

# Adaptive Sliding Mode Attitude and Vibration Control of Flexible Spacecraft Under Unknown Disturbance and Uncertainty

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Abstract: This paper is concerned with the development of a control system for rotational maneuver and vibration suppression of a flexible spacecraft. It is assumed that the system parameters are unknown. The design approach presented here treats the problem of spacecraft attitude control separately from the elastic vibration suppression problem. As a stepping stone, a state feedback sliding mode control command is designed for the reaction wheel to achieve the reference trajectory tracking control of attitude angle. This is followed by the design of an adaptive sliding mode control (ASMC) law using only the output for robust stabilization of spacecraft in the presence of parametric uncertainty and external disturbances. Although this controller has the ability to reject the disturbance, deal with uncertainty and to ensure that the system output errors asymptotically converge to the sliding mode during the commanded motion, it excites the elastic modes of flexible appendages. The undesirable vibration is actively suppressed by applying feedback control voltages to the piezoceramic actuators, in which the control voltages are determined using the modal velocity feedback control method. Both analytical and numerical results are presented to show the theoretical and practical merits of this hybrid approach

# 1. INTRODUCTION

Modern spacecraft often employ large, complex and lightweight structures such as solar arrays to achieve increased functionality at a reduced launch cost. The combination of large and lightweight design results in these space structures being extremely flexible and having lowfrequency fundamental vibration modes. Orbital operations, such as slewing maneuvers, will induce vibrations in these flexible structures that can degrade operational performance. Dynamical models of spacecraft are also nonlinear and include the rigid and flexible mode interaction. Moreover, the parameters of spacecraft are not precisely known. All of these issues create considerable difficulty in the design of control systems for attitude tracking of flexible spacecraft.

To reduce vibrations the use of smart structures with advanced control algorithms are investigated as a potential solution to efficiently maneuver lightweight flexible spacecraft and minimize the excitation of structural resonances during operations. Piezoelectric twist actuators used for this application are based on anisotropic straining of the host structure using directionally attached isotropic actuators or using piezoelectric fibers integrated into the composite structural members. Fanson and Caughey (1990) proposed positive position feedback (PPF) control where the structural position coordinate is fed directly to the compensator and the product of the compensator and a scalar gain is fed back positively to the structure. Hu and Ma (2005, 2006) have extended this approach. The design of active controllers using piezoelectric actuators for vibration control of flexible spacecraft systems has also been considered theoretically by various authors (Singh and Arahio, 1999; Song and Agrawal, 2001; Gennaro, 1998).

Sliding mode control (SMC) schemes have been proposed for spacecraft attitude control system design (Crassidis and Markley, 1996; Lo and Chen, 1995). In a practical situation, the measurement of the flexible mode responses is extremely difficult. Thus, there is a need to design a control system for torque control which does not require the measurement of all of the state variables. Based on output feedback, controllers for maneuvering flexible spacecraft have been designed (Zeng et al., 1999; Singh and Zhang, 2004). However, in this approach, the structure of the uncertainty has to be known for the controller design. Thus the objective is to design an adaptive and robust control law which is simple and has a good tracking precision in the presence of disturbances, yet is independent of system parameters.

The contribution of this paper lies in the design of a simple but effective control system for rotation maneuvers and elastic mode stabilization of a flexible spacecraft. It is assumed that the spacecraft parameters are completely unknown and its model is also of finite but arbitrary order. The control scheme consists of two separate feedback control loops for accomplishing the objectives of vibration reduction and accurate pointing simultaneously. More precisely, the desired control torque is designed based on a continuous version of the adaptive sliding mode control (ASMC) design technique to force the tracking error to asymptotically approach zero, and also to only use the attitude angle and angular rate for feedback. Moreover, a priori bounds on the uncertainty and the disturbance are released by using an adaptive learning law. With the presence of this outer feedback controller, the inner feedback controller based on modal velocity feedback control is then designed to actively suppress the vibration (flexible motion) of the system. This is then extended to incorporate the attitude controller and inner feedback controller to achieve the reference trajectory tracking and vibration suppression, in which a reference trajectory generator of third order is chosen. Numerical simulations performed on a five-mode model of the spacecraft with flexible appendage during attitude tracking demonstrate the effectiveness and feasibility of the method.

## 2. FLEXIBLE SPACECRAFT AND CONTROL PROBLEM

Using the extended Hamilton's principle, the equations of motion for the flexible spacecraft with surface mounted PZT sensors and actuators can be written as (Hu and Ma, 2005)

$$J\ddot{\theta} + 2\dot{\theta}q^T Mq + \tilde{\Phi}\ddot{q} = u + d(t)$$
(1)

$$\tilde{\Phi}^T \ddot{\theta} + M \ddot{q} + C \dot{q} + (K - \dot{\theta}^2 M) q = -\overline{B}v$$
<sup>(2)</sup>

$$\overline{C}v = \overline{B}^T q \tag{3}$$

where  $\tilde{\Phi} = [\phi_1 \ \phi_2 \ \cdots \ \phi_n]$ ,  $q = [q_1 \ q_2 \ \cdots \ q_n]^T$ , and  $\phi_j(x)$  and  $q_j(t)$  (j = 1, 2, ..., n) are the assumed mode shapes and generalized co-ordinates, respectively. Hu and Ma (2005) gave further details of the model derivation.

In this work, we are interested in deriving a control system such that, in the closed-loop system, the attitude angle  $\theta(t)$ tracks the reference trajectory  $\theta_r(t)$  in the presence of external disturbance and parametric uncertainty, and at the same time the induced elastic oscillations are actively damped out. The reference command is generated by the third-order system (Singh and Arahio, 1999)

$$\ddot{\theta}_r + 3\lambda_c \ddot{\theta}_r + 3\lambda_c^2 \dot{\theta}_r + \lambda_c^3 (\theta - \theta^*) = 0$$
(4)

where  $\lambda_c > 0$  and  $\theta^*$  is the desired terminal value of the angle. For a proper choice of reference trajectory, the spacecraft attains the desired orientation, as  $\theta$  converges to  $\theta_r$ . For the design of the controller, it is assumed that the system parameters are not known. Moreover, the controller must only use the measured signals  $\theta$  and  $\dot{\theta}$ , since the elastic modes q and  $\dot{q}$  are not available.

#### 3. CONTROL STRATEGY

#### 3.1. Attitude Control Subsystem

We first formulate the sliding mode controller to determine the flywheel torque such that reference angle trajectory tracking is accomplished and the elastic oscillations remain bounded during the tracking. To do this, as a first step, we rewrite Eqs. (1) and (2) as

$$\ddot{\theta} - C_1 \dot{q} - K_1 q + f(\theta, \dot{\theta}, q) = \frac{u}{J_1}$$
(5)

where 
$$f(\theta, \dot{\theta}, q) = \frac{2\dot{\theta}q^T Mq + \tilde{\Phi}M^{-1}\dot{\theta}^2 Mq - d(t)}{J_1}$$
 is the

lumped perturbation including external disturbance torque, nonlinear couple terms and possible parametric uncertainty,

$$J_1 = \left(J - \tilde{\Phi}M^{-1}\tilde{\Phi}^T\right)$$
,  $C_1 = \frac{\tilde{\Phi}M^{-1}C}{J_1}$  and  $K_1 = \frac{\tilde{\Phi}M^{-1}K}{J_1}$ 

Note that the effect of the inner-loop, which will increase the damping of the flexible structure by the control using the piezoceramic actuators, is neglected here for simplicity, but will be considered later.

Throughout this paper, the following assumptions are used:

Assumption 1: The elastic oscillation and its rate are bounded, that is ||q(t)|| and  $||\dot{q}(t)||$  are bounded during the whole attitude tracking process. The notation ||x|| in this paper denotes the Euclidean norm of vector x, and ||X|| is the induced two-norm of a matrix X.

**Assumption 2**: The external disturbance d(t) to the spacecraft system given in Eq. (1) is bounded.

**Assumption 3**: Given the assumptions 1 and 2, there exist positive constant scalars  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$  such that

$$\left| f\left(\theta, \dot{\theta}, q\right) \right| \le \gamma_2 \left| \dot{\theta} \right| + \gamma_1 \left| \theta \right| + \gamma_0 \tag{6}$$

for all  $t \in \mathbb{R}^+$ .

For the purpose of design, let the attitude tracking error be  $\tilde{\theta} = \theta - \theta_r$ , and select a switching surface in the error state space as

$$\sigma = \dot{\tilde{\theta}} + \lambda_p \tilde{\theta} + \lambda_i \int_0^t \tilde{\theta} \, d\tau \tag{7}$$

where  $\lambda_p > 0$  and  $\lambda_i > 0$ .

To derive the control law such that the surface  $\sigma = 0$  attracts all trajectories, a Lyapunov based approach is used. A candidate Lypunov function is chosen as

$$V_1 = \frac{\sigma^2}{2} \tag{8}$$

The derivative of  $V_1$  along the trajectory of Eq. (7) is given by

$$\dot{V}_{1} = \sigma \left( C_{1} \dot{q} + K_{1} q - f(\bullet) + \frac{u}{J_{1}} \right) + \sigma \left( \lambda_{p} \dot{\tilde{\theta}} + \lambda_{i} \tilde{\theta} - \ddot{\theta}_{r} \right)$$
(9)

In view of Eq. (9), to make  $\dot{V}_1 < 0$ , one option is to choose u(t) of the form

$$u = J_1 \left[ -\beta \sigma - (C_1 \dot{q} + K_1 q) - (\lambda_p \dot{\tilde{\theta}} + \lambda_i \tilde{\theta} - \ddot{\theta}_r) - (\gamma_2 |\dot{\theta}| + \gamma_1 |\theta| + \gamma_0) \operatorname{sgn}(\sigma) \right]$$
(10)

where  $\beta > 0$  is a constant.

Substituting the control u given in Eq. (10) into Eq. (9) yields

$$\dot{V}_{1} = -\beta\sigma^{2} - \sigma f(\bullet) - \left(\gamma_{2} \left|\dot{\theta}\right| + \gamma_{1} \left|\theta\right| + \gamma_{0}\right) \left|\sigma\right|$$

$$\leq -\beta\sigma^{2} \leq 0$$
(11)

Since  $V_1$  is a positive definite function of  $\sigma$  and  $\dot{V}_1$  is negative semi-definite, according to Eq. (12), it follows that  $\sigma$  is bounded function and  $V_1(\infty)$  exists. Integrating both sides of Eq. (11) with respect to time gives

$$\int_0^\infty \beta \sigma^2(\tau) \, d\tau \le V_1(0) - V_1(\infty) < \infty \tag{12}$$

Also, in view of Eqs. (7) and (10),  $\dot{\sigma}$  is bounded. It then follows from Barbalat's lemma (Slotine and Li, 1991) that  $\sigma(t) \rightarrow 0$  as  $t \rightarrow \infty$ , which implies that the tracking error

converges to zero,  $\tilde{\theta} \to 0$  and  $\tilde{\theta} \to 0$  as  $t \to \infty$ . Thus, in the closed-loop system, the attitude angle asymptotically follows the given reference trajectory  $\theta_r$ . Now the following theorem can be stated.

**Theorem 1**: Consider the system given by Eqs. (1)-(3) with the assumptions 1-3. If the control law is designed according to Eq. (10), and the switching surface function is selected as given in Eq. (7), then the switching variable  $\sigma(t)$  will

converge to zero, and the tracking errors  $\tilde{\theta} \to 0$  and  $\tilde{\tilde{\theta}} \to 0$ as  $t \to \infty$ .

**Remark 1.** The synthesis of the control law in Eq. (10) has two problems:

1. It is necessary to measure the elastic vibration deflection and its rate information in the attitude control loop. Although a dynamical compensator can be used to estimate q and  $\dot{q}$ , this will increase the complexity of the control system. Therefore, there is a great benefit in synthesizing a controller using only the attitude measurements, namely the attitude angle and its derivative, for feedback in attitude control loop.

2. It is seen that Eq. (10) is obtained only if the system parameters  $(C_1, K_1, J_1, \gamma_2, \gamma_1 \text{ and } \gamma_0)$  are completely known. When there are parameter variations in the system, exact cancellation of the variation in Eq. (9) is not possible and one cannot guarantee that the derivative of Lyapunov function is semi-definite, as given in Eq. (11).

To overcome these drawbacks, we will design an adaptive output feedback sliding mode controller for the spacecraft system in the presence of parametric uncertainty and external disturbance. A modified control law is obtained from Eq. (10) as follows. It is assumed that scalar  $J_1$  is unknown but that its sign is assumed to be known. Note that the sign of the

scalar parameter  $J_1$  is known by computation using the model parameters.

Let  $\hat{J}_1$  be the estimated of the unknown parameter  $J_1$ , and  $\hat{\gamma}_i$  (i = 1, 2, 3) be the estimates of the unknown bound parameters  $\gamma_i$  (i = 1, 2, 3), in which  $\gamma_3$  is selected to satisfy  $\gamma_0 + |C_1\dot{q} + K_1q| < \gamma_3$ . Note that from assumptions 1-3, there does exist a positive constant  $\gamma_3$  such this inequality is guaranteed to be satisfied in the domain of interest.

Denote

$$\Psi = \begin{bmatrix} \gamma_2 & \gamma_1 & \gamma_3 \end{bmatrix}^T, \quad \Upsilon = \begin{bmatrix} |\dot{\theta}| & |\theta| & 1 \end{bmatrix}^T$$
(13)

This definition will be used later to prove stability of the following proposed control scheme. Now, based on Theorem 1 and this definition, we have the following theorem.

**Theorem 2.** Consider the flexible spacecraft system defined by Eqs. (1)-(3), with the assumptions 1-3 and switching surface Eq. (7). Assume the sign of scalar parameter  $J_1$  is known. If the adaptive sliding mode controller is chosen to be

$$u = -\hat{J}_{1} \left[ \beta \sigma + \left( \lambda_{p} \dot{\tilde{\theta}} + \lambda_{i} \tilde{\theta} - \ddot{\theta}_{r} \right) + \left( \underline{\mathscr{B}} \dot{[\theta]} + \gamma_{1} |\theta| + \gamma_{3} \right) \operatorname{sgn}(\sigma) \right]$$
(14)

with the adaptation law

$$\dot{\hat{J}} = \frac{1}{\alpha} \left[ \beta \sigma + \left( \lambda_p \dot{\tilde{\theta}} + \lambda_i \tilde{\theta} - \ddot{\theta}_r \right) + \left( \frac{\beta c}{2} \left| \dot{\theta} \right| + \gamma_1 \left| \theta \right| + \gamma_3 \right) \operatorname{sgn}(\sigma) \right] \operatorname{sgn}(J_1) \sigma$$
(15a)

$$\hat{\Psi} = \Gamma^{-1} \Upsilon \left| \sigma \right| \tag{15b}$$

where  $\Gamma$  is a positive definite symmetric matrix and  $\alpha > 0$ , then  $\dot{V} \leq -\beta \sigma^2 < 0$  holds and this will in turn lead to the convergence of the switching variable  $\sigma(t) \rightarrow 0$  and attitude tracking error  $\tilde{\theta} \rightarrow 0$  as  $t \rightarrow \infty$ .

**Proof**. To show the resulting control scheme achieves the control objective, consider the following candidate Lyapunov function

$$V_2 = \frac{1}{2} \left( \sigma^2 + \tilde{\Psi}^T \Gamma \tilde{\Psi} + \alpha \left| \frac{1}{J_1} \right| \tilde{J}_1^2 \right)$$
(16)

where  $\tilde{\Psi} = \begin{bmatrix} \tilde{\gamma}_2 & \tilde{\gamma}_1 & \tilde{\gamma}_3 \end{bmatrix}^T = \begin{bmatrix} \gamma_2 - \beta \tilde{\omega} & \gamma_1 - \gamma_1 & \gamma_3 - \gamma_3 \end{bmatrix}^T$  and  $\tilde{J}_1 = J_1 - \hat{J}_1$  denote the parameter errors.

Differentiating  $V_2$  with respect to time and substituting the time derivative of sliding surface  $\dot{\sigma}$  and control law u into Eq. (14) with the adaptation law yields

$$\dot{V}_{2} \leq |\sigma| (\gamma_{2} |\dot{\theta}| + \gamma_{1} |\theta| + \gamma_{3}) - \beta \sigma^{2} - (\not{E} |\dot{\theta}| + \gamma_{1} |\theta| + \gamma_{3}) |\sigma| - \tilde{\Psi}^{T} \Upsilon |\sigma|$$

$$= -\beta \sigma^{2}$$
(17)

where the identity  $\frac{1}{J_1}\hat{J}_1 = 1 - \frac{1}{J_1}\tilde{J}_1$  has been used in the

derivation. Barbalat's lemma (Slotine and Li, 1991) can be used to show  $\sigma$  converges to zero such that the tracking error converges to zero,  $\tilde{\theta} \rightarrow 0$  and  $\dot{\tilde{\theta}} \rightarrow 0$  as  $t \rightarrow \infty$ .

## 3.2. Vibration Control Subsystem

To suppress the vibrations of the flexible structures, we adopt the modal velocity feedback control method (Iyer and Singh, 1991) to determine the control input voltages for the piezoceramic actuators. Here we assume that the elastic displacement and its velocity can be measured by piezoceramic sensors. When the switching variable  $\sigma(t)$  is

identically zero,  $\tilde{\theta} = \theta - \theta_r$  and  $\tilde{\theta} = \dot{\theta} - \dot{\theta}_r$  will approach zero. For simplicity, let us assume  $\theta_r$  is a constant trajectory. Then, the elastic vibration becomes decoupled from the rigid body motion. Imposing this condition, Eq. (2) becomes

$$M\ddot{q} + C\dot{q} + Kq = -\overline{B}v \tag{18}$$

For this case, suppose that the feedback control law for the stabilization of the elastic modes is chosen to be of the form

$$v = \overline{B}^{-1} F \dot{q} \tag{19}$$

where  $F = \text{diag}(f_1, f_2)$  is a diagonal matrix with positive elements  $f_i$ , and  $\overline{B}^{-1}$  is understood as the pseudoinverse of  $\overline{B}$ .

To establish the stability of the structural dynamics given by Eq. (18), we consider a quadratic positive definite Lyapunov function of the form

$$V_3 = \frac{1}{2} \left( \dot{q}^T M \dot{q} + q^T K q \right) \tag{20}$$

The derivative of  $V_3$ , using Eq. (18), is given by

$$\dot{V}_3 = \dot{q}^T M \ddot{q} + q^T K q \le -\dot{q}^T F \dot{q}(t) \le 0$$
(21)

One can conclude that the trajectory of the system given by the solution to Eq. (18) eventually lies in the set  $\overline{E}$  (Iyer and Singh, 1991), where

$$\overline{E} = \left\{ \left( q^T, \dot{q}^T \right) \in \mathbb{R}^{2N}, \ F \dot{q}(t) = 0 \right\}$$
(22)

The motion of Eq. (18), confined to the set  $\overline{E}$  , evolves according to

$$\dot{x}_f = A_f x_f \tag{23}$$

where  $A_f = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}$ 

Associated with the system given by Eq. (18), an output vector may be defined as

$$y = C_v x_f = \begin{bmatrix} 0 & \overline{B}^T \end{bmatrix} x_f \tag{24}$$

According to this definition, y is identically zero whenever  $x_f$  belongs to the set  $\overline{E}$ . Now if the matrix pair  $(C_v, A_f)$  is observable, then y = 0 implies that  $x_f = 0$ , and it follows that the equilibrium point  $x_f = (q^T, \dot{q}^T)^T = 0$  of the equations of motion (18), with the control law Eq. (19), is globally exponentially stable.

**Theorem 3**. Consider the closed-loop system given by Eqs. (1), (2), (3), (14) and (19). Suppose that the hypothesis of Theorem 2 is satisfied and that the system Eq. (18) with controller given by Eq. (19) is stable. Then in the closed-loop system  $\tilde{\theta} \rightarrow 0$ ,  $\dot{\tilde{\theta}} \rightarrow 0$ ,  $q \rightarrow 0$  and  $\dot{q} \rightarrow 0$ , as  $t \rightarrow \infty$ .

#### **Proof**: The proof is omitted.

**Remark 2**: Note that the proposed control scheme has the following characteristics.

1. For the attitude control subsystem, the measurement of the flexible modes is not necessary. Due to the difficulty in measuring q and  $\dot{q}$  this is a great advantage for implementation of the attitude controller.

2. In practice, a suitable estimate for the lumped perturbation is difficult as we are not able to anticipate the variation in the bounds of the uncertainties. Overestimation may result in unnecessarily high gains and large chattering which degrade system performance. Underestimation, on the other hand, is not permitted as it may lead to instability. The proposed adaptation law to estimate the lumped perturbation solves this problem and alleviates the difficulty arising from assumptions about the flexible modes, parameter variations, disturbances and other system uncertainties.

#### 4. SIMULATION AND COMPARISON RESULTS

In order to demonstrate the effectiveness of the proposed control schemes, numerical simulations have been performed and are presented in this section. The complete spacecraft model given by Hu and Ma (2005) is used for the simulations. The first five modal frequencies of the flexible appendage are 3.161, 16.954, 47.233, 94.557, and 153.003 rad/s, and all modal damping ratios are assumed to be 0.004. Only the first two low-order modes of the five modes in the flexible system model are considered for vibration suppression. In the simulation, it is desired to slew the spacecraft to a target angle 70° and the initial conditions are assumed to be  $\theta(0) = 0$ ,  $\dot{\theta}(0) = 0$ , q(0) = 0 and  $\dot{q}(0) = 0$ . In addition, practical implementations will restrict the moment available from the flywheel, and thus the control input is bounded with a saturation value of 1 Nm. Here the vibration level is described by the energy function  $E = \dot{q}^T \dot{q} + q^T K_{qq} q$ . The

external disturbance is assumed to be of the form  $d(t) = 3\sin(0.05t) + 1.5\cos(0.02t)$ .

Fig. 1 gives the simulation plots for the case of ASMC with step input reference. It is noted that an acceptable angle response is achieved and the vibrations achieves a maximum amplitude of 0.004 Nm in energy. Moreover, the settling time is less than 40s. This reflects the effectiveness of the proposed ASMC for atttiude control and flexible structural vibrations suppression.



Fig. 1. Time response with ASMC control with a step input command

The simulation was repeated using the smooth reference command instead of the step command. The simulation results are shown in Fig. 2. Comparing Figs. 1 and 2 shows that the attitude angle response is similar in both cases. The smooth input gave a significantly smaller vibration response, with the maximum amplitude of the energy being less than 0.0003 Nm. The modal vibration response was found to have almost zero vibration after 30s. A step input command will always excite the vibration modes of a structure, and the smooth input command using Eq. (4) acts as a low pass filter. The required attitude angle was achieved with settling time of 30s, and no overshoot was observed.

The case of the adaptive sliding mode controller integrated with a modal velocity feedback (MVF) compensator with the smooth reference command was also studied. Figure 3 shows the results of employing this combination. The imposed desired angular displacement is accurately achieved and the relatively large amplitude vibrations excited by rapid maneuvers can be actively suppressed, as shown in Fig. 3(b). This demonstrates the validity of active vibration reduction based on the modal velocity feedback control technique using piezoelectric materials as sensors/actuators.

Extensive simulations were also performed using different parameter uncertainty and disturbance inputs. These results show that in the closed-loop system attitude control and vibration stabilization were achieved in spite of perturbations in the system. Moreover, the flexibility in the choice of control parameters can be utilized to obtain desirable performance while meeting the constraints on the control magnitude and elastic deflection.



Fig.2. Time response with ASMC control with a smooth input command

## 5. CONCLUSIONS

In this paper, a robust adaptive control system was derived for the attitude tracking and vibration suppression of an orbiting spacecraft with flexible appendages with bonded piezoelectric actuators/sensors. The system parameters are assumed to be uncertain and the truncated model of the spacecraft has a finite but arbitrary dimension. The design approach presented here treats the problem of spacecraft attitude control separately from the elastic vibration suppression problem. A third-order reference command generator was chosen for output tracking. For the attitude tracking subsystem, two controllers are designed based on sliding mode control techniques. The first, a direct sliding mode controller, is a full-state feedback controller and achieves asymptotic attitude tracking, but the system parameters need to be known. This controller is then redesigned such that the need to measure the elastic modes and the system parameters is eliminated, in which parametric uncertainty and additive bounded disturbance are compensated by using an adaptive updating law. This adaptive controller has several design parameters that can be adjusted to obtain desirable response characteristics. For actively damping elastic motion, the vibration compensator is separately designed based on modal velocity feedback control method to determine the control voltage for the piezoelectric actuators. The benefits of the proposed control methodology are demonstrated on a five-mode model of a spacecraft with a flexible appendage. Simulation results show that control of the attitude tracking and the vibration reduction can be accomplished using adaptive output feedback sliding mode in spite of the uncertainties in the system parameters and the external disturbance.



Fig.3. Time response with ASMC control + MVF compensator with the smooth reference command

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