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Extended Moore-Spence Equations Based Reduced Method for Computing Bifurcation Points of Power System Model ⋆

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Abstract: For differential-algebraic power systems, saddle-noddle bifurcation and Hopf bifurcation are both of universally existent phenomena in power systems. Usually Newton iteration method could be applied to the Moore-Spence system to compute saddle-noddle and Hopf bifurcation points directly. But the Moore-Spence system has very high dimension and causes much complexity in Jacobian matrix factorization. By introducing an auxiliary variable and an auxiliary equation to form an extended Moore-Spence system, this paper derives an effective matrix reduction technique. The high dimensionality of Jacobian matrix can thus be reduced and the complexity involved in matrix factorization can be simplified.

1. INTRODUCTION

The dynamics of a power system can be modeled by parameter dependent differential-algebraic equations as:

$$\begin{cases} \dot{x} = f(x,y,\lambda) & f : \mathbf{R}^{n+m+1} \to \mathbf{R}^n \\ 0 = g(x,y,\lambda) & g : \mathbf{R}^{n+m+1} \to \mathbf{R}^m \end{cases} \quad (1)$$

where $x \in \mathbf{R}^n$, $y \in \mathbf{R}^m$, $\lambda \in \mathbf{R}$, x is differential state vector for describing element dynamic behavior of power system, such as generator angel, rotor speed, voltage and phase of load node; instantaneous variable y is usually connected with the network structure and typically including bus voltage and other load flow variables. The parameter space is composed of the system parameters (which describe the system topography, and equipment constants such as inductances, capacitors, transformer ratios, etc.) and operating parameters (such as loads, generations and voltage set-points etc.). The dynamics of the generators, load dynamics and some other control devices together define the differential equations, and the constrains $g(x,y,\lambda) = 0$ are defined by the power balance equations of the transmission system. DAEs (1) has also been used to model the dynamics of many other systems, including robotics biomedicine, electronic engineering and flight control system, referred to Craig (1989), Wang and Chen (1998) and Goman and Khramtsovsky (1998).
For the constrained system (1), define the set E of all stable equilibrium points and Jacobian matrix J as

$$E = \{\ (x,y,\lambda) \in \mathbf{R}^{n+m+1} | f(x,y,\lambda) = 0, \quad g(x,y,\lambda) = 0\} \quad (2)$$

$$J = D_x f - D_y f (D_y g)^{-1} D_x g \quad (3)$$

where $D_x f$ and $D_y f$ denote the matrix of partial derivatives of the components of f with respect to variable x and y respectively.
It is well known that the qualitative change in the behavior of system with variations of one or more parameters is due to bifurcation. The bifurcation theory provides a potential natural plat for studying the system dynamics, moreover, it is very favorable as a mathematic tool in studying nonlinear differential-algebraic system. Since 1970s, many experts have done a lot of research on bifurcation phenomena about stability, controllability etc., but there are few works about computing bifurcation points. The bifurcation refers to the number of equilibrium points, stability or topological structure of the system change suddenly at certain parameters, therefore the corresponding parameters and state variables are named as bifurcation point. The saddle-noddle and Hopf bifurcation are closely connected with voltage stability (Claudio (1995); Peng (2005)), and analysis and calculation of bifurcation points are necessary for comprehending nonlinear critical dynamics. Lots of facts have proved that SNB always leaded to voltage collapse, lose stability or low frequency oscillation (Lu (2003); Kwatny and Pasrija (1986)). Whereas HB is especially interesting for large power systems because it signals the birth of periodic orbits, or more complicated features such as strange attractors and chaos (Yu (1999); Abed and Varaiya (1989)).
At present, there are two methods for computing bifurcation points, i.e. direct method and indirect method. Indirect method depends on at least one known solution, employs a continued predictor-corrector scheme to find the solution path, and decides whether the eigenvalues satisfy bifurcation conditions. However direct method not only keeps the sparsity of the data structure in the dynamics analysis, but also includes the information such as critical eigenvalues and right eigenvectors (Kwatny et al. (1995)). The matrix reduction method which we will bring forward

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belongs to direct method.

The aim of this paper is to propose one matrix reduced new method for factorizing matrix and solving bifurcation points conveniently. The key of this algorithm is to introduce an auxiliary variable and an auxiliary equation to form an extended Moore-Spence system and decompose the high order Jacobian matrix into two low order matrices. Compared with other approaches, firstly, it is more convenient for decomposion; secondly, it makes calculation speed much more quicker; thirdly, it is more easily to realize through programme. All of those are outstanding advantages of the method.

This paper is organized as follows, in section 2, we present saddle-node and Hopf bifurcation theorem; in section 3, we establish the extended Moore-Spence system and describe the matrix reduced algorithm for computing saddle-node and Hopf bifurcation points respectively; the Moore-Spence system of power system is given in section 4; one simple power system model is studied on the relation of voltage stability with the static and dynamic bifurcation in section 5. Detailed procedures and conditions of this new method will be described below.

2. BIFURCATION OF DIFFERENTIAL-ALGEBRAIC SYSTEMS

2.1 Saddle-Node Bifurcation

The saddle-node bifurcation is also called fold bifurcation, which is extensively studied in power systems. SNB occurs when the system (1) has a non-hyperbolic equilibrium with a geometrically simple zero eigenvalue at the bifurcation point and additional transversality conditions as presented in Theorem 1 (Venkatasubramanian and Zaborszky (1995)) are satisfied. In two-dimension space, one is saddle point, and the other is node point, therefore it is named as SNB. The right eigenvector of the zero eigenvalue shows the direction of system evolvement in the state space, and the left eigenvector indicates the influence degree to the state variables by the zero eigenvalue.

Theorem 1. (Saddle-Node Bifurcation Theorem). The determinant $D_y g$ is not singular, therefore, by the implicit function theorem, there is a suitable unique function f_R, so that the DAEs (1) can be reduced to the differential system $\dot{x} = f_R(X, \lambda)$, $X = (x, y)$. If the following transversality conditions are satisfied,

- S1 Jacobian matrix $D_X f_R = D_x f - (D_y f)(D_y g)^{-1}(D_x g)$ has a geometrically simple zero eigenvalue with right eigenvector v and left eigenvector w, and there is no other eigenvalue on the imaginary axis;
- S2 $w^T(D_\lambda f_R) = w^T[D_\lambda f - (D_y f)(D_y g)^{-1}(D_\lambda g)] \neq 0$;
- S3 $w^T[D_x^2 f_R(v, v)] \neq 0$;

thus, the system (1) occurs saddle-node bifurcation, the point which satisfied the conditions is called the saddle-node bifurcation point.

At the saddle-node bifurcation point, stable and unstable equilibrium points meet and disappear, resulting in a loss of equilibrium locally near the bifurcation point.

2.2 Hopf Bifurcation

Hopf bifurcation is one of the typical dynamic bifurcations and often occurs in power systems. It always leads to oscillate periodically and chaos. Hopf bifurcation occurs at the point where the system has a non-hyperbolic equilibrium connected with a pair of conjugate imaginary eigenvalues, but no zero eigenvalues, and the following additional transversality conditions are meet (Venkatasubramanian and Zaborszky (1995)).

Theorem 2. (Hopf Bifurcation Theorem). Suppose the determinant $D_y g$ is not singular, if the following transversality conditions are satisfied,

- H1 the system has an equilibrium point (x_0, y_0, λ_0), i.e. $f(x_0, y_0, \lambda_0) = 0$, $g(x_0, y_0, \lambda_0) = 0$;
- H2 Jacobian matrix J has a pair of conjugate purely imaginary eigenvalues $\mu_{1,2} = \pm iw$ at the point (x_0, y_0, λ_0), and there is no other eigenvalue on the imaginary axis;
- H3 $c = \partial(Re(\mu(\lambda_0)))/\partial\lambda \neq 0$;

thus, the system (1) occurs Hopf bifurcation, the point (x_0, y_0, λ_0) is called Hopf bifurcation point.

Much research proved that if eigenvalues cross the imaginary axis from the left half plane to the right half plane as λ increases, then the system gives birth of limit cycles at the bifurcation point. If the bifurcation stability coefficients $\beta_2 < 0$ and $c > 0$, the bifurcation is called supercritical Hopf bifurcation, and the cycle orbits are asymptotically stable; if the bifurcation stability coefficients $\beta_2 > 0$ and $c > 0$, the bifurcation is called subcritical Hopf bifurcation, and the cycle orbits are unstable (Ajjarapu and Lee (1992)).

2.3 Equilibrium Solution Manifold

By bifurcation theorem, suppose that Jacobian matrix J has a simple zero eigenvalue $\mu_0 = 0$ at SNB point and has a pair of purely imaginary eigenvalues $\mu_{1,2} = \pm iw$ at HB point, the reference (Moore and Spence (1980)) had established Moore-Spence equations as follows,

$$\begin{cases} f(x, y, \lambda) = 0 \\ g(x, y, \lambda) = 0 \\ A \begin{bmatrix} v_n \\ v_m \end{bmatrix} - \mu_j \begin{bmatrix} v_n \\ 0 \end{bmatrix} = 0 \\ v^T v - 1 = 0 \end{cases} \tag{4}$$

where, $j = 0, 1, 2$, $A = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}$ is generalized Jacobian matrix of (1), $v = \begin{bmatrix} v_n \\ v_m \end{bmatrix} \in R^{n+m}$, v_n is right eigenvector of eigenvalues $\mu_0 = 0$ or $\mu_{1,2} = \pm iw$.

The meaning of (4) is clear: the first two equations are equilibrium solution manifold of (1); the third equations indicate that A has a zero eigenvalue when $j = 0$ and has a pair of purely imaginary eigenvalues when $j = 1, 2$; v is standardization through the last equation.

Although the Moore-Spence system provides a good method to calculate bifurcation points directly, it has very high dimension and causes much difficulty in Jacobian matrix factorization when Newton iteration method is used for solving Moore-Spence equations, because the linearization equations (5) need be solved,

$$\begin{bmatrix} f_x & f_y & f_\lambda & 0 & 0 \\ g_x & g_y & g_\lambda & 0 & 0 \\ A_x v & A_y v & A_\lambda v & \begin{bmatrix} f_x - \mu_j \\ g_x \end{bmatrix} & \begin{bmatrix} f_y \\ g_y \end{bmatrix} \\ 0 & 0 & 0 & v_n^T & v_m^T \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \\ \Delta v_n \\ \Delta v_m \end{bmatrix} = - \begin{bmatrix} f(x,y,\lambda) \\ g(x,y,\lambda) \\ Av - \mu_j \begin{bmatrix} v_n \\ 0 \end{bmatrix} \\ v^T v - 1 \end{bmatrix} \quad (5)$$

There are $(2n + 2m + 1)$ equations in (5). For a large power system which has N load nodes, the dimension of coefficient matrix would be $(4N+2m+1)\times(4N+2m+1)$, therefore it is very complicated to solve (5). For this, we need reduce the high dimensionality.

3. MATRIX REDUCED ALGORITHM

3.1 Establish Extended Moore-Spence System

By introducing an auxiliary variable α and an auxiliary equation $v_n^T x - v_m^T y - \alpha = 0$, and inserting in system (4), we establish the extended Moore-Spence equations

$$\begin{cases} f(x,y,\lambda) = 0 \\ g(x,y,\lambda) = 0 \\ v_n^T x - v_m^T y - \alpha = 0 \\ Av - \mu_j \begin{bmatrix} v_n \\ 0 \end{bmatrix} = 0 \\ v^T v - 1 = 0 \end{cases} \quad (6)$$

Obviously, (6) and (5) have the same solution (x, y, λ, v). In order to make the calculation of SNB points and HB points simple and convenient, it is necessary to describe Jacobian matrix reduced method concretely.

3.2 Matrix Reduced Algorithm for Computing SNB Points

When $\mu_0 = 0$, we make use of Newton iteration method to solve the equations (6), thus the linear equations are

$$\begin{bmatrix} f_x & f_y & f_\lambda & 0 & 0 \\ g_x & g_y & g_\lambda & 0 & 0 \\ v_n^T & v_m^T & 0 & 0 & -1 \\ A_x v & A_y v & A_\lambda v & A & 0 \\ 0 & 0 & 0 & v^T & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \\ \Delta v \\ \Delta \alpha \end{bmatrix} = - \begin{bmatrix} f(x,y,\lambda) \\ g(x,y,\lambda) \\ v_n^T x - v_m^T y - \alpha \\ Av \\ v^T v - 1 \end{bmatrix} \quad (7)$$

Equations (7) can be further transformed as follows

$$\begin{bmatrix} f_x & f_y & f_\lambda & 0 & 0 \\ g_x & g_y & g_\lambda & 0 & 0 \\ v_n^T & v_m^T & 0 & 0 & 0 \\ A_x v & A_y v & A_\lambda v & A & [\, f_\lambda \ g_\lambda \,]^T \\ 0 & 0 & 0 & v^T & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \\ \Delta v \\ \Delta \alpha \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \\ [\, f_\lambda \ g_\lambda \,]^T \\ 0 \end{bmatrix} \Delta\alpha = - \begin{bmatrix} f(x,y,\lambda) \\ g(x,y,\lambda) \\ v_n^T x - v_m^T y - \alpha \\ Av \\ v^T v - 1 \end{bmatrix} \quad (8)$$

For convenience, let

$$M = \begin{bmatrix} f_x & f_y & f_\lambda \\ g_x & g_y & g_\lambda \\ v_n^T & v_m^T & 0 \end{bmatrix} = \begin{bmatrix} A & [\, f_\lambda \ g_\lambda \,]^T \\ v^T & 0 \end{bmatrix} \quad (9)$$

$$N = \begin{bmatrix} A_x v & A_y v & A_\lambda v \\ 0 & 0 & 0 \end{bmatrix} \quad (10)$$

then, (8) can be easily decomposed into two equations as follows

$$M \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \Delta\alpha = - \begin{bmatrix} f(x,y,\lambda) \\ g(x,y,\lambda) \\ v_n^T x - v_m^T y - \alpha \end{bmatrix} \quad (11)$$

$$M \begin{bmatrix} \Delta v \\ \Delta \alpha \end{bmatrix} + N \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \end{bmatrix} - \begin{bmatrix} [\, f_\lambda \ g_\lambda \,]^T \\ 0 \end{bmatrix} \Delta\alpha = - \begin{bmatrix} Av \\ v^T v - 1 \end{bmatrix} \quad (12)$$

From (11) and (12), if the matrix M is not singular and the variable $\Delta\alpha$ is known, then the variables Δx, Δy, $\Delta\lambda$, Δv can be solved through (11) and (12). Therefore, in solving (7) by reducing the high order equations to the low order two block matrices M and N, non-singularity of the matrix M and solvability of the variable $\Delta\alpha$ play a key role.

3.3 Matrix Reduced Algorithm for Computing HB Points

When $\mu_1 = iw$ ($\mu_2 = -iw$ is as the same as $\mu_1 = iw$), using Newton iteration method to solve the equations (6), the corresponding linearization equations are

$$\begin{bmatrix} f_x & f_y & f_\lambda & 0 & 0 & 0 \\ g_x & g_y & g_\lambda & 0 & 0 & 0 \\ v_n^T & v_m^T & 0 & 0 & 0 & -1 \\ A_x v & A_y v & A_\lambda v & \begin{bmatrix} f_x - iw \\ g_x \end{bmatrix} & \begin{bmatrix} f_y \\ g_y \end{bmatrix} & 0 \\ 0 & 0 & 0 & v_n^T & v_m^T & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \\ \Delta v_n \\ \Delta v_m \\ \Delta \alpha \end{bmatrix} = - \begin{bmatrix} f(x,y,\lambda) \\ g(x,y,\lambda) \\ v_n^T x - v_m^T y - \alpha \\ Av - iw \begin{bmatrix} v_n \\ 0 \end{bmatrix} \\ v^T v - 1 \end{bmatrix} \quad (13)$$

Above formula can be further written as

$$\begin{bmatrix} f_x & f_y & f_\lambda & 0 & 0 & 0 \\ g_x & g_y & g_\lambda & 0 & 0 & 0 \\ v_n^T & v_m^T & 0 & 0 & 0 & 0 \\ A_x v & A_y v & A_\lambda v & \begin{bmatrix} f_x \\ g_x \end{bmatrix} & \begin{bmatrix} f_y \\ g_y \end{bmatrix} & \begin{bmatrix} f_\lambda \\ g_\lambda \end{bmatrix} \\ 0 & 0 & 0 & v_n^T & v_m^T & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \\ \Delta v_n \\ \Delta v_m \\ \Delta \alpha \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \\ \begin{bmatrix} f_\lambda \\ g_\lambda \end{bmatrix} \\ 0 \end{bmatrix} \Delta\alpha - \begin{bmatrix} 0 \\ 0 \\ 0 \\ \begin{bmatrix} iw \\ 0 \end{bmatrix} \\ 0 \end{bmatrix} \Delta v_n$$

$$= - \begin{bmatrix} f(x,y,\lambda) \\ g(x,y,\lambda) \\ v_n^T x - v_m^T y - \alpha \\ Av - iw \begin{bmatrix} v_n \\ 0 \end{bmatrix} \\ v^T v - 1 \end{bmatrix} \tag{14}$$

Marked the matrixes M and N as the same as (9) and (10), therefore, (14) can be decomposed into two equations, one is as the same as (11), the other is

$$M \begin{bmatrix} \Delta v \\ \Delta \alpha \end{bmatrix} + N \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \end{bmatrix} - \begin{bmatrix} [\, f_\lambda \ g_\lambda \,]^T \\ 0 \end{bmatrix} \Delta\alpha$$

$$- \begin{bmatrix} [\, iw \ 0 \,]^T \\ 0 \end{bmatrix} \Delta v_n = - \begin{bmatrix} Av - iw \begin{bmatrix} v_n \\ 0 \end{bmatrix} \\ v^T v - 1 \end{bmatrix} \tag{15}$$

Similarly, if the matrix M is not singular and the variable $\Delta\alpha$ is known, we can obtain variables Δx, Δy, $\Delta\lambda$, Δv_n, Δv_m by solving (11) and (15), therefore, we need prove M is not singular at bifurcation points.

3.4 Computing the Bifurcation Points

Theorem 3. The matrix M is non-singular at the saddle-node bifurcation point and Hopf bifurcation point.

Proof. Assume one equations with coefficient matrix M as follows,

$$\begin{bmatrix} f_x & f_y & f_\lambda \\ g_x & g_y & g_\lambda \\ v_n^T & v_m^T & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

where, $p \in R^n$, $q \in R^m$ and $r \in R$, if there is only zero solution satisfied the equations, then the theorem is proved. The first two equations are

$$f_x p + f_y q + f_\lambda r = 0 \tag{17}$$

$$g_x p + g_y q + g_\lambda r = 0 \tag{18}$$

Because g_y is invertible, by solving (18), q can be expressed in terms of p and r as $q = -g_y^{-1}(g_x p + g_\lambda r)$. Substituting q into (17) results in

$$f_x p - f_y g_y^{-1} g_x p - f_y g_y^{-1} g_\lambda r + f_\lambda r = 0 \tag{19}$$

According to (3) and transversality conditions of theorem 1 and 2, (19) can be also writen as

$$Jp + (D_\lambda f_R) r = 0 \tag{20}$$

If we wish (20) is identical, then p and r must be zero, moreover $q = 0$ can be deduced from the last equation of (16). This completes the proof of theorem.

And then, since M is non-singular, we can get the solutions Δx, Δy, $\Delta\lambda$ of (11),

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_\lambda \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \\ w_\lambda \end{bmatrix} \Delta\alpha \tag{21}$$

where, $u_x, w_x \in R^n$, $u_y, w_y \in R^m$, $u_\lambda, w_\lambda \in R$, and the following equations (22) and (23) are satisfied.

$$\begin{bmatrix} u_x \\ u_y \\ u_\lambda \end{bmatrix} = -M^{-1} \begin{bmatrix} f(x,y,\lambda) \\ g(x,y,\lambda) \\ v_n^T x - v_m^T y - \alpha \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} w_x \\ w_y \\ w_\lambda \end{bmatrix} = M^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{23}$$

If calculating saddle-node bifurcation point, we substitutie (21) into (12), then

$$\begin{bmatrix} \Delta v \\ \Delta \alpha \end{bmatrix} = \begin{bmatrix} u_v \\ u_\alpha \end{bmatrix} + \begin{bmatrix} w_v \\ w_\alpha \end{bmatrix} \Delta\alpha \tag{24}$$

where $u_v, w_v \in R^{n+m}$, $u_\alpha, w_\alpha \in R$ satisfing the following equations (25) and (26).

$$\begin{bmatrix} u_v \\ u_\alpha \end{bmatrix} = -M^{-1} N \begin{bmatrix} u_x \\ u_y \\ u_\lambda \end{bmatrix} - M^{-1} \begin{bmatrix} Av \\ v^T v - 1 \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} w_v \\ w_\alpha \end{bmatrix} = M^{-1} \begin{bmatrix} [\, f_\lambda \ g_\lambda \,]^T \\ 0 \end{bmatrix} - M^{-1} N \begin{bmatrix} w_x \\ w_y \\ w_\lambda \end{bmatrix} \tag{26}$$

Therefore, we can obtain the variable $\Delta\alpha$ by solving (24),

$$\Delta\alpha = \frac{u_\alpha}{1 - w_\alpha} \tag{27}$$

then, substituting (27) into (21) and (24), we can get solutions of Δx, Δy, $\Delta\lambda$.
When calculating Hopf bifurcation points, similarly, substituting (21) into (15), we can get

$$\begin{bmatrix} \Delta v_n \\ \Delta v_m \\ \Delta \alpha \end{bmatrix} = \begin{bmatrix} u_{v_n} \\ u_{v_m} \\ u_\alpha \end{bmatrix} + \begin{bmatrix} w_{v_n} \\ w_{v_m} \\ w_\alpha \end{bmatrix} \Delta\alpha$$

$$+ M^{-1} \begin{bmatrix} [\, iw \ 0 \,]^T \\ 0 \end{bmatrix} \Delta v_n \tag{28}$$

where $u_{v_n}, w_{v_n} \in R^n$, $u_{v_m}, w_{v_m} \in R^m$, and $u_\lambda, w_\lambda \in R$ are solutions of (29) and (30)

$$\begin{bmatrix} u_{v_n} \\ u_{v_m} \\ u_\alpha \end{bmatrix} = -M^{-1} N \begin{bmatrix} u_x \\ u_y \\ u_\lambda \end{bmatrix} - M^{-1} \begin{bmatrix} Av - iw \begin{bmatrix} v_n \\ 0 \end{bmatrix} \\ v^T v - 1 \end{bmatrix} \tag{29}$$

$$\begin{bmatrix} w_{v_n} \\ w_{v_m} \\ w_\alpha \end{bmatrix} = M^{-1} \begin{bmatrix} [\, f_\lambda \ g_\lambda \,]^T \\ 0 \end{bmatrix} - M^{-1} N \begin{bmatrix} w_x \\ w_y \\ w_\lambda \end{bmatrix} \tag{30}$$

Thereby, the variable $\Delta\alpha$ can be solved through (28),

$$\Delta\alpha = \frac{u_\alpha + (*) \Delta v_n}{1 - w_\alpha} \tag{31}$$

then, substituting (31) into (21) and (28), we can get solutions of Δx, Δy, $\Delta\lambda$, Δv_n, Δv_m.
From above detailed analysis, we just need to solve the invertible matrix M^{-1} and the equations of which the dimension is $(n+m+1)$, therefore it is much more feasible than computation $(2n+m+1)$ order Jacobian matrix of large power systems.

4. COMPUTING THE BIFURCATION POINTS OF POWER SYSTEM

When studying the bifurcation of large power systems, several procedures for obersvation the bifurcation behavior would involved, the first step is to solve and trace the equilibrium curve; the second step is to indetify bifurcation points. For the power system has N nodes, the equilibrium manifold can be modeled by

$$\begin{cases} P(V,\theta,\lambda) = 0 \\ Q(V,\theta,\lambda) = 0 \end{cases} \tag{32}$$

where, $V \in R^N$ and $\theta \in R^N$ are voltage and angle respectively, $\lambda \in R$ is parameter; $P = 0$ and $Q = 0$ are equilibrium manifolds of active power and reactive power respectively. The right eigenvector U of Jacobian matrix at saddle-node bifurcation points or Hopf bifurcation points satisfies

$$\begin{bmatrix} P_V & P_\theta \\ Q_V & Q_\theta \end{bmatrix} \begin{bmatrix} U_P \\ U_Q \end{bmatrix} - \mu_j \begin{bmatrix} U_P \\ 0 \end{bmatrix} = 0 \tag{33}$$

for one and only eigenvector, $U = \begin{bmatrix} U_P \\ U_Q \end{bmatrix}$ is standardization by

$$U_P^T U_P + U_Q^T U_Q = 1 \tag{34}$$

Equations (32)∼(34) together constitute the Moore-Spence equations of power systems, according to section 3, it is very easy to calculate bifurcation points by using of matrix reduced algorithm.

5. ILLUSTRATIVE EXAMPLE

We consider one simple power system model (3 bus system) shown in Fig.1. It consists of two generators feeding a load, which is represented by an induction motor in parallel with a capacitor and a constant impedance PQ load. One generator is an infinite bus and the other generator has a constant voltage magnitude E_m. The equations that govern the power system model are

$$\begin{cases} \dot{\delta}_m = \omega \\ \dot{\omega} = [-d_m\omega + P_m + E_M Y_m V \sin(\delta - \delta_m - \theta_m) \\ \qquad + (E_m^2 Y_m \sin\theta_m)]/M \\ \dot{\delta} = [-K_{qv2}V^2 - K_{qv}V + Q(\delta, V) - Q_0 \\ \qquad - Q_1]/K_{q\omega} \\ \dot{V} = \{K_{p\omega}K_{qv2}V^2 + (K_{p\omega}K_{qv2} - K_{q\omega}K_{pv})V \\ \qquad + K_{q\omega}[P(\delta,V) - P_0 - P_1] - K_{p\omega}[Q(\delta,V) \\ \qquad - Q_0 - Q_1]\}/(TK_{q\omega}K_{pv}) \end{cases} \tag{35}$$

For the meaning of all variables and detailed deduction of

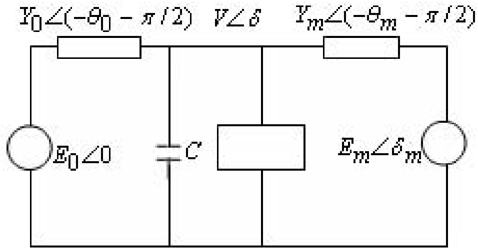

Fig. 1. A sample three bus power system model

the equations, see Dobson and Chiang (1989). There are four state variables, namely, generator angle δ_m, generator angular velocity ω, the angle δ and magnitude V of load voltage. The load reactive power variable Q_1 is a parameter corresponded to increase the load reactive power demand, and active power P and reactive power Q of system are

$$P(\delta,V) = -E_0'Y_0'V\sin(\delta+\theta_0') - E_mY_mV\sin(\delta - \delta_m + \theta_m) + (Y_0'\sin\theta_0' + Y_m\sin\theta_m)V^2$$

$$Q(\delta,V) = E_0'Y_0'V\cos(\delta+\theta_0') + E_mY_mV\cos(\delta - \delta_m + \theta_m) - (Y_0'\cos\theta_0' + Y_m\cos\theta_m)V^2$$

the other parameters are also as the same as aforementioned reference, i.e. $K_{p\omega} = 0.04$, $K_{pv} = 0.3$, $K_{q\omega} = -0.03$, $K_{qv} = -2.8$, $K_{qv2} = 2.1$, $T = 8.5$, $P_0 = 0.6$, $Q_0 = 1.3$, $P_1 = 0.0$, $Y_0 = 20.0$, $\theta_0 = 5.0$, $E_0 = 1.0$, $Y_0' = 8.0$, $\theta_0' = -12.0$, $E_0' = 2.5$, $Y_m = 5.0$, $\theta_m = -5.0$, $E_m = 1.0$, $P_m = 1.0$, $d_m = 0.05$, $M = 0.3$.
Making use of matrix reduced method, we can directly calculate Hopf bifurcation point is $(\delta_m, \omega, \delta, V) = (0.137, 0.0, 0.155, 0.759)$, when parameter $Q_1 = 13.728$, denoted HB_1; but when parameter $Q_1 = 14.258$, denoted HB_2, Hopf bifurcation point is $(\delta_m, \omega, \delta, V) = (0.439, 0.0, 0.168, 0.620)$; simultaneously, when parameter $Q_1 = 14.260$, marked as SNB, saddle-node bifurcation point is $(\delta_m, \omega, \delta, V) = (0.442, 0.0, 0.169, 0.612)$. We would verify above computing results through time domain simulation Fig.2-4.
From Fig.2, we know that an unstable limited loop appears when $Q_1 = 13.72 < HB_1$. At the same time, the voltage loses stability and results in periodically oscillation as shown as Fig.3, therefore power system would occur subcritical Hopf bifurcation when $Q_1 = 13.728$. But when the parameter $Q_1 = 14.259 > HB_2$, the voltage of load resumes to stabilize as shown in Fig.4. If Q_1 decreases less than HB_2, a small and stable limited loop would appear, which indicates that the system would occur supercritical Hopf bifurcation when $Q_1 = 14.258$.
As Q_1 increases further to $14.261 > SNB$, the Fig.5 illuminates that the voltage would drop suddenly to negative after a spell; clearly, voltage happens reverse and leads into voltage collapse. All above show that the computing results and the time domain simulation results are absolutely consistent, therefore it is feasible and effective of using the matrix reduced method to compute the saddle-node bifurcation points and Hopf bifurcation points of large power systems.

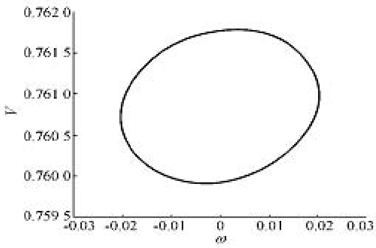

Fig. 2. The projection on $\omega - V$ plane of system orbit ($Q_1 = 13.72$)

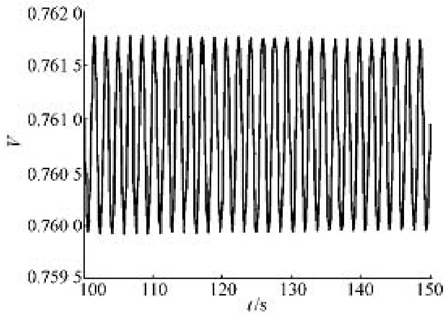

Fig. 3. Time domain simulation result of load voltage V ($Q_1 = 13.72$)

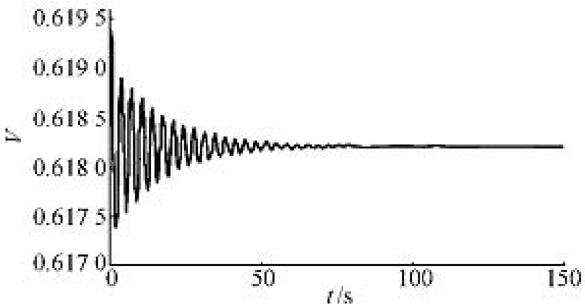

Fig. 4. Time domain simulation result of load voltage V ($Q_1 = 14.259$)

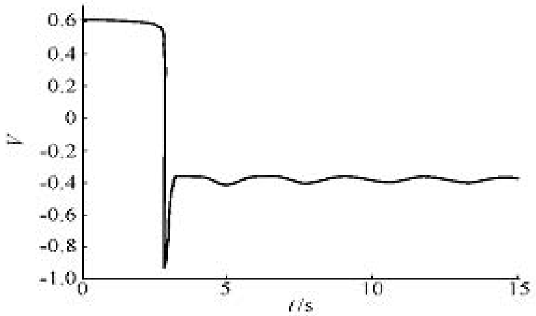

Fig. 5. Time domain simulation result of load voltage V ($Q_1 = 14.261$)

6. CONCLUSION

For large power systems, SNB and HB are both of important reasons for voltage collapse and osciation. The bifurcation theory and nonlinear system theory play a major role in understanding the nonlinear dynamical behavior of the power system. The paper has been derived an effective matrix reduced method through introducing an auxiliary variable and an auxiliary equation. Finally, we considered the 3-bus power system model and the time simulation results verified the validity of the method. Work is under way to extend the analysis to include larger and more detailed power system network model.

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