A FICTITIOUS REFERENCE ITERATIVE TUNING (FRIT) IN THE TWO-DEGREE OF FREEDOM CONTROL SCHEME AND ITS APPLICATION TO CLOSED LOOP SYSTEM IDENTIFICATION

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Abstract: This paper provides a new and powerful tuning method of controllers in the two-degree of freedom control scheme in the sense of that we require only one-shot experimental data of the closed loop. The key concept of our method is to use the fictitious reference (From this reason, we refer to our method as fictitious reference iterative tuning (FRIT)). Moreover, by using this tuning method, we also provide a new and powerful identification method in the sense of that we require only one-shot closed experimental data. Finally, we give an experimental result in order to show the validity of the proposed method in this paper.

Keywords: Controller parameter tuning, Fictitious Reference Iterative Tuning (FRIT), Two-degree of freedom control scheme, Closed loop system idetification One-shot experiment

1. INTRODUCTION

Recently, the control system synthesis based on the direct use of the measured input/output data attracts attentions with respect to practical view points. Since the real measured input/output data of a plant includes fruitful informations on the dynamics of the plant more directly than mathematical models obtained in system identifications, it is to be expected that such direct approaches provide effective controllers reflecting the dynamics of a plant. In the case of that the structure of a controller is fixed (e.g.,PID controllers), this direct approach is regarded as the parameter tuning based on the direct use of the data without mathematical model of a plant. As one of these tuning schemes, iterative feedback tuning (which is abbreviated to IFT in the following) was proposed by Hjalmarsson et.al(cf.(Hjalmarsson *et al.*, 1998), (Hjalmarsson *et al.*, 2002), and so on) and was studied in (Hjalmarsson *et al.*, 2002), (Bruyne *et al.*, 1999), (Hamamoto *et al.*, 2003), (Nakamoto, 2003) and so on.

IFT is the tuning method that iteratively updates the variable parameter of an implemented controller so as to minimize a performance index, e.g., the sum of squared error signal between the desired reference signal and the real output, by using the input/output data obtained in the iterative closed loop experiments. This minimization can be computed by performing non-linear optimization technique like Gauss-Newton method in which Hessian and Jacobian consist of the experimental data. This means that IFT enables us to obtain an appropriate parameter reflecting the plant information through the experimental data for the sake of achieving the specification. Thus, IFT is very effective approach in the case of that a controller with a variable parameter has been already implemented in the real plant with unknown dynamics.

On the other hand, however, the fact that non-linear optimization methods used in IFT require the input-output data also means that *many* experiments must be performed in order to update the parameter of controllers so as to achieve the minimization of the performance index. Thus there may be many cases in which the use of IFT spends considerable cost and time, which is a crucial problem with respect to practical points of view.

From these backgrounds, we have provided a new method of iterative parameter tuning by using *only one-shot* experiment for the sake of reducing cost and time required for arriving at the optimum parameter of the controller in the iterative tuning in (Souma *et al.*, 2004). As another similar method for parameter tuning of controller based on one-shot experimental data, Campi, Lecchini and Savaresi have proposed and developed Virtual Reference Feedback Tuning (VRFT) in (Campi *et al.*, 2002), (Campi *et al.*, 2003), and (Lecchini *et al.*, 2002) by using novel concept based on adaptive identification. Independently of VRFT, the key concept of our approach is to use the fictitious reference signal for the off-line tuning of the parameter of the controller. Thus, we refer to our new method as *fictitious reference iterative tuning* (it is abbreviated to *FRIT* in the following). The fictitious reference signal appears in the unfalsified control which is a novel concept proposed and developed by Safonov in (Safonov and Tsao, 1997).

In this paper, we expand FRIT into the two-degree of freedom control scheme. At the same time, it is well-known that a feedforward-controller in the two-degree of freedom control scheme is regarded as a filter consisting of a desired reference model and the (quasi-) inverse model of a plant. In the case of that the dynamics of a plant is unknown, this means that applying our FRIT to the feedforward controller yields the plant model on the off-line iterative computation by using one-shot experimental data in the case of that the dynamics of a plant is unknown. Thus, the method we provide in this paper is also used to develop a new and powerful identification method in the sense of that we require only one-shot experimental data of the closed loop system.

The contributions of this paper are the following two points: First, as stated in the above, since our method require only one-shot experimental data of the closed loop system, one can drastically reduce cost and time in the closed loop identification experiment . The issue on a system indetification in the closed loop is known as one of the important topics in the control and system theory (e.g.,cf.(Forssel and Ljung, 1999),(Lee, 1995),(Lee *et al.*, 1995),(Verhaegen, 1994) etc.), so our method can provide the contribution to this area in the practical sense. Second, fictitious reference signals used for the parameter tuning in this paper is introduced in the unfalsified control theory (cf. (Safonov and Tsao, 1997), (Safonov and Cabral, 2001)), which is one of the nice extensions of the behavioral approach proposed by Willems (cf. (Willems, 1991) (Willems, 1997)). Thus, this work is also regarded as one of the applications of the unfalsified control and shows that the notion of the behavior is also a powerful concept to solve the real practical problems.

[Assumptions and Notations]:

A plant in this paper is assumed to be a single-input single-output, linear, time-invariant, finite dimensional system. For a time series (data) w, in order to describe the value at the time t, we use the notation w_t. Let q denote the shift operator defined by $qw_t := w_{t+1}$ for a time series w. Let u and y denote the input and output data, respectively, obtained in a finite time. Let N denote the number of the sampled data. Let $\mathbb{R}^n$ denote the set of real vectors of size n. In order to describe y at time t as the output of an operator $G(q)$ for the input u, one must write the convolution in the form of $y_t = \sum_{k=0}^{t} g_k u_{k-i}$ normally, where $G(q) = \sum_{k=0}^{\infty} g_k q^{-k}$ is the Markov parameters description of $G(q)$. However, we use an abbreviated description as $y_t = (G(q)u)_t$ for the sake of enhancing the readability. In addition, in order to explain the basic concept of FRIT and the role in the framework of closed loop system identification in the two-degree of freedom control scheme, we disregard the noise in the following explanation.

2. FRIT IN THE TWO-DEGREE FREEDOM CONTROL SCHEME AND ITS APPLICATION TO CLOSED LOOP SYSTEM IDENTIFICATION

First, we give a brief review of well-known two-degree of freedom control scheme (cf. (Desoer and Gustafson, 1984) etc.) Consider a two-degree of freedom control system in Fig.1.

Here, assume that they are parameterized by using $\rho_e \in \mathbb{R}^{n_e}$ and $\rho_r \in \mathbb{R}^{n_r}$ (where n_e and n_r are the number of the parameters of the feedback controller and the feedforward controller, respectively). In the following, we often use the notation

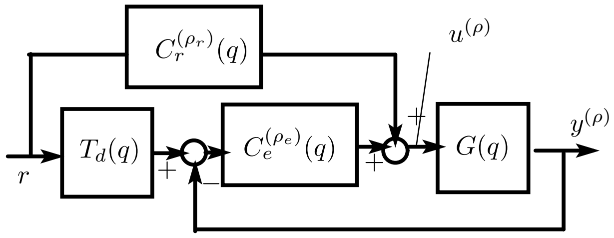

Fig. 1. A two-degree of freedom control system

$\rho = (\rho_e, \rho_r) \in \mathbb{R}^{n_r+n_e}$. Moreover, for example, $\rho_e(k)$ denotes the k-th element of ρ_e . Let $y^{(\rho)}$ and $u^{(\rho)}$ denote the output and the input obtained in the closed loop with the controller parameter ρ. And assume that we are given the desired response model $T_d(q)$. It is preferable that the feedforward controller is designed as

$$C_r^{(\rho_r)}(q) = G(q)^{-1} T_d(q), \tag{1}$$

if we have a nominal model $G(q)$. By changing tracks, we focus on this point for deriving our closed-loop system identification method in the case of that the dynamics of a plant is unknown.

2.1 FRIT(Fictitious Reference Iterative Tuning) in the two-degree of freedom control scheme

As stated in the previous section, IFT method must perform a lot of experiments for updating the controller parameter so as to achieve the optimum value in the sense that the performance index is minimized. By contrast, our FRIT requires *only one-shot experiment*, then the *off-line* Gauss-Newton method by using the fictitious reference signal yields the optimum parameter in the fictitious space. Moreover, this optimum parameter corresponds to the optimum one in the real closed loop system. Now, we suppose that $G(q)$ is unknown, and our aim is to obtain $G(q)$. For this purpose, if we apply IFT (cf.(Hjalmarsson *et al.*, 1998)) or our FRIT, *the feedforward controller can be parameterized so as to be described by $G(q)^{-1}T_d(q)$ from the experimental data.* Moreover, *our FRIT requires only one-shot experiment for arriving at the optimal parameter* while IFT requires many experiments for the same purpose. Hence, our FRIT scheme for two-degree of freedom control systems with unknown plant enables us to obtain the inverse model $G(q)^{-1}$, which means that the model $G(q)$ can also be obtained, by using only one-shot experimental data.

Suppose that the aim of the controller parameter tuning is to find the optimum parameter ρ^* in the sense that

$$\rho^* = \arg\min_{\rho} \sum_{t=1}^{N} \|y_t^{(\rho)} - (T_d(q)r)_t\|^2. \tag{2}$$

Note that the achievement of the above minimization yields the appropriate parameter of a controller, which yields also $G(q)^{-1}$. In the following, we explain how off-line computation yields the optimal parameter.

Let $\rho_i(\rho_{ei}$ and $\rho_{ri})$ denote the i-th step parameter in the following explanation. First, by using the initial parameter ρ_0, perform the first experiment on the closed loop system with ρ_0 and obtain the first data $(u^{(\rho_0)}, y^{(\rho_0)})$. By using $(u^{(\rho_0)}, y^{(\rho_0)})$, compute the fictitious reference signal $\tilde{r}(\rho_i)$ at the i-th step as

$$\tilde{r}^{(\rho_i)} = \frac{u^{(\rho_0)} + C_e^{(\rho_i)}(q) y^{(\rho_0)}}{C_r^{(\rho_i)}(q) + T_d(q) C_e^{(\rho_i)}(q)} \tag{3}$$

and compute the error written by

$$\tilde{e}^{(\rho_i)} := (y^{(\rho_0)} - T_d(q)\tilde{r}(\rho_i)). \tag{4}$$

Observe that $\tilde{r}^{(\rho_i)}$ is a reference signal that yields $u^{(\rho_0)}$ and $y^{(\rho_0)}$. Note that $\tilde{e}^{(\rho_i)}$ can be computed off-line at each $i-$the step. Consider the following performance index in the fictitious domain:

$$J_{\tilde{e}}(\rho) = \sum_{t=1}^{N} \|\tilde{e}_t^{(\rho)}\|^2. \tag{5}$$

Note that Eq.(5) consists of already-known information ($y^{(\rho_0)}$, $T_d(q)$, $\tilde{r}^{(\rho_i)}$). For the sake of minimization of $J_{\tilde{e}}(\rho)$, that is, in order to find

$$\tilde{\rho}^* = \arg\min_{\rho} J_{\tilde{e}}(\rho), \tag{6}$$

we perform the following Gauss-Newton algorithm. Consider the gradient

$$\left.\frac{\partial J_{\tilde{e}}(\rho)}{\partial \rho}\right|_{\rho=\rho_i} = \sum_{t=1}^{N} \tilde{e}(\rho)_t \left.\left(\frac{\partial \tilde{e}(\rho)}{\partial \rho}\right)_t\right|_{\rho=\rho_i} \tag{7}$$

and the following update equation

$$\rho_{i+1} = \rho_i - \gamma R_i^{-1} \left.\frac{\partial J_{\tilde{e}}(\rho)}{\partial \rho}\right|_{\rho=\rho_i}, \tag{8}$$

where R_i is a Hessian approximated by

$$R_i = \left(\left.\frac{\partial J_{\tilde{e}}(\rho)}{\partial \rho}\right|_{\rho=\rho_i} \right)^T \left(\left.\frac{\partial J_{\tilde{e}}(\rho)}{\partial \rho}\right|_{\rho=\rho_i} \right) \tag{9}$$

and γ is a parameter that tunes the speed of the convergence. In Eq.(8) and Eq.(7), $\left.\frac{\partial \tilde{e}(\rho)}{\partial \rho}\right|_{\rho=\rho_i}$ can be computed as

$$\left.\frac{\partial \tilde{e}^{(\rho)}}{\partial \rho}\right|_{\rho=\rho_i} = \frac{T_d(q)}{T_d(q)C_e^{(\rho_e)}(q) + C_r^{(\rho_r)}(q)} \times \left\{ \frac{\left(T_d(q)\frac{\partial C_e^{(\rho_e)}(q)}{\partial \rho} + \frac{\partial C_r^{(\rho_r)}(q)}{\partial \rho}\right)}{T_d(q)C_e^{(\rho_e)}(q) + C_r^{(\rho_r)}(q)} \times (C_e^{(\rho_e)}(q)y^{(\rho_0)} + u^{(\rho_0)}) - \left(\frac{\partial C_e^{(\rho_e)}(q)}{\partial \rho} y^{(\rho_0)}\right) \right\} \Bigg|_{\rho=\rho_i} \quad (10)$$

In this point, it should be noted that *the off-line computation can yield $\tilde{e}(\rho_i)$ and $\frac{\partial \tilde{e}^{(\rho_i)}}{\partial \rho}$, so we do not have to perform an experiment at each step in the Gauss-Newton method, differently from IFT.*

The remained problem is to see whether Eq.(5) is equivalent to that of Eq.(2). The following theorem gives a solution to this problem.

Theorem 2.1. Let $(u(\rho_0), y(\rho_0))$ denote the input/output data in the feedback system described in Figure 1 by using the initial parameter ρ_0. Assume that $(u(\rho_0), y(\rho_0))$ are not trivial zero data. Then $\lim_{\rho_i \to \rho'} \sum_{t=1}^{N} \|(y(\rho_i)_t - (T_d(q)r)_t\|^2 = 0$ if and only if $\lim_{\rho_i \to \rho'} \sum_{t=1}^{N} \|\tilde{e}(\rho_i)_t\|^2 = 0$. □

Proof: We omit the proof here. The detailed proof will be shown in our forthcoming paper (Kaneko *et al.*, 2004).

The above theorem guarantees that the minimization of $J_{\tilde{e}}(\rho_i)$ by using fictitious reference in the off-line computation yields the optimum parameter ρ^* in the real closed loop.

From the practical points of view, if one can find the optimum parameter, then there are many cases in which it is possible to achieve the sum of the squared error is almost equal to zero. Thus, the above theorem gives the practical solution for the question on whether the minimizations of the fictitious cost and that of the real cost are almost equivalent. Of course, it is preferable to guerantee that then minimization of the real cost function is equivalent to that of the fictiotious one. This point is one of our future studies.

Remark 2.1. At first glance, our proposed method "FRIT" seems to be similar to "VRFT" proposed by Campi in (Campi *et al.*, 2002) and (Campi *et al.*, 2003) the sense that we require only one-shot experimental data and the fictitious or virtual signals are used for the parameter tuning in the off-line. However, in VRFT, the virtual signal is computed with respect to the input while our fictitious signal is computed with respect to the output. This is somewhat indirect in the sense that the aim of the controller is to let the output follow the desired output. Moreover, VRFT requires the power density spectrum of the input from the first experiment in order to design "shaping filter "(see also (Campi *et al.*, 2002)) used for the iterative tuning. This means that the first experiment in VRFT must be open-loop experiment. The reason for this is that it is very difficult to obtain the power density spectrum of the input in the closed loop data because of the existence of the correlation between the input and the output in the feedback system. Particularly, this also means that VRFT can not be applied to an unstable system and an already-operated closed loop systems in the real industries. On the contrary to VRFT, our method FRIT can be applied to the case in which the first experimental data is obtained in the closed loop system. Hence, our method FRIT has advantages in these senses. □

2.2 Closed loop identification using FRIT

Combining the role of the feedforward controller in the two-degree of freedom control scheme and the concept of FRIT for two-degree of freedom control structure, we can obtain one of the system identification methods of a system embeded in the closed loop so as to enable us to save a lot of costs for the idetification. The algorithm of closed loop system idetification with FRIT in Fig.1 can be summarized as follows:

(a). Initial setting: Give a desired transfer function $T_d(q)$ from r to y and the initial controllers $C_e^{(\rho_{e0})}(q)$ and $C_r^{(\rho_{r0})}(q)$ (the relative degree is zero) with the initial parameter $\rho_0 = (\rho_{e0}, \rho_{r0})$.

(b). The first experiment: Perform one-shot experiment and obtain the finite input/output data $u^{(\rho_0)}$ and $y^{(\rho_0)}$.

(c). The off-line tuning: Compute the following Gauss-Newton method.
   - (1). Set $i = 0$.
   - (2). Compute the fictitious reference signal $\tilde{r}^{(\rho_i)}$ in Eq.(3).
   - (3). Compute the update equation Eq.(8) by using Eq.(7) and Eq.(10).
   - (4). Check $\|\rho_{i+1} - \rho_i\|^2 < \epsilon$ (where ϵ is a sufficiently small positive real number).
     - * No: $i = i + 1$ and go back to (2).
     - * Yes: $\tilde{\rho}^* := \rho_i$ and go to (d).

(d). Calculate the plant model by using

$$G(q) = T_d(q)C_r^{(\tilde{\rho}^*)}(q)^{-1} \quad (11)$$

with obtained the optimal parameter $\tilde{\rho}^*$.

Remark 2.2. The obtained model depends on $T_d(q)$, because it determines the frequency band in which we can obtain the accurate mathematical model. Moreover, since the initial experimental data reflects on not only the plant dynamics but also the feedback loop property, the initial feedback controller also plays a crucial role in this algorithm. They are our further studies. □

3. EXPERIMENTAL RESULT

In this section, we give an experimental result for showing the validity of our approach. The system we address here is described by Figure 2. The cart

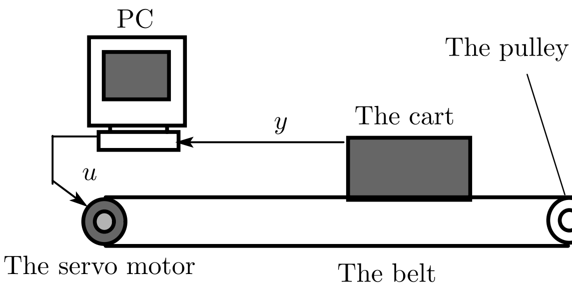

Fig. 2. The cart system

is attached to the belt and the belt is moving by the rolling of the servo motor. The location y (output) from the initial position of the cart is measured by the potentiometer and send to the personal computer (PC). And the servo motor is driven by the voltage u (input) from PC. The controllers are

$$C_e^{(\rho_i)}(q) = \rho_{ei}(1) + \rho_{ei}(2)\frac{q}{q-1}$$

and

$$C_r^{(\rho_i)}(q) = \frac{\rho_{ri}(5)q^2 + \rho_{ri}(4)q + \rho_{ri}(3)}{q^2 + \rho_{ri}(2)q + \rho_{ri}(1)}$$

(The closed loop controller is a well known P.I, Controller). We have no information on the dynamics of this system a priori except the relative degree of the plant The desired response model is to

$$T_d(q) = \left(\frac{1}{1 - q^{-1}e^{-0.1\Delta}}\right)^2$$

which corresponds to $\left(\frac{1}{0.1s+1}\right)^2$ in continuous time. We use the reference signal as the sum of some sinusoidal waves described by $r = \sum_{i=1}^{3} sin(10^{i-1}t)$. Moreover, Δ is the sampling time 0.01[sec], the experimental time is 10[sec] (i.e., $N = 1000$), and $\gamma = 1.0 \times 10^{-6}$. Firstly, we obtained the initial experimental data $y^{(\rho_0)}$ and $u^{(\rho_0)}$ described by Fig.3 and Fig.4, respectively.
Next, we perform our FRIT algorithm described by the previous section. As the optimal controller, we obtain

$$C_r^{(\tilde{\rho}^*)}(q) = \frac{q^2 - 1.299q + 0.302}{36.7q^2 - 73.22q + 36.52}$$

and

$$C_e^{(\tilde{\rho}^*)}(q) = 2.5573 + 2.0347\frac{q}{q-1}.$$

As a result, we obtain a model of the plant described by

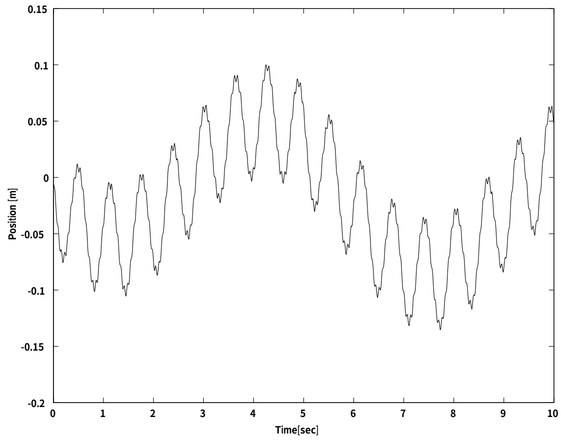

Fig. 3. The intial output data

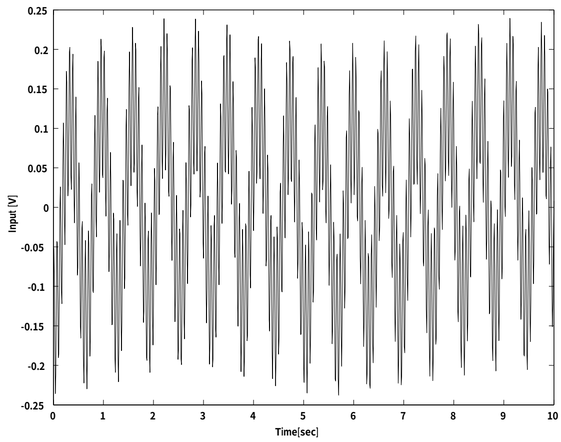

Fig. 4. The intial input data

$$G(q) = \left(\frac{1}{1 - q^{-1}e^{-0.1\Delta}}\right)^2 \times \frac{36.7q^2 - 73.22q + 36.52}{q^2 - 1.299q + 0.302} \quad (12)$$

We perform the validation test by using the step response (see Fig.5) and the response for the sum of some sinusoidal waves (see Fig.6). From Fig.5

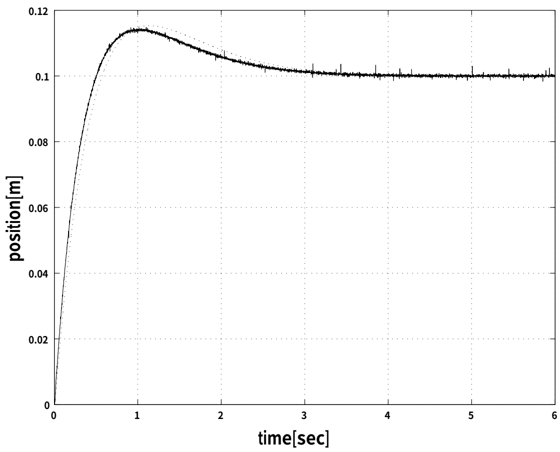

Fig. 5. The validation using the step signal (The real line:The obtained model, The dotted line: experimental data)

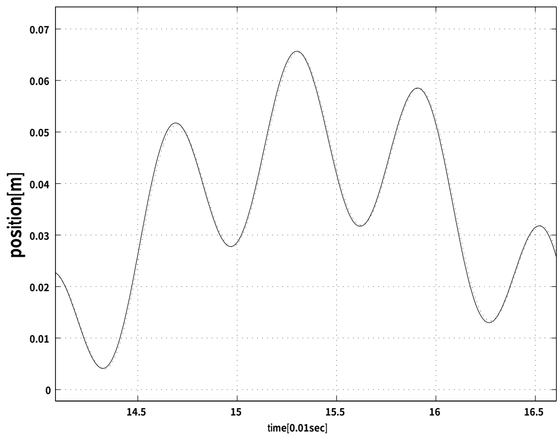

Fig. 6. The validation using the sum of sinusoidal waves (The real line:The obtained model, The dotted line: experimental data)

and Fig.6, we can observe that the obtained model describe the dynamics (particularly, in the low-frequency).

4. CONCLUSIONS

In this paper, we have provided a new and powerful method of closed loop system identification from the practical points of view. The key concept is fictitious reference iterative tuning in the two-degree of freedom control scheme. The detailed discussions, proofs and remarks will be shown in our forthcoming paper (Kaneko *et al.*, 2004).

Of course, the study in this paper is the first step of the research topic on fictitious reference itertive tuning and its applications. The difference between FRIT and VRFT by Campi et.al. need to be clarified. Moreover, the effectiveness of the noise we neglet in this paper is also considered in order to develop our method as one of the practical tools for system identification.

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