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Structural approach for sensor placement with diagnosability purpose

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Abstract: Maintenance and diagnosis of complex systems are common activities in the industrial world. Technological advances have led to a continuously increasing complexity of industrial systems. This complexity, which is due to an increasing number of components reduces in turn the reliability of plants. Therefore, fault diagnosis is becoming a growing field of interest. But fault diagnosis relies on sensors: efficient fault diagnosis procedures require a relevant sensor placement. This paper presents fundamental results for sensor placement based on diagnosability criteria. These results contribute to the design of sensor placement algorithms, which satisfies diagnosability specifications.

Keywords: fault diagnosis, diagnosability, sensor placement

1. INTRODUCTION

Sensor placement decisions depend on expected objectives. For instance, in control theory, the sensor placement is used to provide sufficient information for the control of systems. Criteria deal with observability and controllability of the variables. Madron and Veverka [1992] has proposed a sensor placement method which deals with linear system. This method makes use of the Gauss-Jordan elimination to find a minimum set of variables to be measured. This ensures the observability of variables while simultaneously minimizing the cost of sensors. In this theory, the observable variables include the measurable variables plus the unmeasured but deductible variables. Another method for sensor placement has been proposed in Maquin et al. [1997]. This method aims at guaranteeing the detectability and isolability of sensor failures. The proposed method is based on the concept of redundancy degree in a variable and the structural analysis of the system model. The sensor placement can be solved with a matricial analysis of a cycle matrix or using the technique of mixed linear programming. Commault et al. [2006] has proposed a method of sensor location. In this method, they defined a new set of separators (Irreducible Input Separators), which generates sets of system variables in which additional sensors must be implemented to solve the considered problem.

However, in fault diagnosis, the goal of sensor placement should be to satisfy detectability and diagnosability properties. Detectability is the possibility of detecting a fault on a component and diagnosability is the possibility of identifying a fault on a component without this creating ambiguity with any other fault.

Travé-Massuyès et al. [2001] has proposed a method based on consecutive additions of sensors, which takes into account diagnosability criteria. The principle of this method is to analyze the physical model of a system from a structural point of view. This structural approach is based on Analytical Redundancy Relations (ARR) Cassar and Staroswiecki [1997], which can be obtained from combinations of model constraints using bipartite graph Blanke et al. [2003] or elimination rules Ploix et al. [2005], and on the corresponding signature table Patton and Chen [1991]. In a signature table, rows and columns represent respectively, the set of analytical redundancy relations and the set of considered faults. However, this method requires an a priori design of all the ARR for a given set of sensors.

This paper presents results for the design of sensor placement algorithms. Thanks to these results, the sensor placement satisfying diagnosability objectives becomes possible without designing ARR a priori. It is an important feature since it is no longer necessary to design all the possible ARR assuming all the variables are measured.

2. PROBLEM FORMULATION

In the following, the set of variables appearing in a constraint k is denoted: $var(k)$ and the set of variables appearing in the set of constraints K: $var(K) = \bigcup_{k\in K} var(k)$. A system Σ can be described by a tuple (K_Σ, C_Σ). $var(K_\Sigma)$ is the set of variables that models phenomena influenced by Σ. The behavior is represented by constraints $K_\Sigma = \{\dots, k_i, \dots\}$ that establish relationships between variables of $var(K_\Sigma)$. It can be represented by a structural matrix $\mathcal{M}_\Sigma$, which is an incidence matrix representing the application $\mathcal{M}_\Sigma : var(K_\Sigma) \to K_\Sigma$. $C_\Sigma = \{\dots, c_j, \dots\}$ is a set of independent components constituting Σ. Each constraint in K_Σ models one component and, conversely, a component can be modeled by at most one constraint:

$\forall k \in K_\Sigma, comp(k) \in C_\Sigma$ [1] where $comp(k)$ refers to the component corresponding to the constraint k. Let us introduce the concept of testable subsystem (TSS) and its relationship with the concept of ARR.

Definition 1. Let K be a set of constraints and v a variable in $var(K)$ characterized by its domain $dom(v)$. K is a solving constraint set for v if using K, it is possible to find a value set S for v such that $S \subset dom(v)$. A solving constraint set for v is minimal if there is no subset of K, which is also a solving constraint set for v. A minimal solving constraint set K for v is denoted: $K \vdash v$.

Definition 2. Let K be a set of constraints. K is testable if and only if there are two distinct subsets $K_1 \subset K$, $K_2 \subset K$ such that $K_1 \nsubseteq K_2$ and $K_2 \nsubseteq K_1$, and a variable $v \in var(K)$ such that $K_1 \vdash v$ and $K_2 \vdash v$. If this property is satisfied, it is indeed possible to check if the value set S_1 deduced from K_1 is consistent with the value set S_2 deduced from K_2: $S_1 \cap S_2 \neq \emptyset$.

This definition also applies to models containing ordinary differential equations. Indeed, testable state space representations, including state space observers, always have equivalent parity space representations Staroswiecki et al. [1991].

Adding any constraint to a testable set leads also to a testable set of constraints. Only minimal testable sets are interesting.

Definition 3. A testable set of constraints is minimal if it is not possible to keep testability when removing a constraint.

A global testable constraint that can be deduced from a TSS is called ARR. Let $R_\Sigma = \{\ldots, r_k, \ldots\}$ be the set of all the testable subsystems that can be deduced from K_Σ according to Blanke et al. [2003], Ploix et al. [2005], Staroswiecki and Declerck [1989]. Because of the one-to-one relationships between constraints and components, notions of detectability and discriminability can be extended to constraints. Therefore, usual definitions Struss et al. [2002] related to continuous systems can be extended from faults to constraints. Let R be a set of TSS coming from (K_Σ, C_Σ) [2].

Definition 4. A constraint $k \in K_\Sigma$ is detectable (see Struss et al. [2002]) in R if $\exists r_i \in R / k \in r_i$. By extension, the constraints $K \subset K_\Sigma$ are detectable in R if $\forall k_i \in K$, k_i is detectable in R.

Definition 5. Two constraints $(k_1, k_2) \in K_\Sigma^2$ are discriminable (see Struss et al. [2002]) in R if: $\exists r_i \in R / k_1 \in r_i \, and \, k_2 \notin r_i$ or if $\exists r_j \in R / k_2 \in r_j \, and \, k_1 \notin r_j$. By extension, the constraints of a set $K \subset K_\Sigma$ are discriminable in R if: $\forall (k_i, k_j) \in K^2$, k_i and k_j are discriminable in R with $k_i \neq k_j$.

Obviously, non detectability of both constraints (k_1, k_2) implies non discriminability of (k_1, k_2).

Definition 6. A constraint $k \in K_\Sigma$ is diagnosable (see Struss et al. [2002]) in R if: it is detectable and if $\forall k_j \in (K_\Sigma \setminus k)$, (k, k_j) are discriminable in R. By extension, the constraints $K \in K_\Sigma$ are diagnosable in R if: $\forall k_i \in K$, k_i are diagnosable in R.

In order to formulate the sensor placement problem, the notion of terminal constraint has to be introduced.

Definition 7. A terminal constraint k is a constraint that satisfies: $card(var(k)) = 1$. A terminal constraint usually models a sensor or an actuator: $var(k)$ is generally a measured or a controlled variable. It may also be a variable for which the value is assumed (such ambiant temperature). It is thus a major concept in sensor placement.

In fault diagnosis, sensor placement has to satisfy specifications dealing with detectability and diagnosability. Because of the one-to-one relation between components and constraints, what is true for components is also true for constraints. Therefore, the components C_Σ and the corresponding constraints K_Σ may be decomposed into several sets:

- the set of components C_{diag} / constraints K_{diag} that has to be diagnosable
- the set of subsets of components $\mathbb{C}_{nondis} = \{\ldots, C_i, \ldots\}$ / constraints $\mathbb{K}_{nondis} = \{\ldots, K_i, \ldots\}$ that have to be non discriminable but detectable for each set C_i or K_i
- the set of components C_{nondet} / constraints K_{nondet} that has to be non detectable

Specifications C_{diag}, $\mathbb{C}_{nondis}$ and C_{nondet} of sensor placement problems are meaningful if the two following properties are satisfied:

(1) Sets in specifications must not to overlap one each other to make sense: constraint sets have to satisfy: $C_{nondet} \cap C_{diag} = \phi$, $\forall C_i \in \mathbb{C}_{nondis}, C_i \cap C_{nondet} = \phi$, $\forall C_i \in \mathbb{C}_{nondis}, C_i \cap C_{diag} = \phi$ and $\forall (C_i, C_j) \in \mathbb{C}^2_{nondis}$, $C_i \cap C_j = \phi$ if $C_i \neq C_j$ (no overlapping property).
(2) The union of all the components appearing in C_{diag}, $\mathbb{C}_{nondis}$ and C_{nondet} has to correspond to C_Σ: $C_\Sigma = C_{diag} \cup C_{nondet} \cup \bigcup_{C_i \in \mathbb{C}_{nondis}} C_i$ (completeness property).

If these properties are satisfied the specifications are qualified as consistent in C_Σ . Replacing components by corresponding constraints leads to the same properties for specifications K_{diag}, $\mathbb{K}_{nondis}$ and K_{nondet} to be consistent in K_Σ.

Satisfying the specifications requires information delivered by sensors. Let Σ' represent the system Σ with the additional sensors. Σ' can be described by a tuple $(K_{\Sigma'}, C_{\Sigma'})$ where $C_{\Sigma'}$ represents the components of system Σ plus the additional sensors and $K_{\Sigma'}$ represents the constraints of system Σ plus the additional terminal constraints which model the sensors. The sensor placement problem consists in determining the additional terminal constraints in $K_{\Sigma'}$ that lead to the satisfaction of the specification K_{diag}, $\mathbb{K}_{nondis}$ and K_{nondet}. Because of the relations between constraints and components, the results can be extended to components.

In the next sections, fundamental results are proposed for the design of sensor placement satisfying diagnosability

[1] A component may also be modeled by several constraints but, for the sake of simplicity, it has not been considered in this paper.
[2] C_Σ is not used at this stage.

and detectability specifications. Algorithms are not detailed in this paper.

3. PRELIMINARY CONCEPTS

Before deducing diagnosability properties of constraint sets, some concepts have to be introduced.

3.1 Value propagation as a theoretical tool

According to definition 3, a TSS is a minimal set of constraints K such that there exists a constraint $k \in K$ for which all the variables of $var(k)$ can be instantiated, starting from terminal constraints. An ARR corresponding to a TSS can be seen in different ways. The most common approach is to consider an ARR as a global constraint. Another way is to think of an ARR as a complete value propagation Fron [1994] w.r.t. variables i.e. a propagation that leads to information about the consistency of a set of constraints, including terminal constraints that contain known data. This approach has been adopted as a theoretical tool to develop proofs. Relationship between value propagation and ARR is detailed in this section.

Let k_1 and k_2 be two constraints. The propagation of a variable v between k_1 and k_2 is possible only if $v \in var(k_1) \cap var(k_2)$. The variable v is qualified as propagable between k_1 and k_2. Consider a system, defined by $K_\Sigma = \{k_1, k_2, k_3, k_4, k_5\}$ with $var(k_1) = \{v_1, v_3\}$, $var(k_2) = \{v_1, v_2\}$, $var(k_3) = \{v_2, v_3\}$, $var(k_4) = \{v_2\}$ and $var(k_5) = \{v_3\}$. Terminal constraints k_4 and k_5 model sensors or actuators. Each terminal constraint contains known data. The set of all TSS that can be tested is represented by the propagations drawn in figure 1.

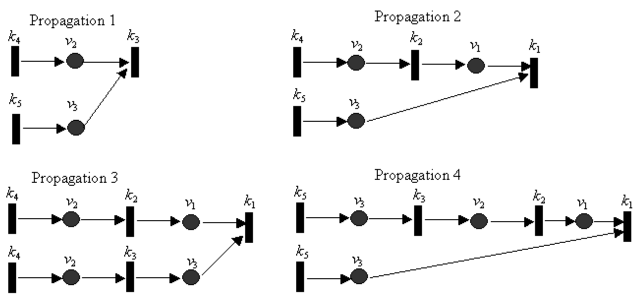

Fig. 1. Set of propagations

A propagation starts by a terminal constraint, which means that "a variable is equal to a known value". In this example, propagations start either with k_4 or k_5. Thanks to these constraints, a value can be respectively assigned to v_2 and v_3. Once values have been assigned to these variables, new variables can then be instantiated. Propagation continues until no more assignments are possible because terminal constraints or instantiated variables have been reached. The set of constraints that appears in a propagation, corresponds to a testable subsystem. These constraints can be combined into a unique global constraint named ARR. Depending on the constraints chosen for propagating values, different ARR may be obtained (see figure 1). In the continuation of this paper, value propagation is implicitly used and appears in the proofs of the different lemmas and theorems.

3.2 Some characteristics of constraint sets

The concept of *linked constraints* is introduced because it is important regarding sensor placement. Indeed, discriminability depends on this concept.

As mentioned in Blanke et al. [2003], the constraints of a system Σ may be modeled by a non directed bipartite graph $(K_\Sigma, var(K_\Sigma), E_\Sigma)$ where E_Σ is the set of edges. Each edge $e = (k, v)$ models that $v \in var(k)$. Let us introduce new definitions useful for sensor placement.

Definition 8. A set of constraints $K \subset K_\Sigma$ is interconnected by a set of variables $V \subset var(K_\Sigma)$ iff there is a tree $(K, V, E) \subset (K_\Sigma, var(K_\Sigma), E_\Sigma)$ with constraints at extremities (see Bollobás [1998] for example), which satisfies $card(V) = card(K) - 1$.

Definition 9. A set of constraints $K \subset K_\Sigma$ is linked in K_Σ by a set of variables $V \subseteq var(K_\Sigma)$ iff K is interconnected by V and iff the other constraints of K_Σ (i.e. $K_\Sigma \backslash K$) do not contain any variable of V. The variables of V are called linking variables for K. They are denoted: $var_{linking}(K, K_\Sigma)$.

The shape of a structural matrix dealing with linked constraints is drawn in figure 2.

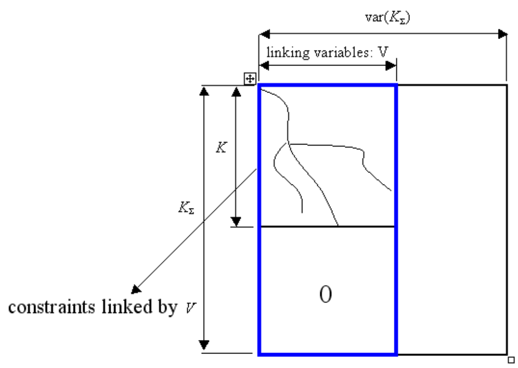

Fig. 2. Structural matrix of a constraint set, which is linked by path

The concept of linked constraints is strongly connected with discriminability.

Lemma 10. A set of constraints $K \subset K_\Sigma$ linked by a set of variables $V \subset V_\Sigma$ is necessarily non discriminable.

Proof. Indeed,

(1) because variables in V only appear in the constraints in K, the only way of propagating variables is to use the constraints in K and the variables in V,
(2) because there is a tree $(K, V, E) \subset (K_\Sigma, var(K_\Sigma), E_\Sigma)$ with constraints at extremities, instantiating all the variables in V involves at least the achievement of the propagations defined by the tree.

Therefore, all the constraints are invariably found together in TSS: K is non discriminable.

In order to improve the clarity of these explanations, let us introduce the notion of stump variables.

Definition 11. A set of variables $var(K)$ appearing in a set of constraints K but not in the other constraints of

K_Σ (i.e. $K_\Sigma \backslash K$) are named stump variables in K_Σ. They are denoted: $var_{stump}(K, K_\Sigma)$. For instance, the set of variables V that links a set of constraints K belong to the stump variables $var_{stump}(K, K_\Sigma)$.

A set of constraints cannot be used to generate a TSS if they are linked and if there are additional variables that cannot be propagated. These constraints are qualified as isolated. Detectability depends on this concept.

Definition 12. A set of several constraints $K \subset K_\Sigma$ is isolated in K_Σ by a set of variables $V \subset var(K_\Sigma)$ if they are linked by V and if there is at least one variable in $var(K) \backslash V$ that does not belong to other constraints of K_Σ (i.e. $K_\Sigma \backslash K$). If the set contains only one constraint, the link condition disappears but the other remains.

The shape of a structural matrix dealing with isolated constraints is drawn in figure 3.

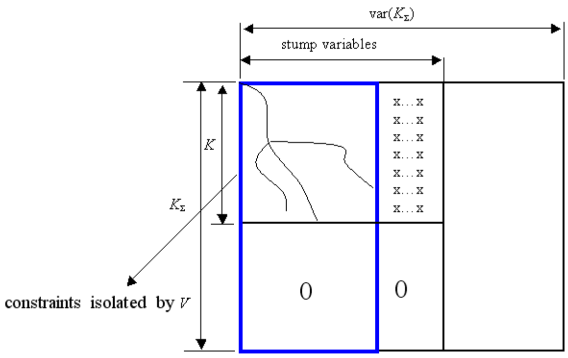

Fig. 3. Structural matrix of a constraint set, which is isolated by the set of variables V

The concept of isolated constraints is strongly linked with detectability.

Lemma 13. A set of constraints $K \subset K_\Sigma$ isolated in K_Σ by V is necessarily non detectable.

Proof. The constraints K isolated in K_Σ by V will always come together in TSS because, by definition, they are linked by V. Because of the fact that, in isolated constraints, there is at least one additional variable in $var(K)$ which does not appear in other constraints (i.e. $K_\Sigma \backslash K$), it is not possible to instantiate this value and, therefore, this set of constraints cannot be involved into a TSS: K is non detectable.

4. CONSTRAINT SET AND DIAGNOSABILITY PROPERTIES

This section aims at setting up a direct link from sets of constraints to detectability and diagnosability properties. Firstly, it is obvious that adding additional constraints connected to all the variables $var(k)$ appearing in a constraint k, ensures the diagnosability of k.

Lemma 14. Let $k \in K_\Sigma$ be a constraint. If additional terminal constraints dealing with all the variables in $var(k)$ are added, then the constraint k is diagnosable.

Proof. Because there are additional terminal constraints connected to each variable in $V(k)$, a value can be assigned for each variable. Consequently, there is one TSS containing k plus additional terminal constraints connected to variables in $var(k)$. Therefore, the constraint $k \in K$ is necessarily diagnosable because there is one TSS that does not contain other constraints of K_Σ (i.e. $K_\Sigma \backslash \{k\}$).

Lemma 14 can be directly applied to all the constraints of a constraint set.

Corollary 15. If additional terminal constraints dealing with all the variables $var(K)$ of a constraint set $K \in K_\Sigma$, then each constraint $k \in K$ is diagnosable.

In lemma 13, a relationship between isolated constraints and the detectability property has been presented. The next lemma generalizes the previous results.

Lemma 16. A sufficient condition for a subset of constraints $K \subset K_\Sigma$ to be non detectable is that there is a tuple $(K_1, \ldots, K_m)$ of m sets of constraints making up a partition $\mathcal{P}(K)$ of K such that each K_i is isolated in $K_\Sigma \backslash \bigcup_{j<i} K_j$ (K_1 is a limit case: it should be isolated in K_Σ).

Proof. The case of K_1 has been discussed in lemma 13: because the constraints in K_1 are isolated in K_Σ, they are non detectable and therefore cannot be included in TSS. Then, the remaining candidate constraints for TSS belong to $K_\Sigma \backslash K_1$. Because K_2 is isolated in $K_\Sigma \backslash K_1$, they are non detectable. The reasoning can be extended to any i. Consequently, the constraints in $K = \bigcup_i K_i$ are non detectable.

Figure 4 indicates the shape of a structural matrix of non detectable constraints.

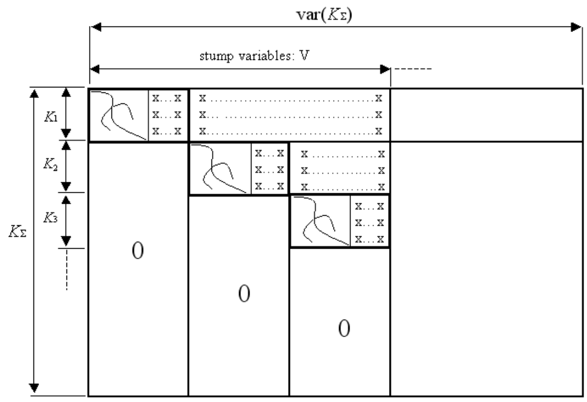

Fig. 4. Structural matrix of non detectable constraints

Consider, for example, a system modeled by the following structural matrix:

	v_1	v_2	v_3	v_4	v_5	v_6
k_1	1	0	0	1	0	0
k_2	0	1	1	0	1	0
k_3	0	1	1	0	1	0
k_4	0	0	0	1	0	1
k_5	0	0	0	1	1	1

Assume that the set $K = \{k_1, k_2, k_3\}$ is required to be non detectable. In this example, there exists a tuple $(\{k_1\}, \{k_2, k_3\})$ such that each element K_i satisfies lemma

16. If there are no additional terminal constraints containing v_1, v_2 and v_3, the subset K is non detectable.

Lemma 17. A sufficient condition for each set $K_i \subset K$ belonging to a set of m constraint sets $\mathbb{K} = \{K_1, \dots, K_m\}$ such that $\forall K_i \neq K_j, K_i \cap K_j = \emptyset$, to be non discriminable is that each K_i is linked by a set of variables V_i.

Proof. This lemma is a direct application of lemma 10 to several sets of constraints.

Consider, for example, a system modeled by the following structural matrix:

	v_1	v_2	v_3	v_4	v_5
k_1	1	0	1	1	1
k_2	1	1	1	1	0
k_3	1	1	1	0	1
k_4	0	1	1	0	0
k_5	0	0	0	1	1

Assume that $K = \{k_1, k_2, k_3, k_4\}$ is a constraint subset that should be non discriminable. Because the constraints k_1, k_2, k_3 and k_4 are linked by $V = \{v_1, v_2, v_3\}$, lemma 17 is satisfied. Therefore, k_1, k_2, k_3 and k_4 are non discriminable provided that no additional terminal constraints contain a variable of V.

The following theorem groups lemmas 14, 16 and 17.

Theorem 18. Let K_Σ be a set of constraints and K_{nondet}, $\mathbb{K}_{nondis}$ and K_{diag} be the specifications of a sensor placement problem consistent in K_Σ. Sufficient conditions for the specifications to be fulfilled are:

(1) there exists a tuple $(K_1, \dots, K_p)$ of p sets of constraints making up a partition $\mathcal{P}(K_{nondet})$ of K_{nondet} such that each K_i is isolated in $K_\Sigma \backslash \bigcup_{j<i} K_j$ (K_1 is a limit case: it should be isolated in K_Σ) see figure 4.
(2) each set K_i belonging to $\mathbb{K}_{nondis} = \{K_1, \dots, K_m\}$ such that $\forall K_i \neq K_j, K_i \cap K_j = \emptyset$, is linked by a set of variables V_i in considering only the constraints $K_\Sigma \backslash K_{nondet}$
(3) Additional terminal constraints are added on the variables: $V_{candidate} = var(K_\Sigma) \backslash (var_{stump}(K_{nondet}, K_\Sigma) \cup \bigcup_{K_j \in \mathbb{K}_{nondis}} var_{linking}(K_j, K_\Sigma \backslash K_{nondet}))$ (see figure 5).

Proof. The proof relies on the resulting structure of the structural matrix, which directly stems from corollary 15 and lemmas 16 and 17. Note that point 2 could also be stated for the whole set of constraints K_Σ. However, it is not useful to include non detectable constraints, which will not appear in resulting TSS: it would be less conservative.

Because of lemma 16 and 17, the variables of $var(K_{diag})$ cannot contain variables appearing in the variables involved in (1) and (2) i.e. in $var_{stump}(K_{nondet}, K_\Sigma)$ and in $\bigcup_{K_j \in \mathbb{K}_{nondis}} var_{linking}(K_j, K_\Sigma \backslash K_{nondet})$. Then, $var(K_{diag})$ satisfies: $var(K_{diag}) \subset V_{candidate}$. Because the variables of $V_{candidate}$ can be instantiated with measured values, all the constraints of K_{diag} are diagnosable following corollary 15.

The point that has to be proved is that, in specifications, $\mathbb{K}_{nondis}$ defines non discriminable but detectable sets and not only non discriminable sets as in lemma 17: the detectability of sets in $\mathbb{K}_{nondis}$ has to be proved.

The variables $var(K_i)$ of a constraint set $K_i \in \mathbb{K}_{nondis}$ can be decomposed into two sets: V_i^- and V_i^+ where $V_i^- = var_{linking}(K_i, K_\Sigma \backslash K_{nondet})$ contains the linking variables and V_i^+ contains the remaining variables $V_i^+ = var(K_i) \backslash V_i^-$. Because of lemma 16 and 17, the set V_i^+ cannot contain variables in $var_{stump}(K_{nondet}, K_\Sigma)$ and in $\bigcup_{K_j \in \mathbb{K}_{nondis}; K_j \neq K_i} var_{linking}(K_j, K_\Sigma)$. Therefore, V_i^+ satisfies: $V_i^+ \subset V_{candidate}$

Because of the third point of the theorem, all the variables of $V_{candidate}$ are known: additional terminal constraints are indeed added, there is necessarily a TSS dealing with all the constraints in K_i. It proves that the constraint set K_i is necessarily detectable. Because this result holds for any $K_i \in \mathbb{K}_{nondis}$, it proves the theorem.

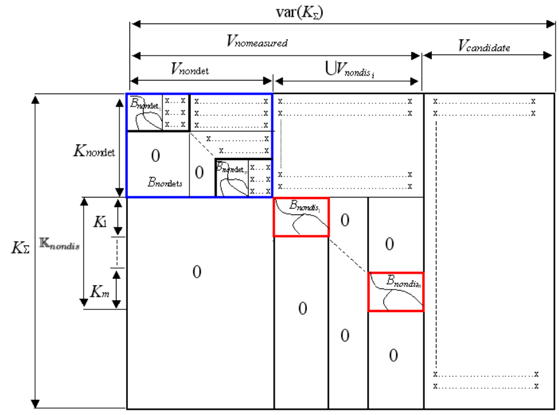

Fig. 5. Shape of a structural matrix Satisfying theorem 18

Satisfying theorem 18 guarantees that the specifications are satisfied. However, because the theorem provides only a sufficient condition for diagnosability, the number of additional terminal constraints is not necessarily minimal. It has to be checked afterwards.

The sensor placement problem has been studied without considering components. Let us now take components into account. Components of a system may be divided into three sets: the components on which faults need to be isolated, the components on which faults need to be detected but not necessarily localized and the components on which faults need to be non detectable. Because it has been assumed that each component is modeled by only one constraint, the results obtained for constraints can be extended to components using the application $\Phi_\Sigma : K_\Sigma \rightarrow C_\Sigma$.

5. APPLICATION TO DAMADICS BENCHMARK

Several methods for fault isolation have been benchmarked on a pneumatic servo-motor actuated valve named DAMADICS (Development and Application of Methods for Actuator diagnosis in Industrial Control Systems). Spanache and Escobet [2004] has designed a sensor placement method for this problem that optimize the diagnosability level of the system. In this section, the method proposed in this paper, is applied on this benchmark. The system is defined by the following equations:

$$k_1 : X = r_1(P_s, \triangle P)$$
$$k_2 : F_V = r_2(X, \triangle P)$$
$$k_3 : CVI = r_3(SP, PV)$$
$$k_4 : P_s = r_4(X, CVI, P_z)$$
$$k_5 : PV = r_5(X)$$

The corresponding structural matrix is given in table 1.

Table 1. structural matrix of DAMADICS

	X	P_s	CVI	PV	F_V	P_z	SP	$\triangle P$
k_1	1	1	0	0	0	0	0	1
k_2	1	0	0	0	1	0	0	1
k_3	0	0	1	1	0	0	1	0
k_4	1	1	1	0	0	1	0	0
k_5	1	0	0	1	0	0	0	0

Let's fix these specifications: $K_{nondet} = \{k_1, k_2\}$, $\mathbb{K}_{nondis} = \{\{k_3, k_4\}\}$ and $K_{diag} = \{k_5\}$.

The set of constraints $K_{nondet} = \{k_1, k_2\}$ is linked by the path $\{k_1, \triangle P, k_2\}$. Because of variable F_V, K_{nondet} is isolated by the path $\{k_1, \triangle P, k_2\}$.

The set of constraints $K = \{k_3, k_4\} \in \mathbb{K}_{nondis}$ is linked by the path $\{k_3, CVI, k_5\}$. Then, according to theorem 18, no terminal constraints containing a variable from $\{\triangle P, F_V, CVI\}$ have to be added i.e. these variables have not to be measured.

In order to satisfy the last item of theorem 18, all the variables of the system except $\{\triangle P, F_V, CVI\}$ have to be measured.

The method proposed in Ploix et al. [2005] has been used to design all the ARR. It has led to the fault signature matrix drawn in table 2.

Table 2. Fault Signature Matrix of DAMADICS

TSS	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
TSS_1	0	0	1	1	1	1	1	0	1	1
TSS_2	0	0	1	1	1	0	1	1	1	1
TSS_3	0	0	0	0	1	1	0	1	0	0
TSS_4	0	0	1	1	0	1	1	1	1	0

According to these results, the constraints that cannot be discriminated are: $\{k_3, k_4\}$, the constraints that cannot be detected are: $\{k_1, k_2\}$ and the diagnosable constraint is: $\{k_5\}$. Applying the function $\Phi : K_\Sigma \longrightarrow C_\Sigma$, it is obvious that the components, which cannot be discriminated are: $\{c_3, c_4\}$ and the components, which cannot be detected is: $\{c_1, c_2\}$. The diagnosable component is: $\{c_5\}$.

The results presented in this paper demonstrate that it is possible to design sensor placements which satisfy diagnosability criteria without designing ARR a priori.

6. CONCLUSION

New results for the design of sensor placement algorithms has been proposed. It manages, the specifications dealing with sets of constraints that have to be diagnosable, non discriminable or non detectable. These results apply to any system depicted by constraints, which may only be described by the variables appearing in them. Thanks to these results, sensor placements satisfying diagnosability specifications become possible without designing ARR a priori. It is a very important feature since it is no longer necessary to design all the possible ARR assuming that some variables are measured. An algorithm providing solutions to the sensor placement problem that contains a minimum number of sensors will be provided in the near future.

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