Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Stabilizing Multimachine Systems with Decentralized and Nonlinear Feedbacks

J. Zhou * **Y. Ohsawa** **

* *Department of Electrical Engineering, Kyoto University, Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8510, Japan; Tel: 0081-75-3832260; e-mail: zhouj@kuee.kyoto-u.ac.jp.*
** *Department of Electrical Engineering, Kyoto University, Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8510, Japan.*

Abstract: Stabilization in multimachine (synchronous generators) power systems is dealt with through a class of decentralized and nonlinear state feedback laws that can be separately designed in a generator-wise fashion, based on what we call the improved swing equations. Stability of the closed-loop power systems is robust with regard to perturbations in electric torques happening in the synchronous generators. The stabilization algorithm consists of decentralized control laws, and provides us with many more freedoms for accomodating various control indices.

1. INTRODUCTION

In the paper, we consider stabilization problems related to swing dynamics in a class of multimachine power systems. It is well known in the power system field that swing dynamics of synchronous generators are closely related to electric power transmission, and can be described by swing equations, which are nonlinear and probably nonautonomous. Conventional or simplified swing equations (see Haque (2005) and Zaborszky (1988)) are usually modeled by neglecting many seemingly trivial but important factors of synchronous generators to surmount nonlinearities and singularities in differential equations (Anderson (2003), Kimbark (1995), Padiyar (1996), Pavella (1994), and Saccomanno (2003)). When multiple synchronous generators are inter-connected, a complicated multimachine network is created (see Gupta (2005)). In such a large-scale network, stabilization is one of the key tasks we must be confronted with, in which most of the classical Lyapunov methods do not work well generally.

As an answer to the stabilization problem in multimachine power systems, we introduce the improved swing equations without significant approximations. Working with the improved swing equations, a class of nonlinear feedback laws are developed by exploiting some feedback design methods suggested by Isidori. Stability thus resulted in the closed-loop multimachine power system is robust against various perturbations in form of electrical torque variations.

By the best knowledge of the authors, there are numerous efforts to attach stability problems in the literature about multimachine power systems. For example, stability analysis and stabilization in both single machine power systems and multimachine ones through the conventional swing equations (Chiang (1988), Gupta (2005), Haque (2005), Liu (1994), Mitani (1994), Salam (1986), Zaborszky (1988)); or improved modelings by Parashar (2004) and Ueda (2004) suggest that can reflect swing processes more precisely in multi-machine power systems. This study also got benefits from the decentralized control technique (Gupta (2005), Yang (1987), Yang *et al.* (1994)), and the partial stability concept (Vorotnikov, 1998).

2. PRELIMINARIES TO MULTIMACHINE POWER SYSTEMS

In this section, we first quickly review what we call the improved swing equaions and its approximations for describing the dynamics of synchronous generators in a mutlimachine power system. The standing assumptions include: there are n generators, in each of which the rotor winding flux is constant and without voltage regulator.

2.1 Improved Swing Equations

Firstly, we recall that the swing equation (Anderson (2003), Padiyar (1996)) governs the dynamics of a synchronous generator rotor, which can conventionally be written as follows.

$$J_i \frac{d^2\theta_i}{dt^2} + D_i \frac{d\theta_i}{dt} + \Delta_i = T_{mi} - T_{ei}, \qquad (1)$$

where the subscript i denotes the genertor i, and

θ_i: mechanical angular displacement of the generator poles (rad);

J_i: combined inertia moment of the corresponding generator rotor (kg·m^2);

D_i: viscous damping constant of the corresponding generator rotor;

T_{mi}: equivalent mechanical torque minus the electrical load torque at the generator i (N·m);

T_{ei}: equivalent electrical swing torque opposing the mechanical torque (N·m);

Δ_i: electrical torques other than T_{ei} (N·m); we assume that $\Delta_i \in \mathcal{F}$, where $\mathcal{F}$ is a closed perturbation set. There are

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10.3182/20080706-5-KR-1001.0050

various electrical torques in synchronous generators, which are complicatedly dependent on the generator internal voltage, the machine reactances and so on. Their effect on the dynamics is approximately reflected by Δ_i.

Secondly, we express θ_i through a so-called center-of-angle (COA) reference framework that is rotating at the system angular frequency ω_0 (rad/s).

$$\theta_i = \omega_0 t + \delta_i, \quad t \geq t_0,$$

where t_0 denotes the initial time, and δ_i represents the rotor angular displacement of the generator i with respect to the COA reference framework. Furthermore, let us define

$$\omega_i = \frac{d\delta_i}{dt}, \quad \frac{d\omega_i}{dt} = \frac{d^2\delta_i}{dt^2} \tag{2}$$

Hence, ω_i and $d\omega_i/dt$ are the rotor angular displacement speed and acceleration, respectively, and thus $d\theta_i/dt = \omega_0 + d\delta_i/dt$. In the sequel, we denote the mechanical input power (minus the output load power) by P_{mi} and the electrical swing power by P_{ei}, respectively, and we write

$$P_{mi} =: T_{mi}(\omega_0 + \omega_i), \quad P_{ei} =: T_{ei}(\omega_0 + \omega_i),$$

Using the above notations and (2) in (1), we obtain readily that

$$\begin{aligned} & J_i(\omega_0 + \omega_i)\frac{d\omega_i}{dt} + D(2\omega_0 + \omega_i)\omega_i + \omega_i \Delta_i \\ & = P_{mi} - P_{ei} - D_i\omega_0^2 - \omega_0\Delta_i \end{aligned} \tag{3}$$

Thirdly, we see (Anderson, p. 22–23; Salam) that the electrical swing power is given by

$$P_{ei} = \sum\nolimits_{k=1}^{n} b_{ik} \sin(\delta_i - \delta_k - \psi_{i,k}) \tag{4}$$

where b_{ik} is the maximal real power transferred between the internal nodes of the synchronous generators i and k, while ψ_{ik} stands for the complement of transfer admittance phase between the internal nodes i and k. By the definition, $b_{ik} = b_{ki}$. For simplicity, we assume that ψ_{ik} are small constants.

Finally, we summarize the above deductions around (2), (3) and (4). More precisely, under the assumption of $\omega_0 + \omega_i \neq 0$, Eqs. (2), (3) and (4) can be expressed by

$$\begin{cases} \dot{\delta}_i = \omega_i \\ \dot{\omega}_i = -\displaystyle\sum_{k=1}^{n} \frac{b_{ik}\sin(\delta_i - \delta_k - \psi_{ik})}{J_i(\omega_0 + \omega_i)} \\ \qquad -\dfrac{D_i(2\omega_0 + \omega_i) + \Delta_i}{J_i(\omega_0 + \omega_i)}\omega_i + \dfrac{P_i(\Delta_i, t)}{J_i(\omega_0 + \omega_i)} \end{cases} \tag{5}$$

where $P_i(\Delta_i, t) =: P_{mi} - D_i\omega_0^2 - \omega_0\Delta_i$. Eq. (5) is called the improved swing equation for the generator i in the multimachine power system.

In what follows, we are interested in swing behaviour of the mutlimachine power system when ω_i is sufficiently small, compared with ω_0; in less rigorous words, all the generators involved are swinging slowly. Then the improved swing equation (5) can be reduced as follows by assuming that all the mechanical torques are constants.

$$\begin{cases} \dot{\delta}_i = \omega_i \\ \dot{\omega}_i = -\displaystyle\sum_{k=1}^{n} \frac{b_{ik}\sin(\delta_i - \delta_k - \psi_{ik})}{M_i} \\ \qquad -\dfrac{2D_i\omega_0 + \Delta_i}{M_i}\omega_i + \dfrac{P_i(\Delta_i, t)}{M_i} \end{cases} \tag{6}$$

where $M_i = J_i\omega_0$ and $P_i(\Delta_i, t) = P_{mi} - D_i\omega_0^2 - \omega_0\Delta_i$. For simplicity, Eq. (6) is also called the improved swing equation for the generator i.

2.2 Concise Expression of Multimachine Power Systems

To express the improved swing equation (6) in a form that is more convenient for our latter discussion, let us introduce the following variable transformation.

$$\begin{cases} \hat{\delta}_{i1} = \delta_i - \delta_1 - \psi_{i1} \\ \cdots\cdots\cdots \\ \hat{\delta}_{ii-1} = \delta_i - \delta_{i-1} - \psi_{ii-1} \\ \hat{\delta}_{ii+1} = \delta_i - \delta_{i+1} - \psi_{ii+1} \\ \cdots\cdots\cdots \\ \hat{\delta}_{in} = \delta_i - \delta_n - \psi_{in} \end{cases}$$

which leads that

$$\begin{cases} \dot{\hat{\delta}}_{i1} = \dot{\delta}_i - \dot{\delta}_1 = \omega_i - \omega_1 \\ \cdots\cdots\cdots \\ \dot{\hat{\delta}}_{ii-1} = \dot{\delta}_i - \dot{\delta}_{i-1} = \omega_i - \omega_{i-1} \\ \dot{\hat{\delta}}_{ii+1} = \dot{\delta}_i - \dot{\delta}_{i+1} = \omega_i - \omega_{i+1} \\ \cdots\cdots\cdots \\ \dot{\hat{\delta}}_{in} = \dot{\delta}_i - \dot{\delta}_n = \omega_i - \omega_n \end{cases}$$

Now we are ready to express the improve swing equation (6) for the generator i in the multimachine power system as follows.

$$\begin{cases} \dot{\hat{\delta}}_i = G\omega_i - \hat{\omega}_i \\ \dot{\omega}_i = H_i(\hat{\delta}_i)\hat{\delta}_i + K_i(\Delta_i)\omega_i + B_i P_i(\Delta_i, t) \end{cases} \tag{7}$$

where

$$\hat{\delta}_i =: \begin{bmatrix} \hat{\delta}_{i1} \\ \vdots \\ \hat{\delta}_{ii-1} \\ \hat{\delta}_{ii+1} \\ \vdots \\ \hat{\delta}_{in} \end{bmatrix}, \quad \hat{\omega}_i =: \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_{i-1} \\ \omega_{i+1} \\ \vdots \\ \omega_n \end{bmatrix}, \quad G =: \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\begin{aligned} H_i(\hat{\delta}_i) = -\Bigg[& \frac{b_{i1}\sin(\hat{\delta}_{i1})}{M_i\hat{\delta}_{i1}}, \ldots, \frac{b_{in}\sin(\hat{\delta}_{ii-1})}{M_i\hat{\delta}_{ii-1}} \Bigg| \\ & \Bigg| \frac{b_{ii+1}\sin(\hat{\delta}_{ii+1})}{M_i\hat{\delta}_{ii+1}}, \ldots, \frac{b_{in}\sin(\hat{\delta}_{in})}{M_i\hat{\delta}_{in}} \Bigg] \end{aligned}$$

$$K_i(\Delta_i) = -\frac{2D_i\omega_0 + \Delta_i}{M_i}, \quad B_i = \frac{1}{M_i}$$

Clearly, $H_i(\hat{\delta}_i)$ and $K_i(\Delta_i)$ are smooth functions with respect to their arguments. Namely, these functions are continuously differentiable with respect to $\hat{\delta}_i$ and Δ_i. Our problem of this study is to find possible feedback control laws to stabilize the improved swing equation (7).

3. STABILIZING MULTIMACHINE POWER SYSTEMS

To understand the suggested design algorithm for the nonlinear feedback laws for stabilizing the multimachine power system, we first construct some auxiliary swing equations, for which we descibe the desired feedback control laws. After that, we apply the resulted feedback control laws to the real multimachine power systems.

3.1 Stabilizing an Auxiliary Multimachine Model

Let us begin with the following auxiliary swing equation.

$$\begin{cases} \dot{\hat{\delta}}_i = F_i^{(\cdot)}\hat{\delta}_i + G\omega_i - \hat{\omega}_i \\ \dot{\omega}_i = H_i(\hat{\delta}_i)\hat{\delta}_i + K_i(\Delta_i)\omega_i + B_iP_i(\Delta_i, t) \end{cases} \tag{8}$$

where $F_i^{(\cdot)}$ is a $(n-1)\times(n-1)$ real matrix with all its eigenvalues having negative real parts (we simply say such $F_i^{(\cdot)}$ is Hurwitz) that will be determined sequentially. The superscript $(\cdot)$ indicates the sequential step. To include $F_i^{(\cdot)}\hat{\delta}_i$ in (8) guarantees that the stabilization design procedure by Isidori applies on (8) as well. Eq. (8) is termed the $F_i^{(\cdot)}$-auxiliary swing equation.

Now we are ready to show the main results.

Theorem 1. Consider the auxiliary multimachine power system represented by the $F_i^{(\cdot)}$-auxiliary swing equations (8), $i = 1, 2, \ldots, n$. For each i, there always exists a Hurwitz matrix sequence $\{F_i^{(l)} : l = 0, 1, \ldots,\}$ such that the solution $Q(F_i^{(l)})$ to the Lyapunov matrix equation

$$F_i^{(l)^T} Q(F_i^{(l)}) + Q(F_i^{(l)})F_i^{(l)} = -I,$$

satisfies $Q(F_i^{(l)}) > 0$ and $||Q(F_i^{(l)})|| < 1$. Accordingly, there exist n sequences of functions $\{u_i(\hat{\delta}_i, F_i^{(l)}) : l = 0, 1, \ldots\}$ and n piecewise continuous function $v_i(\hat{\omega}_i, \omega_i)$ such that by applying all the n feedback laws

$$P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(\hat{\delta}_i, F_i^{(l)}), \quad i = 1, \ldots, n \tag{9}$$

separately to the generator i, the closed-loop auxiliary system as a whole is uniformly and locally asymptotically stable as long as $\Delta_i \in \mathcal{F}$ and the initial vector $[\hat{\delta}_i(t_0), \omega_i(t_0)]^T$ is an interior point of $\mathcal{B}$ for each i. Namely, $\hat{\delta}_i(t) \to 0$ and $\omega_i(t) \to 0$ as $t(> t_0) \to \infty$ for each i. Here, $\mathcal{B} \subset \mathcal{R}^{n+1}$ is a closed ball centering at the origin.

Sketched Proof of Theorem 1 Let $F_i^{(l)}$ be Hurwitz such that by the Lyapunov theorem (see Theorem 5.3.55 of Vidyasagar(1978)), there exists uniquely a solution $Q(F_i^{(l)}) = Q^T(F_i^{(l)})$ satisfying the Lyapunov equation $F_i^{(l)^T} Q(F_i^{(l)}) + Q(F_i^{(l)})F_i^{(l)} = -I$ with $Q(F_i^{(l)}) > 0$ and $||Q(F_i^{(l)})|| < 1$.

Now construct the Lyapunov function candidate $V_i(\hat{\delta}_i, \omega_i)$ given below, which is a locally positive definite function (we write l.p.d.f for simplicity) over $[\hat{\delta}_i, \omega_i]^T \in \mathcal{B}$.

$$V_i(\hat{\delta}_i, \omega_i) =: \hat{\delta}^T Q(F_i^{(l)})\hat{\delta}_i + \omega_i^T\omega_i.$$

Clearly, $V_i(\hat{\delta}_i, \omega_i) > 0$ for any $[\hat{\delta}_i, \omega_i]^T \neq 0$ and $V_i(0, 0) = 0$. Indeed, $V_i(\hat{\delta}_i, \omega_i)$ is a continuously differentiable decrescent l.p.d.f by Definition 5.1.52 and Definition 5.1.65 in the book by Vidyasagar (1978). The proof will be accomplished if it is shown that there exist a function sequence $\{u_i(\hat{\delta}_i, F_i^{(l)}) : l = 0, 1, \ldots\}$ and a function $v_i(\hat{\omega}_i, \omega_i)$ such that in the closed-loop auxiliary system formed by applying the feedback law (9) to (8), it holds that $dV_i(\hat{\delta}_i, \omega_i)/dt < 0$ for any $[\hat{\delta}_i, \omega_i]^T \neq 0 \in \mathcal{B}$ over $t \geq t_0$.

To this end, we observe by (8) and (9) that

$$\begin{aligned}
\frac{dV_i(\hat{\delta}_i, \omega_i)}{dt} &= \frac{\partial V_i(\hat{\delta}_i, \omega_i)}{\partial \hat{\delta}_i}\frac{d\hat{\delta}_i}{dt} + \frac{\partial V_i(\hat{\delta}_i, \omega_i)}{\partial \omega_i}\frac{d\omega_i}{dt} \\
&= \hat{\delta}_i^T[Q(F_i^{(l)})F_i^{(l)} + F_i^{(l)^T}Q(F_i^{(l)})]\hat{\delta}_i \\
&\quad + \hat{\delta}_i^T Q(F_i^{(l)})G\omega_i - \hat{\delta}_i^T Q(F_i^{(l)})\hat{\omega}_i \\
&\quad + \omega_i G^T Q(F_i^{(l)})\hat{\delta}_i - \hat{\omega}_i^T Q(F_i^{(l)})\hat{\delta}_i \\
&\quad + 2\omega_i H_i(\hat{\delta}_i)\hat{\delta}_i + 2K_i(\Delta_i)\omega_i^2 \\
&\quad + 2\omega_i B_i\Big(v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(\hat{\delta}_i, F_i^{(l)}) \\
&\quad - D_i\omega_0^2 - \omega_0\Delta_i\Big) \\
&= -\hat{\delta}_i^T\hat{\delta}_i + \hat{\delta}_i^T\Big(Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i)\Big)\omega_i \\
&\quad - \hat{\delta}_i^T Q(F_i^{(l)})\hat{\omega}_i \\
&\quad + \omega_i\Big(G^T Q(F_i^{(l)}) + H_i(\hat{\delta}_i)\Big)\hat{\delta}_i - \hat{\omega}_i^T Q(F_i^{(l)})\hat{\delta}_i \\
&\quad + 2\Big(K_i(\Delta_i) - B_i u_i(\hat{\delta}_i, F_i^{(l)})\Big)\omega_i^2 \\
&\quad - \hat{\omega}_i^T\hat{\omega}_i + \hat{\omega}_i^T\hat{\omega}_i \\
&\quad + 2\omega_i B_i\Big(v_i(\hat{\omega}_i, \omega_i) - D_i\omega_0^2 - \omega_0\Delta_i\Big) \\
&=: -\begin{bmatrix} \hat{\delta}_i^T & \hat{\omega}_i^T & \omega_i \end{bmatrix} W_i(\hat{\delta}_i, \Delta_i, F_i^{(l)}) \begin{bmatrix} \hat{\delta}_i \\ \hat{\omega}_i \\ \omega_i \end{bmatrix} + \hat{\omega}_i^T\hat{\omega}_i \\
&\quad + 2\omega_i B_i\Big(v_i(\hat{\omega}_i, \omega_i) - D_i\omega_0^2 - \omega_0\Delta_i\Big),
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
& W_i(\hat{\delta}_i, \Delta_i, F_i^{(l)}) \\
&= \left[\begin{array}{cc|c} I & -Q(F_i^{(l)}) & -Q(F_i^{(l)})G - H_i^T(\hat{\delta}_i) \\ -Q(F_i^{(l)}) & I & 0 \\ \hline -G^T Q(F_i^{(l)}) - H_i(\hat{\delta}_i) & 0 & 2(B_i u_i(\hat{\delta}_i, F_i^{(l)}) - K_i(\Delta_i)) \end{array}\right] \\
&=: \left[\begin{array}{c|c} W_{i1} & * \\ \hline * & * \end{array}\right]
\end{aligned}$$

where W_{i1} has the obvious definition formula, while the asterisks represent the terms whose exact definitions are not needed in the arguments.

By the matrix expression of W_{i1}, if $||Q(F_i^{(l)})|| < 1$, then W_{i1} is strictly positive definite and thus nonsingular. In particular, all the principal minors in W_{i1} must be positive. Recalling that W_{i1} is $2(n-1) \times 2(n-1)$ while $W_i(\hat{\delta}_i, \Delta_i, F_i^{(l)})$ is $(2(n-1)+1) \times (2(n-1)+1)$, we can conclude that $W_i(\hat{\delta}_i, \Delta_i, F_i^{(l)})$ is strictly positive definite over $[\hat{\delta}_i, \omega_i]^T \neq 0$ and $\Delta_i \in \mathcal{F}$ if and only if $\det(W_i(\hat{\delta}_i, \Delta_i, F_i^{(l)})) > 0$ in the same sense, if and only if

$$\begin{aligned}
&\det(W_i(\hat{\delta}_i, \Delta_i, F_i^{(l)})) \\
&= \det\left(I - Q^2(F_i^{(l)})\right)\Big(2(B_i u_i(\hat{\delta}_i, F_i^{(l)}) - K_i(\Delta_i)) \\
&\quad -(G^T Q(F_i^{(l)}) + H_i(\hat{\delta}_i))(I - Q^2(F_i^{(l)}))^{-1} \\
&\quad \cdot(Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i))\Big) > 0,
\end{aligned}$$

over $[\hat{\delta}_i, \omega_i]^T \neq 0$ and $\Delta_i \in \mathcal{F}$. Clearly, one can assert readily that $\det(I - Q^2(F_i^{(l)})) > 0$ since $Q^2(F_i^{(l)})$ is symmetric and $||Q^2(F_i^{(l)})|| < 1$. Hence, $\det(W_i(\hat{\delta}_i, \Delta_i, F_i^{(l)})) > 0$ if and only if

$$\begin{aligned}
&2(B_i u_i(\hat{\delta}_i, F_i^{(l)}) - K_i(\Delta_i)) \\
&-(G^T Q(F_i) + H_i(\hat{\delta}_i))(I - Q^2(F_i^{(l)}))^{-1} \\
&\cdot(Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i)) > 0
\end{aligned}$$

Thus, if $u_i(\hat{\delta}_i, F_i^{(l)})$ and $v_i(\hat{\omega}_i, \omega_i)$ can be chosen such that

$$\begin{cases}
B_i u_i(\hat{\delta}_i, F_i^{(l)}) > K_i(\Delta_i) \\
\quad +\dfrac{1}{2}(G^T Q(F_i^{(l)}) + H_i(\hat{\delta}_i)) \\
\quad \cdot(I - Q^2(F_i^{(l)}))^{-1}(Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i)) \\
\omega_i B_i(v_i(\hat{\omega}_i, \omega_i) - D_i\omega_0^2 - \omega_0\Delta_i) \\
\quad +\hat{\omega}_i^T\hat{\omega}_i \leq 0
\end{cases} \tag{11}$$

then $dV(\hat{\delta}_i, \omega_i)/dt < 0$ for any $[\hat{\delta}_i, \omega_i]^T \neq 0$ and $t \geq 0$. Therefore, Theorem 5.2.45 of Vidyasagar implies immediately that the closed-loop auxiliary system, which is formed by applying each feedback law $P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(\hat{\delta}_i, F_i^{(l)})$ to a corresponding generator described by (8), is uniformly and locally asymptotically stable for any initial vector $[\hat{\delta}_i(t_0), \omega_i(t_0)]^T \in \mathcal{B}$ and $\Delta_i \in \mathcal{F}$, as long as $[\hat{\delta}_i, \omega_i]^T = 0$ for all i is an equilibrium of the closed-loop auxiliary system as a whole.

In what follows, we show such $u_i(\hat{\delta}_i, F_i^{(l)})$ and $v_i(\hat{\omega}_i, \omega_i)$ do exist, according to ω_i.

On the one hand, the first inequality in (11) holds true if $u_i(\hat{\delta}_i, F_i^{(l)})$ is chosen under the prescribed matrix $F_i^{(l)}$ such that

$$\begin{aligned}
&u_i(\hat{\delta}_i, F_i^{(l)}) > 2D_i\omega_0 + \Delta_i \\
&+\frac{M_i}{2}(G^T Q(F_i^{(l)}) + H_i(\hat{\delta}_i))(I - Q^2(F_i^{(l)}))^{-1} \\
&\cdot(Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i))
\end{aligned}$$

Note that $\sup_{\Delta_i \in \mathcal{F}} |\Delta_i| \geq \Delta_i$. We choose $u_i(\hat{\delta}_i, F_i^{(l)})$ to satisfy

$$\begin{aligned}
&u_i(\hat{\delta}_i, F_i^{(l)}) > -2D_i\omega_0 + \sup_{\Delta_i \in \mathcal{F}} |\Delta_i| \\
&+\frac{M_i}{2}(G^T Q(F_i^{(l)}) + H_i(\hat{\delta}_i))(I - Q^2(F_i^{(l)}))^{-1} \\
&\cdot(Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i))
\end{aligned} \tag{12}$$

On the other hand, it is straightforward to see that the second inequality of (11) will hold true for all $\omega_i \neq 0$ if $v_i(\hat{\omega}_i, \omega_i)$ is chosen to satisfy

$$\begin{cases}
v_i(\hat{\omega}_i, \omega_i) < -\dfrac{M_i}{\omega_i}\hat{\omega}_i^T\hat{\omega}_i + D_i\omega_0^2 + \omega_0\Delta_i, & \omega_i > 0 \\
V_i(\hat{\omega}_i, \omega_i) \ \text{ can be arbitrary} & \omega_i = 0 \\
v_i(\hat{\omega}_i, \omega_i) > -\dfrac{M_i}{\omega_i}\hat{\omega}_i^T\hat{\omega}_i + D_i\omega_0^2 + \omega_0\Delta_i, & \omega_i < 0
\end{cases}$$

It is easy to see that when $\omega_i = 0$, the second inequality of (11) may not be true no matter how $v_i(\hat{\omega}_i, \omega_i)$ is taken (this is why we assume that $v_i(\hat{\omega}_i, \omega_i)$ can be arbitrary at $\omega_i = 0$). If this is really the case, $dV_i(\hat{\delta}_i, \omega_i)/dt|_{\omega_i=0,\hat{\delta}_i\neq 0} < 0$ may not be true by taking $v_i(\hat{\omega}_i, \omega_i)$. To solve such a problem, we need to alter the auxiliary matrix $F_i^{(l)}$, say to $F_i^{(l+1)}$, to guarantee that $dV_i(\hat{\delta}_i, \omega_i)/dt < 0$ when $\omega_i = 0$ but $\hat{\delta}_i \neq 0$. We need to show that such $F_i^{(l+1)}$ really exists. In fact, we can choose $F_i^{(l+1)}$ according to the last equation of (10) when $\omega_i = 0$ but $\hat{\delta}_i \neq 0$. That is,

$$-\hat{\delta}_i^T\hat{\delta}_i - 2\hat{\omega}_i^T Q(F_i^{(l+1)})\hat{\delta}_i < 0 \tag{13}$$

It is not hard to see by Theorem 2.3 of Gajic that such $F_i^{(l+1)}$ exists and is not unique.

Letting $l = l + 1$ and thus $F_i^{(l)} = F_i^{(l+1)}$ and returning to (12), we see that $u_i(\hat{\delta}_i, F_i^{(l+1)})$ can be determined, and so can $v_i(\hat{\omega}_i, \omega_i)$. In other words, implementing the feedback control $P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(\hat{\delta}_i, F_i^{(l)})$ to the $F_i^{(\cdot)}$-auxiliary improved swing equation (8), we can find a sequence of l.p.d.f Lyapunov candidates $V_i(\hat{\delta}_i, \omega_i) > 0$ for any $[\hat{\delta}_i, \omega_i]^T \neq 0 \in \mathcal{B}$ such that $dV_i(\hat{\delta}_i, \omega_i)/dt < 0$ for any $[\hat{\delta}_i, \omega_i]^T \neq 0 \in \mathcal{B}^{(l)}$ and $\Delta_i \in \mathcal{F}$. Here $\mathcal{B}^{(l)}$ is a subset of $\mathcal{B}$. Clearly, the above arguments can be repeated for each step l.

From these facts, Theorem 5.2.45 of Vidyasagar (1978) indicates that the closed-loop auxiliary system as a whole, which is formed by applying all the n feedback laws $P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(\hat{\delta}_i, F_i^{(l)})$, respectively, is uniformly and locally asymptotically stable for any $\Delta_i \in \mathcal{F}$. □

3.2 Stabilizing the Real Multimachine Power System

Theorem 1 suggests a group of nonlinear feedback laws that can stabilize the closed-loop auxiliary system formed by applying it to the $F_i^{(\cdot)}$-auxiliary swing equation (8). In this section, we construct these feedback laws independently of $\hat{\delta}_i$; such feedback laws are denoted simply by

$P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(F_i^{(l)})$. We will see that if such nonlinear feedback laws are implemented into the real multimachine power system that is modeled through the improved swing equation (6), then the system can at least be partially stabilized.

First, we show that such $u_i(F_i^{(l)})$ does exist under the assumptions of Theorem 1. We notice from the proof of Theorem 1 and the fact of $|\sin(\delta)/\delta| \leq 1$ for any δ that

$$\begin{aligned}
&(G^T Q(F_i^{(l)}) + H_i(\hat{\delta}_i))(I - Q^2(F_i^{(l)}))^{-1} \\
&\cdot(Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i)) \\
\leq\, &\sigma_{\max}((I - Q^2(F_i^{(l)}))^{-1})||Q(F_i^{(l)})G + H_i^T(\hat{\delta}_i)||^2 \\
\leq\, &\sigma_{\min}^{-1}(I - Q^2(F_i^{(l)}))\Big(||Q(F_i^{(l)})G|| + ||H_i(\hat{\delta}_i)||\Big)^2 \\
=\, &\sigma_{\min}^{-1}(I - Q^2(F_i^{(l)}))\Big(||Q(F_i^{(l)})G|| \\
&+\Big[\sum_{k=1;\neq i}^{n}\Big(\frac{b_{ik}\sin(\hat{\delta}_{ik})}{M_i\hat{\delta}_{ik}}\Big)^2\Big]^{1/2}\Big)^2 \\
=\, &\sigma_{\min}^{-1}(I - Q^2(F_i^{(l)}))(||Q(F_i^{(l)})G|| + ||H_i(0)||)^2
\end{aligned}$$

This, together with (12), implies that if $u_i(F_i^{(l)})(= u_i(\hat{\delta}_i, F_i^{(l)}))$ is chosen such that

$$\begin{aligned}
u_i(F_i^{(l)}) > &-2D_i\omega_0 + \sup_{\Delta_i\in\mathcal{F}}|\Delta_i| \\
&+\frac{M_i(||Q(F_i^{(l)})G|| + ||H_i(0)||)^2}{2\sigma_{\min}(I - Q^2(F_i^{(l)}))}, \qquad (14)
\end{aligned}$$

then the first inequality of (11) is satisfied. Note that the right-hand side of (14) has nothing to do with $\hat{\delta}_i$. It follows that $u_i(F_i^{(l)})$ exists. Needless to say, such $u_i(F_i^{(l)})$ is not unique, either. This implies that the feedback law $P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(F_i^{(l)})$ can be determined independently of $\hat{\delta}_i$.

With implementing $P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(F_i^{(l)})$ in the $F_i^{(\cdot)}$-auxiliary swing equation (8), the corresponding closed-loop auxiliary system, denoted by $\Sigma(F_i^{(\cdot)})$, is uniformly and locally asymptotically stable by Theorem 1; namely, $\hat{\delta}_i(t) \to 0$ and $\omega_i(t) \to 0$ as $t \to \infty$ as long as the initial vector $[\hat{\delta}_i(t_0), \omega_i(t_0)]^T \in \mathcal{B}$ and $\Delta_i \in \mathcal{F}$. Now we examine the differential equation about $\dot{\omega}_i$ in $\Sigma(F_i^{(\cdot)})$. Simple manipulations about (8) show that it can be written as follows.

$$\begin{aligned}
\dot{\omega}_i = &-\sum_{k=1;\neq i}^{n}\frac{b_{ik}\sin(\hat{\delta}_{ik})}{M_i} - \frac{2D_i\omega_0 + \Delta_i}{M_i} \\
&+\frac{1}{M_i}\Big(v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(F_i^{(l)}) \\
&-D_i\omega_0^2 - \omega_0\Delta_i\Big), \qquad (15)
\end{aligned}$$

where $\hat{\delta}_{ik}, (k = 1, 2, \ldots, n; k \neq i)$ are determined by the first $n-1$ differential equations in (8). Since $[\hat{\delta}_i(t_0), \omega_i(t_0)]$ is arbitrary in $\mathcal{B}$ and $\hat{\delta}_{ik}$ is continuous to $[\hat{\delta}_i(t_0), \omega_i(t_0)]$, $\sin(\hat{\delta}_{ik})$ must be continuous trajectories that overlapping the whole interval $[-1, 1]$. That is, any trajectory of $\sin(\hat{\delta}_{ik})$ must lay in $[-1, 1]$ for an initial vector $[\hat{\delta}_i(t_0), \omega_i(t_0)]$. Then, the asymptotic stability assertion in $\Sigma(F_i^{(\cdot)})$, together with the fact that $v_i(\hat{\omega}_i, \omega_i)$ and $u_i(F_i^{(l)})$ have nothing to do with $\hat{\delta}_i$, can be interpreted as that for any initial $\omega_i(t_0)$ as appropriately, the solution ω_i to (15) converges to zero as $t \to \infty$ uniformly in t_0 for any trajectory of $\sin(\hat{\delta}_{ik})$ in $[-1, 1]$ and $\Delta_i \in \mathcal{F}$.

Now let us introduce the same nonlinear feedback law $P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(F_i^{(l)})$ to the improved swing equation (6). In the corresponding closed-loop system, denoted by Σ, the differential equation about $\dot{\omega}_i$ is completely the same as (15) in form, except that $\hat{\delta}_{ik}$ are determined by the first $n-1$ differential equations in (6). Apparently, any trajectory of $\sin(\hat{\delta}_{ik})$ also lays in $[-1, 1]$ even if $\hat{\delta}_{ik}$ are determined by the first $n-1$ differential equations in (6).

Combining the observation with the arguments around (15), we are led that in the closed-loop system Σ, it holds at least that $\omega_i(t) \to 0$ as $t \to \infty$ uniformly for any initial vector $[\hat{\delta}_i(t_0), \omega_i(t_0)]^T \in \mathcal{B}$ and $\Delta_i \in \mathcal{F}$. This in particular says that the rotor displacement δ_i must come to a halt in the closed-loop system Σ, since $\omega_i = \dot{\delta}_i$ by the first relationship in (5).

Theorem 2. Consider the multimachine power system represented by the improved swing equation (6). For each i, there always exists a Hurwitz matrix sequence $\{F_i^{(l)} : l = 0, 1, \ldots, \}$ such that the solution $Q(F_i^{(l)})$ to the Lyapunov equation

$$F_i^{(l)^T} Q(F_i^{(l)}) + Q(F_i^{(l)})F_i^{(l)} = -I,$$

with $Q(F_i^{(l)}) > 0$ and $||Q(F_i^{(l)})|| < 1$. Accordingly, there exist n sequences of functions $\{u_i(\hat{\delta}_i, F_i^{(l)}) : l = 0, 1, \ldots\}$ and n piecewise continuous function $v_i(\hat{\omega}_i, \omega_i)$ such that by applying the feedback laws

$$P_{mi} = v_i(\hat{\omega}_i, \omega_i) - \omega_i u_i(F_i^{(l)}), i = 1, \ldots, n \qquad (16)$$

separately to all the generators, the dynamical behavior of ω_i in the generator i is uniformly and locally asymptotically stable for any $\Delta_i \in \mathcal{F}$ and initial vector $[\hat{\delta}_i(t_0), \omega_i(t_0)]^T \in \mathcal{B}$; that is, $\omega_i(t) \to 0$ as $t \to \infty$; and thus $\delta_i(t) \to \text{const}$ as $t \to \infty$.

4. CONCLUSIONS

Based on what we call the improved swing equations, stabilization feedback design problems in a class of multimachine power systems are attacked through decentralized and nonlinear state feedback laws in the paper. It is revealed that stability of the closed-loop power systems is robust with regard to perturbations of electrical torques in the synchronous generators. The suggested stabilization technique consists of a group of decentralized algorithms in a generator-wise fashion and the resulted feedback laws possess more freedoms available for accommendating various control performance requirements. To illustrate their efficacy in practical multimachine power systems is one of our subsequent research topics.

REFERENCES

L. F. C. Alberto and N. G. Bretas, Synchronism versus stability in power systems, *Internation Journal of Electrical Power & Energy Systems*, vol 21, no. 4, pp. 261–267, 1999.

P. M. Anderson and A. A. Fouad, *Power System Control and Stability*, 2nd Ed, Wiley-Interscience, New York, 2003.

H. D. Chiang and C. C. Chu, Stability regions of nonlinear autonomous dynamical systems, *IEEE Transactions on Automatic Control*, vol. 33, no. 1, pp. 16–17, 1988.

Z. Gajić and M.T. J. Qureshi, *Lyapunov Matrix Equation in System Stability and Control*, Academic Press, San Diegon, 1995.

R. Gupta, B. Bandyopadhyay and A. M. Kulkarni, Power system stabilizer for multimachine power system using robust decentralised periodic output feedback, *IEE Proceedings of Control Theory and Applications*, vol. 152, no. 1, pp. 3–8, 2005.

M. H. Haque, Application of energy function to assess the first-swing stability of a power system with an SVC, *IEE Proc.-Gener. Transm. Distrib.,* vol. 152, no. 6, pp. 806–812, 2005.

A. Isidori, *Nonlinear Control Systems II*, Springer, London, 1999.

H. K. Khalil, *Nonlinear Systems*, 3rd Ed, Prentice-Hall, New Jersey, 2000.

E. W. Kimbark, *Power System Stability*, Vol. I, Wiley-Interscience, New York, 1995.

V. Lakshmikantham, V. M. Matrosov and S. Sivasundaram, *Vector Lyapunov Functions and Stability Analysis of Nonlinear Systems*, Kluwer Academic Publishers, Dordrecht, 1991.

C. W. Liu, J. S. Thorp, J. Lu, R. J. Thomas and H. D. Chiang, Detection of transiently chaotic swings in power-systems using real-time phasor measurements, *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1285–1292, 1994.

Y. Mitani and K. Tsuji, A method for analysis of backswing phenomena of synchronous machine in multimachine power-systems, *Electrical Engineering in Japan*, vol. 114, no. 6, pp. 13–23, 1994.

D. O′Regan, *Existence Theory for Nonlinear Ordinary Differential Equations*, Kluwer Academic Publishers, Dordrecht, 1997.

K. R. Padiyar, *Power System Dynamics*, John Wiley & Sons Ltd, Singapore, 1996.

M. Parashar, J. S. Thorp and C. E. Seyler, Continuum modeling of ecectromechanical dynamics in large-scale power sysetms, *IEEE Transactions on Circuits and Systems I*, vol. 51, no. 9, pp. 1848–1858, 2004.

M. Pavella and P. G. Murthy, *Transient Stability of Power Systems-Theory and Practice*, John Wiley & Sons, Chichester, 1994.

F. Saccomanno, *Electric Power Systems: Analysis and Control*, Wiley-Interscience, New York, 2003.

F. M. A. Salam, Asymptotic stability and estimating the region of attraction for the swing equations, *Systems & Control Letters*, vol. 7, no. 4, pp. 309–312, 1986.

Y. Ueda, M. Hirano and R. H. Abraham, Attractor and basin portraits of a double swing power system, *International Journal of Bifurcation and Chaos*, vol. 14, no. 9, pp. 3135–3152, 2004.

M. Vidyasagar, *Nonlinear Systems Analysis*, Prentice-Hall, Englewood Cliffs, 1978.

V. I. Vorotnikov, *Partial Stabiltiy and Control*, Birkhauser, Boston, 1998.

T. C. Yang, N. Munro and A. Brameller, A new decentralized stabilization method with application to power system stabilizer design for multimachine systems. *International Journal of Electrical Power & Energy Systems*, vol. 9, pp. 206–261, 1987.

T. C. Yang, Synchronous generator stabilizer design through incomplete state feedback, *International Journal of Electrical Power & Energy Systems*, vol 16, no. 2, pp. 91–95, 1994.

J. Zaborszky, G. Huang, B. Zheng and T. C. Leung, On the phase portrait of a clas of large nonlinear dynamic systems such as the power power, *IEEE Trans. Automat. Contr.,* vol. 33, no. 1, pp. 4–16, 1988.