R_{n-1}$ and source resistor $R_s$. The last row has elements of $C_1$, $L_1$, $R_1$, load resistor $R_0$ and capacitor $C_0$.

**Remark 3.3:** The system sub-matrix $A_{22}$ is an upper-triangle matrix. Its elements relate to the ratios of $R_i/L_i$ and the ratios of $C_i/C_j$ as shown in (9). These characteristics reflect the structure of distributed interconnect and transmission line with its element index sequence.

**Corollary 3.1.** An evenly distributed RLC interconnect circuit Model 4 with (3) in Fig. 2 has its state space model $\{A,B,C,D\}$ with (4) in the closed form of (7) and

$$A_{21} = \frac{1}{cl}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 \\ 1 & -2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -2 & 1 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}, A_{22} = -\frac{r}{l}I, \; n > 1 \tag{12}$$

$$B = (1/cl)[0 \cdots 0 \vdots 1 \; 0 \cdots 0]^T, \; C = [0 \cdots 0 \; 1 \vdots 0 \cdots 0], \; D = 0 \tag{13}$$

where their dimensions are in (11), and the computation complexity is only *O*(*1*), independent of system order 2*n*.

**Remark 3.4.** Theorem 3.1 and Corollary 3.1 present the closed forms of the state space models of the distributed RLC circuits. All these closed forms do not involve any matrix inverse, LU factorization, or matrix multiplication, where the matrix size is in very large scale. They not only reduce the computation complexity, but also provide accurate distributed RLC interconnect state space models. For the evenly distributed Model 4, it has computation complexity $O(1)$ only, i.e., it involves only constant number of scalar multiplications and divisions for any orders of the models!

**Corollary 3.2.** A general distributed RLC interconnect line circuit Model 3 has its state space closed form of $\{A,B,C,D\}$ as matrix *A* in (7), matrices *B, C* and *D* in (10), and

$$A_{21} = \begin{bmatrix} -\frac{1}{C_n}(\frac{1}{L_n}+\frac{1}{L_{n-1}}) & \frac{1}{C_n L_{n-1}} & 0 & \cdots & 0 \\ \frac{1}{C_{n-1}L_{n-1}} & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -\frac{1}{C_2}(\frac{1}{L_2}+\frac{1}{L_1}) & \frac{1}{C_2 L_1} \\ 0 & \cdots & 0 & \frac{1}{C_1 L_1} & -\frac{1}{C_1 L_1} \end{bmatrix} \tag{14}$$

$$A_{22} = \begin{bmatrix} -\frac{R_n}{L_n} & \frac{C_{n-1}}{C_n}(\frac{R_{n-1}}{L_{n-1}}-\frac{R_n}{L_n}) & \cdots & \frac{C_1}{C_n}(\frac{R_{n-1}}{L_{n-1}}-\frac{R_n}{L_n}) \\ 0 & -\frac{R_{n-1}}{L_{n-1}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{C_1}{C_2}(\frac{R_1}{L_1}-\frac{R_2}{L_2}) \\ 0 & \cdots & 0 & -\frac{R_1}{L_1} \end{bmatrix}. \tag{15}$$

**Theorem 3.2.** For unevenly distributed interconnect, if we may, really we may always divide this unevenly distributed interconnect line into some *m* evenly distributed sections (i.e., sub-interconnect lines), then the total computation complexity of the state space model will be dramatically reduced further to O(*m*), where $m < n$ (or $m << n$), in view of Corollary 3.1. The loading effect from one section to another is taken into account, i.e., the complexity $O(m)$ is valid for computation of the line composed of *m* even sub-lines.

**Remark 3.5.** As a simple example, we divide it into two even sub-lines $(m = 2)$ with orders $2n_1$ and $2n_2$ respectively. Then, we have its state space model in (4), (7), and

$$A_{21} = \begin{bmatrix} A_{21}^{11} & A_{21}^{12} \\ A_{21}^{21} & A_{21}^{22} \end{bmatrix}, \quad A_{22} = \begin{bmatrix} A_{22}^{11} & A_{22}^{12} \\ 0 & A_{22}^{22} \end{bmatrix} \tag{16}$$

$$A_{21}^{11} = \frac{1}{C_2 L_2}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 \\ 1 & -2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -2 & 1 \\ 0 & \cdots & 0 & 1 & -1-\frac{L_2}{L_1} \end{bmatrix}, \; A_{21}^{12} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 1/C_2 L_1 & \cdots & 0 \end{bmatrix}$$

$$A_{21}^{21} = \begin{bmatrix} 0 & \cdots & 1/C_1 L_1 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}, \; A_{21}^{22} = \frac{1}{C_1 L_1}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 \\ 1 & -2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -2 & 1 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}$$

$$A_{22}^{11} = -\frac{R_2}{L_2}I, \; A_{22}^{22} = -\frac{R_1}{L_1}I, \; A_{22}^{12} = \frac{C_1}{C_2}(\frac{R_1}{L_1}-\frac{R_2}{L_2})\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \cdots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \tag{17}$$

matrices *B, C* and *D* are in (13) with respective appropriate dimensions, and $n_1 + n_2 = n$.

## 4. TRANSFER FUNCTION RECURSIVE ALGORITHM

This section presents the recursive algorithms of transfer functions of the distributed RLC interconnect circuits.

**Theorem 4.1.** The unevenly distributed RLC circuit Model 1 in Fig. 1 has its transfer function $T_n(s) = N_n(s)/D_n(s)$ in (5) from a recursive algorithm:

$$N_n(s) = 1 \tag{18}$$

$$\Delta_j = sC_j D_{j-1}(s) + \Delta_{j-1}, \quad D_j(s) = (L_j s + R_j)\Delta_j + D_{j-1}(s)$$
$$j = 2, \cdots, n-1 \tag{19}$$

$$\Delta_n = sC_n D_{n-1}(s) + \Delta_{n-1}$$
$$D_n(s) = (L_n s + R_n + R_s)\Delta_n + D_{n-1}(s) \tag{20}$$

where the initial values for (19)-(20) are

$\Delta_1 = (C_1 + C_0)s + 1/R_0$, and

$$D_1(s) = s(L_1 s + R_1)(C_1 + C_0) + 1 + (L_1 s + R_1)/R_0 \tag{21}$$

and $2n$ is the system order.

**Corollary 4.1.** In the recursive algorithm of Theorem 4.1, all factors $\Delta_j(s)$ and denominators $D_j(s)$ of the $j$-th transfer function, $j = 1, \cdots, n$, are polynomials.

**Corollary 4.2.** The transfer function (5) of the evenly distributed RLC circuit Model 4 in Fig. 2 has a recursive algorithm as follows:

$$N_n(s) = 1 \tag{22}$$

$$\Delta_j = s \cdot cD_{j-1}(s) + \Delta_{j-1}, \quad D_j(s) = (ls + r)\Delta_j + D_{j-1}(s)$$
$$j = 2, \cdots, n \tag{23}$$

$$\Delta_1 = cs, \text{ and } D_1(s) = s(ls + r)c + 1. \tag{24}$$

**Theorem 4.2.** The transfer function from the input to any intermediate output node, say the *k-th* node, of the interconnect line in any Models 1– 4 is

$$T_{nk}(s) = D_k(s)/D_n(s) \tag{25}$$

where $D_n(s)$ and $D_k(s)$ are derived from the above recursive algorithm in Theorem 4.1 or Corollary 4.2, respectively.

**Remark 4.1.** The recursive formulas for the circuits with or without external connection parameters are similar, except that the $n$-th step is different due to the source parameter, and the initial values for $\Delta_1(s)$ and $D_1(s)$ are different due to the load parameters.

**Remark 4.2.** For general unevenly distributed RLC interconnect line models of the 2n-th order, the transfer function recursive algorithms have a computation complexity of $O(n^2)$ scalar multiplications. On the other hand, computing the first few (relevant) moments can be done with complexity less than $O(n^2)$ by limiting the polynomial multiplication order. For evenly distributed RLC interconnect, the recursive algorithm can be further programmed in a low computation complexity.

## 5. EXAMPLES

Two examples are used to demonstrate the results. Simulations are executed for their step responses and ramp responses in the time domain and Bode plots in the frequency domain. The ramp response is usually used in chips.

For comparison, the time response results via our approach and the PSpice are presented in figures, the former in blue solid line and the latter in red dash lines. However, they are nearly identical as shown, hardly to see the difference. During the simulations, we also observe that the red dash curves of PSpice approach to their corresponding blue solid curves of our models as the PSpice step length is further reduced. It implies that the new method may reduce the simulation time in view of the step length difference. For example, the runtimes of the step responses in Example 1 are 3.955 s from our closed form via MATLAB including the modeling time 3.419 ms, and 57.58 s from the PSpice with the step size for the same accuracy, but not including the netlist or schematic time in the PSpice modeling.

**Example 1.** Consider an evenly distributed RLC interconnect circuit of 0.01cm long in Fig. 2 with the distribution characteristic data of resistor 5.5kΩ/m and capacitor 94.2pF/m, and the external parameter data of source resistor $R_s$ = 500Ω, and load resistor $R_0$ = 1MΩ. A 200th order model is used as its original model with $n$ = 100, $r$ = 5.5m Ω, and $c$ = $9.42\cdot10^{-5}$ pF, while the inductor value is $l$ = $2.831\times10^{-1}$pH, calculated from the light speed in the material and the capacitor value $c$. By Theorem 3.1, its 200th order state space model $S = \{A, B, C, D\}$ is:

$$A = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix}, \; A_{21} = 3.75 \cdot 10^{28} \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & -5\cdot10^{-4} \\ 1 & -2 & 1 & \ddots & \ddots & 0 \\ 0 & 1 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & -2 & 1 \\ 0 & \cdots & \cdots & 0 & 1 & -1 \end{bmatrix}$$

$$A_{22} = -1.943\cdot10^{10} \begin{bmatrix} 9.091\cdot10^4 & 9.091\cdot10^4 & \cdots & 9.091\cdot10^4 & 9.091\cdot10^4 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & 0 & 1.546 \end{bmatrix}$$

$B = [0\, 0 \cdots 0 \,\vdots\, 3.75\cdot10^{28}\; 0 \cdots 0]^T$, $C = [0 \cdots 0\, 1 \,\vdots\, 0 \cdots 0]$ and $D = 0$.

Figures 3 and 4 show its step response and ramp responses. From Fig. 3 we see the oscillation behavior of the distributed RLC interconnect. It is clear from Figures 3 and 4 that the results from our approaches are nearly identical to the ones from the PSpice. Fig. 5 shows the Bode plot via the recursive algorithm of Theorem 4.1.

**Example 2.** Consider an example in Reed and Rohrer (1999) and extend it as follows. It is an aluminum IC interconnect, 2mm long, 1.0μm thick, and 1.5μm wide, connecting two gates over an 100-nm $SiO_2$ dielectric layer. The permittivity of $SiO_2$ is $\varepsilon_{SiO_2}$=$3.37\cdot10^{-13}$ F/cm and the resistivity of aluminum is $\rho_{A1} = 2.8\mu\Omega\cdot cm$. It leads to its "total" resistance $R_{int}$= 37.3Ω, "total" capacitance $C_{int}$=1.011pF, and "total" inductor $L_{int}$ = 105.61pH, but they are really distribution parameters.

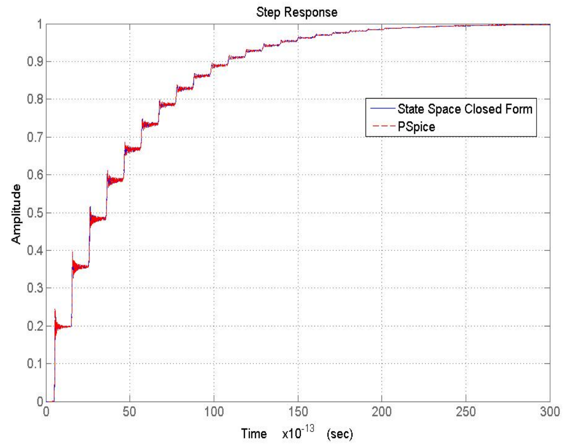


Fig. 3. Step response of evenly distributed RLC interconnect

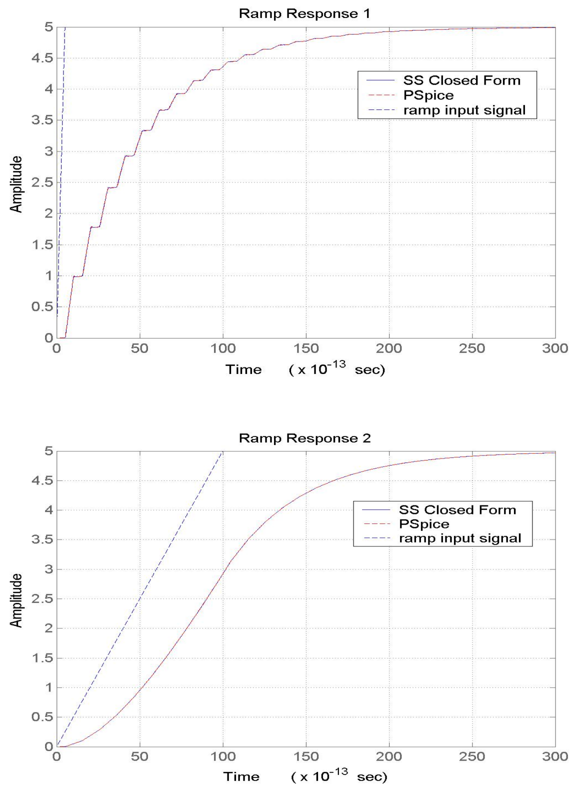


Fig. 4. Ramp responses 1 & 2 of an even RLC interconnect

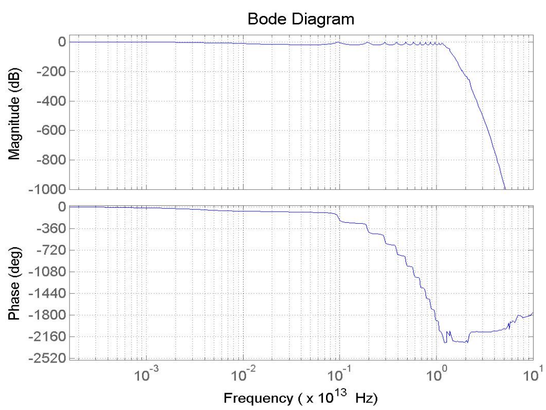


Fig. 5. Bode plot of an evenly distributed RLC interconnect

Here, we demonstrate our methods. In view of the page limit, a 20th order model ($n$ = 10) is used with $r$ = 3.73 Ω, $c$ = 0.1011 pF and $l$ = 10.561 pH in Fig. 2. By Corollary 3.1, its 20th order state space model $\{A, B, C, D\}$ is:

$$A = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix}, \; A_{21} = 9.3750 \cdot 10^{23} \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 \\ 1 & -2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -2 & 1 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}$$

$$A_{22} = (r/l) \cdot I_{10\times10} = -3.5375 \cdot 10^{11} \cdot I_{10\times10}$$

$$B = [0 \cdots 0 \;\; 9.375 \cdot 10^{23} \; 0 \cdots 0]^T, \; C = [0 \cdots 0 \;\; 1 \; \vdots \; 0 \cdots 0], \; D = 0.$$

For the transfer function, we have $T(s) = 1/D(s)$ via the recursive algorithm in Corollary 4.2, and

$$\begin{aligned} D(s) = {} & 1.9066 \cdot 10^{-240} \cdot s^{20} + 6.7411 \cdot 10^{-228} \cdot s^{19} + 4.4687 \cdot 10^{-215} \cdot s^{18} \\ & + 1.1818 \cdot 10^{-202} \cdot s^{17} + 4.1549 \cdot 10^{-190} \cdot s^{16} + 8.5395 \cdot 10^{-178} \cdot s^{15} \\ & + 2.0334 \cdot 10^{-165} \cdot s^{14} + 3.3024 \cdot 10^{-153} \cdot s^{13} + 5.7711 \cdot 10^{-141} \cdot s^{12} \\ & + 7.4194 \cdot 10^{-129} \cdot s^{11} + 9.7714 \cdot 10^{-117} \cdot s^{10} + 9.8253 \cdot 10^{-105} \cdot s^{9} \\ & + 9.7140 \cdot 10^{-93} \cdot s^{8} + 7.4201 \cdot 10^{-81} \cdot s^{7} + 5.3275 \cdot 10^{-69} \cdot s^{6} \\ & + 2.9191 \cdot 10^{-57} \cdot s^{5} + 1.4050 \cdot 10^{-45} \cdot s^{4} + 4.9030 \cdot 10^{-34} \cdot s^{3} \\ & + 1.2907 \cdot 10^{-22} \cdot s^{2} + 2.0742 \cdot 10^{-11} \cdot s + 1.0000 . \end{aligned}$$

Since a 200th model has 201 coefficients in its transfer function, so we take the 20th order model as an example for a short list. However, it is easy to compute any size of the state space model and transfer function by the theorems and corollaries. In figures 6 and 8 we show simulations on the 200th model. Furthermore, it is possible and easy to use scaling skill for modification of the above coefficients orders and for simulations as shown in the unit scaling of the figures.

Figure 6 shows the step responses of this distributed RLC interconnect of order 200. It shows oscillations around the middle amplitude zone, which may not be seen for its low order models. Fig. 7 displays two ramp responses with order 20 to different ramp rates. Fig. 8 shows the Bode plot of the transfer function model with order 200 via the recursive algorithm in Corollary 4.2. It is seen that the new results and the PSpice ones are identical for various system orders.

The simulation results clearly show the correctness of the new approaches by the comparison with the PSpice. The correctness of our results can also be theoretically proved.

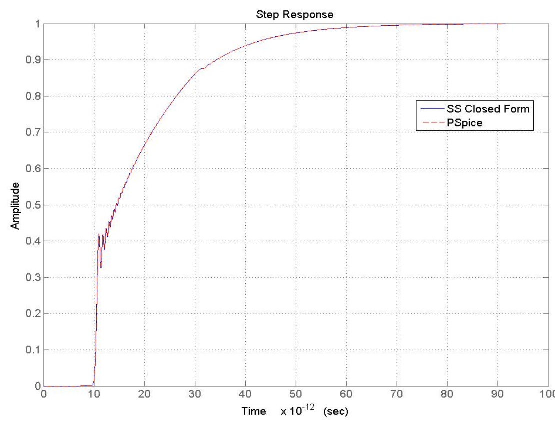


Fig. 6. Step response of an even RLC interconnect

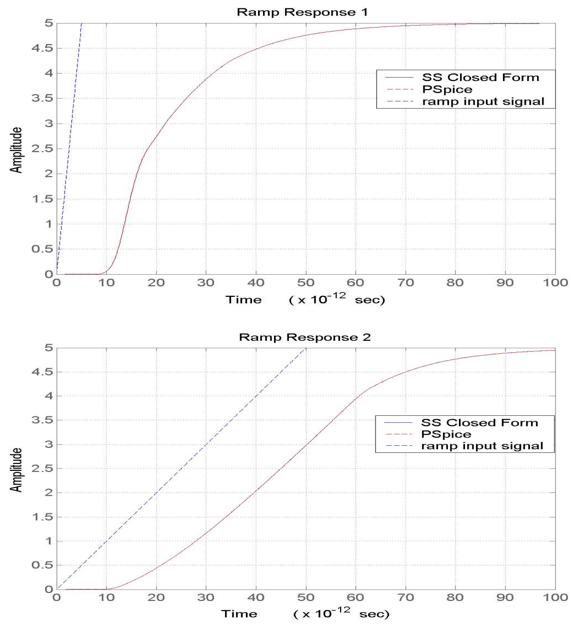


Fig. 7. Ramp responses 1 & 2 of an even RLC interconnect

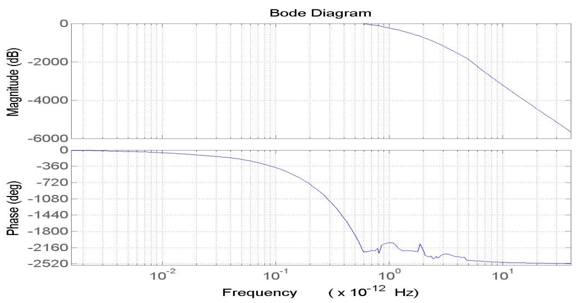


Fig. 8. Bode plot of an evenly distributed RLC interconnect

It is observed that the proposed methods are effective with high accuracy and low computation cost. Since the PSpice needs to type a net-list for its model, or build its circuit schematic plot, thus the state space model closed-form and the transfer function recursive algorithm are valued for many model reduction methods, e.g., the BTM, ELO, and others.

## 6. CONCLUSIONS

The main feature of the closed-form of state space model is its very low computation complexity. The key characteristics of the transfer function recursive algorithm are effectively to utilize a new internal s-polynomial function and multiplications with only a 1st order or 2nd order s-polynomial, different from the s-rational functions in previous ones to treat with an RLC interconnect line, a very special tree.

We first present the novel closed form of the state space model and the effective recursive algorithm of the transfer function for the distributed RLC line circuits in the literature based on Wang 2005. Next step is for the tree structures.

The results of distributed RLC models presented in this paper and Wang 2005 can be used for an alternate incorporation with the standard circuit simulation engines for the associated time savings. The results may also be used to control systems and communication systems with long distributed transmission lines, internet, or delay lines.

## ACKNOWLDGEMENT

Prof. Sheng-Guo Wang's work was supported in part by the NSF under Grant CCR 0098275.

## REFERENCES

Antoulas, A.C. and D.C. Sorensen (2003). *Approximation of large-scale dynamical systems: An overview*. MTNS, Perpignan.

Feldmann, P. and R.W. Freund (1995). Efficient linear circuit analysis by Padé via Lanczos process. *IEEE Trans. CAD*, **14**, 639-649.

Glover, K (1984). All optimal Hankel-norm approximations of linear multivariable systems and their $L^{\infty}$-error bounds. *Int. J. Control*, **39**, 1115-1193.

Heydari, P. and M. Pedram (2006). Model-order reduction using variational balanced truncation with spectral shaping. *IEEE Trans. CAS-I*, **53**, 879-891.

Ismail, Y. and E.G. Friedman (2002). DTT: direct truncation of the transfer function – an alternative to moment matching for tree structured interconnect. *IEEE Trans. CAD*, **21**, 131-144.

Li, J.R., F. Wang and J. White (1999). An efficient equation based approach for generating reduced-order models of interconnect. *Proc. DAC'99*.

Kalman, R. (2005). The evolution of system theory: my memories and hopes. Plenary Speech, *16th IFAC World Congress.* Prague, Czech.

Pillage, L.T. and R.A. Rohrer (1990). Asymptotic waveform evaluation for timing analysis. *IEEE Trans. CAD*, **9**, 352-366.

Reed, M. and R. Rohrer (1999). *Applied Introductory Circuit Analysis.* Prentice Hall, Upper Saddle River, NJ.

Wang, B., S.-G. Wang and B. Yuan (2007). Evenly distributed RC interconnect ELO model simplification and its simulation. *Int. J. Modeling & Simulation*, **27**, 324-332.

Wang, B., S.-G. Wang and B. Yuan (2005). ELO model reduction and case study of evenly distributed RC interconnect. *Proc. 16th IFAC World Congress*. Prague, Czech.

Wang, J.M., C.-C. Chu, Q. Yu and E.S. Kuh (2002). On projection-based algorithms for model-order reduction of interconnects. *IEEE Trans. CAS-I*, **49**, 1563-1585.

Wang, S.-G. (2005). US Patent Application, *USPTO*, 2005.

Yuan, B., B. Wang and S.-G. Wang (2004). RC interconnect circuits and its balanced truncated models. *IEEE Proc. 5th World Congress Intelligent Control & Automation* (pp. 190-194). Hongzhou, China.

Yuan, B., B. Wang and S.-G. Wang (2005). Balanced truncated models of RC interconnect circuits and their simulation. *J. Electronics*, **22**, 403-408.

Zhou, D., F.P. Preparata and S.M. Kang (1991). Interconnection delay in very high-speed VLSI. *IEEE Trans. Circuits and Systems*, **38**(7).

Zhou, K. (1998). *Essentials of Robust Control*. Prentice Hall, Upper Saddle River, NJ.