A LIGHTWEIGHT CONTROL METHODOLOGY FOR FORMATION CONTROL OF VEHICLE SWARMS

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Abstract: Multi-vehicle swarms offer the potential for increased performance and robustness in several key robotic and autonomous applications. Emergent swarm behavior demonstrated in biological systems shows performance that far outstrips the abilities of the individual members. We demonstrate a lightweight formation control methodology that uses conservative potential functions to ensure group cohesion, and yet has very modest communication and control requirements for the individual nodes. Any arbitrary formation can be formed and held, even while navigating through an unstructured obstacle environment. Simulation studies demonstrate that the formation control is robust, stable, and easily implemented in a distributed fashion.

Keywords: Vehicle Swarm, Formation Control, Potential Field, Artificial Potential, Virtual Leader

1. INTRODUCTION

As the cost and capability of small autonomous vehicles have improved, research efforts have focused on the distributed control of multi-agent systems. A biologically inspired swarm of vehicles has the potential to deliver inexpensive yet robust performance, especially in the case of hostile and adverse environments. Typically, the computational capability of the embedded processors and the limited communications bandwidth between individual vehicles makes large scale control optimization difficult. In this paper we describe a very lightweight framework for moving a group of homogenous vehicles through an obstacle field as a cohesive flock that is based on a simplified liquid surface tension abstraction.

Examples of cohesive group movement can be found throughout the natural world. Ants, fish, birds, and even sub atomic particles exhibit astonishingly elegant group coordination in the midst of complex and highly dynamic environments. Mathematical biologists have attempted to model this swarming behavior for some time [Breder (1954), Okubo (1986), Warburton and Lazarus (1991), G. Flierl and Olson (1999)], referring to the swarm behavior that is greater than the sum of its parts as "emergent behavior." In many cases the behavior can be reduced to rules of attraction and repulsion between neighbors [Breder (1954)].

In 1986 Craig Reynolds created a computer graphics model of coordinated animal motion based on the behavior of fish schools and bird flocks [Reynolds (1987)]. These "boids" used only knowledge of flock-mates in their immediate vicinity and motion based upon three simple rules: (1) avoid crowding (separation), (2) match heading (alignment), and (3) avoid separation (cohesion). Using these rules, the boids demonstrated motion in a remarkably cooperative and fluid way, showing that rule-based distributed group motion control is indeed feasible.

The overarching theme in this paper is to model the vehicle swarm in a simplified manner as a liquid droplet balanced between gravity and surface tension. To simplify the mathematical formulation, in this paper we use an artificial potential relationship to control vehicle separation and cohesion [Khatib (1986), Latombe (1991)]. As a technique, artificial potential has recently has come to prominence in the area of distributed control [Olfati-Saber and Murray (2002)].

In this paper, we define three types of artificial potential relationships: (1) vehicle to vehicle, (2) vehicle to *virtual leader*, and (3) vehicle to obstacle. The vehicle to vehicle relationship is used to control separation and cohesion, keeping the flock members from drifting too far or close to one another, and are modeled mathematically as a linear force relationship with an equilibrium position, much like a mechanical spring.

It is critical to note that within this model, the swarm has no mission goal or destination; the individual vehicles will simply move towards an equilibrium point. In essence, each vehicle follows a steepest descent towards the geometric point at which the sum of the virtual forces become zero. Motion of the group is based on the motion of the *virtual leader*, whose trajectory is controlled independently of the swarm. When the virtual leader is motionless (static environment), the swarm moves towards its *equilibrium formation*. Note that this equilibrium formation can be either the natural equilibrium or an arbitrary formation, as detailed in Section 3.

We use *virtual leader* in much the same way as Leonard from whom we have adapted the term [Leonard and Friorelli (2001)]. Note that the virtual leader is not a vehicle, but simply an imaginary point used to guide group movement. The simplicity of this approach is that the group is pulled by the virtual leader, and the formation is held together simply by the individual vehicles seeking their own equilibrium positions with respect to each other and the virtual leader.

Each vehicle has an artificial potential relationship with the virtual leader which is, mathematically, nearly identical to that between vehicles. When the virtual leader shifts from its equilibrium position it takes the entire flock with it causing a mass movement of the swarm to regain its equilibrium configuration. This global dependence on virtual leader motion reduces the task of planning multiple collision free paths for many vehicles to planning just one collision free path.

This paper is organized into the following sections: Section 2 describes the basic force relationships of individual members of the swarm, Section 3 describes the initial group formation based on a simple geometrical approach and the nature of the various artificial potential, Section 4 covers obstacle force formulation, Section 5 the virtual leader motion and swarm feedback control laws, Section 6 discusses the simulation studies, and Section 7 concludes with a summary of the results and indications for future work.

2. BASIC FORMULATION

As described in Section 1, the basic formulation of the swarming problem is modeled upon a simplified liquid surface tension. While this analysis can be extended to three dimensions, for the purposes of this paper, we limit the problem to a two-dimensional plane. The initial observation that led to this formulation of the flocking problem was the way that a liquid droplet of water is able to flow between and around various obstacles. Thus, various formulations were attempted in order to generate a set of mathematical relations that would produce a similar macroscopic behavior.

The initial formulation is to have each identical node in the swarm attracted to a center of mass (cohesion), yet repelled from each other (separation). In order to simplify the analysis, the formulas are kept linear, thus given N vehicles, the center of mass is defined as:

$$x_{cm} = \frac{1}{N}\sum_{1}^{N} x_i \quad \textit{and} \quad y_{cm} = \frac{1}{N}\sum_{1}^{N} y_i \tag{1}$$

For the attractive force to the center of mass (the gravity type force), the basic formulation is a linear force that grows larger with distance, much like a spring stretched from the center of mass to each node. Mathematically, the force is expressed as:

$$\boldsymbol{F}_{cm,i} = K_{cm}\boldsymbol{d}_{cm,i} \tag{2}$$

where K_{cm} is the equivalent spring constant, and $d_{cm,i}$ is the distance between the i^{th} node and the center of mass. Rewriting this in its coordinate form yields equation:

$$\begin{bmatrix} F_x^{cm} \\ F_y^{cm} \end{bmatrix} = K_{cm} \begin{bmatrix} x_{cm} - x_i \\ y_{cm} - y_i \end{bmatrix} \tag{3}$$

which always points towards the center of mass of the group. The repulsive force between each node of the group is also modeled as a simple force, in this case:

$$\boldsymbol{F}_{ij} = \frac{K}{|\boldsymbol{d_{ij}}|}\left(-\hat{e}_{ij}\right) \tag{4}$$

where F_{ij} is the repulsive force, K is a spring-type constant, d_{ij} is the scalar distance between nodes i and j, and $\hat{e}_{ij}$ is the unit vector from i to j. Given the definitions of $\boldsymbol{d}_{\mathrm{ij}}$, Eq. 4 can be rewritten in coordinate form as:

$$\begin{bmatrix} F_x^{ij} \\ F_y^{ij} \end{bmatrix} = \frac{K}{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} \tag{5}$$

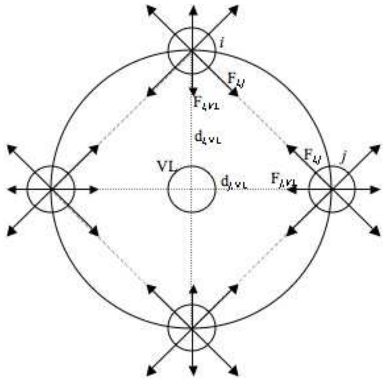

Fig. 1. Initial formation for a four node formation with distances and artificial potential forces labeled.

Note that all of the forces generated on the individual nodes are either of the attractive type, Eq. 2, or the repulsive type, Eq. 4. Obstacle forces are dealt with in Section 4.

3. INITIAL FORMATION

Given the above formulation for the basic forces that provide the cohesion and separation of the swarm, it is instructive to define the equilibrium initial formation, as well as demonstrate how arbitrary formations can be held. Intuitively, we can see that the initial formation will be a uniform ring with nodes equally spaced around the circumference of the ring. That is, the for N nodes, they are distributed equally in polar coordinates, such that the i^{th} angle, θ is $\frac{2\pi}{N}i$, where i ranges from 0 to $N-1$. Assuming the center of mass is at the origin, and each node is located at a radius, R_0 away, then the distance vector between the i^{th} node and the center of mass is:

$$d_{cm,i} = R_0 \begin{bmatrix} -\cos\left(\dfrac{2\pi}{N}i\right) \\ -\sin\left(\dfrac{2\pi}{N}i\right) \end{bmatrix} \tag{6}$$

again where i ranges from 0 to $N-1$. The geometry of the node to node distances is slightly more complex with the magnitude of the distance between the 0^{th} and the i^{th} node as:

$$d_{0,i} = 2R_0 \sin\left(\frac{\pi}{N}i\right) \tag{7}$$

and the unit vector from the 0^{th} to the i^{th} node is:

$$\hat{e}_{0i} = \begin{bmatrix} \sin\left(\dfrac{\pi}{N}i\right) \\ \cos\left(\dfrac{\pi}{N}i\right) \end{bmatrix} \tag{8}$$

putting these distance formulas together with the force relationships defined above and solving for the equilibrium point yields relationship for the base radius, R_0 that is only a function of the two "spring" constants:

$$R_0 = \sqrt{\frac{(N-1)\,K}{2K_{cm}}} \tag{9}$$

where the negative root is discarded.

In order to hold arbitrary formations, the nodes are placed into the desired formation and the equilibrium forces are calculated upon each node. Then, and equal and opposite "formation force" is assigned to each node to bring the net force on each node to zero. This formation force is a constant that is held on each node as long as this formation is desired. Note that in order to transition from one formation to another, these formation forces can be linearly varied and the resultant gradual force change will more the individual swarm vehicles from one formation to the other.

Based on these original forces, another formulation for the basic forces demonstrates identical properties while simplifying the calculations somewhat. In this case, the cohesion force is the same as the attraction to the center of mass (now replaced with a virtual leader), and the separation forces are based on a nominal distance from the nearest neighbors. Thus, the vehicle to vehicle interactions are defined by a linear equation analogous to the spring equation;

$$\boldsymbol{F}_{ij} = K_{ij} \begin{bmatrix} \boldsymbol{d}_x^{ij} - \boldsymbol{d}_{x_o}^{ij} \\ \boldsymbol{d}_y^{ij} - \boldsymbol{d}_{y_o}^{ij} \end{bmatrix} \tag{10}$$

$$\begin{aligned} d_x^{ij} &= \left(\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}\right)\cos(\theta) \\ d_y^{ij} &= \left(\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}\right)\sin(\theta) \end{aligned} \tag{11}$$

where d_o^{ij} is a constant and represents the initial or equilibrium distance between two vehicles. Two vehicles are at their d_o^{ij} when the force between them is zero. The force is conservative, such that it attracts the vehicles together as the distance increases, and repels the vehicles from each other as distance decreases. K_{ij} serves as the new spring constant, determining the "stiffness" of the relationship between the nodes.

In this alternate formulation, arbitrary formations are held by altering the d_o^{ij} values for each node. Again, formation transition can be performed by linearly varying the d_o^{ij} values from one formation to the other and the force computation will smoothly move the swarm from the original to the new formation. Note that the two formulations are very close to identical, with only the addition of an extra degree of freedom due to the independent spring constants.

Once the equilibrium positions have been determined it is straightforward to find d_o^{ij} and d_o^{VL}. Eq. 10 and Eq. 13 demand that d and d_o for all vehicles be equal when the vehicles are in equilibrium. Thus:

$$F_{i-1}^{ij} + F_{i+1}^{ij} + F_{VL} = 0 \qquad (12)$$

where F_{i-1}^{ij} and F_{i+1}^{ij} are the force between i^{th} node's left and right neighbors respectively. Any arbitrary formation can be created in this way as long as a realistic set of distances is used.

The vehicle to virtual leader relationship is very similar to that between vehicles except for one important distinction: the spring constant, K_{ij} is chosen separately.

$$\boldsymbol{F}_{VL} = K_{VL} \begin{bmatrix} d_x^{VL} - d_{x_o}^{VL} \\ d_y^{VL} - d_{y_o}^{VL} \end{bmatrix} \qquad (13)$$

$$\begin{aligned} d_x^{VL} &= x_{VL} - x_i \\ d_y^{VL} &= y_{VL} - y_i \end{aligned} \qquad (14)$$

The difference between these two methods of computing the virtual leader forces is in the direction of the resulting force.

4. OBSTACLES

In order to deal with obstacles, we enclose them in bounding convex polygons and impose a repulsive force relationship between the vehicles and polygon edges that is inversely proportional to the distance between them. Because vehicles continually recalculate their forces locally without any assumptions, both dynamic (moving) and static obstacles can be handled equally well. In this work, we do not deal with obstacle detection and assume that we have knowledge of obstacle position and shape.

Fig. 2 illustrates how obstacles are enclosed by convex polygons. Note that this method of defining obstacles has several advantages. Firstly, convex polygons simplify the task of obstacle avoidance by the swarm. This is done by determining which face of the polygon the individual vehicle lies in front of, and how far away from that surface the vehicle is.

Secondly, the reduction of a complex obstacle field to a simple set of convex polygons simplifies both the calculation and the communication of the location of the obstacles in the field to the other vehicles. This is important in terms of the bandwidth requirements of the communication channel between the vehicles, as well as future extensions to simultaneous localization and mapping (SLAM) applications.

The vehicle obstacle potential function is inversely proportional to the distance between them and approaches infinity as distance decreases, thus ensuring that individual vehicles do not collide with the obstacles. A typical function that embodies this feature is:

$$\boldsymbol{F}_{ob} = K_{ob} / \left(\boldsymbol{d}_{ob}\right) \qquad (15)$$

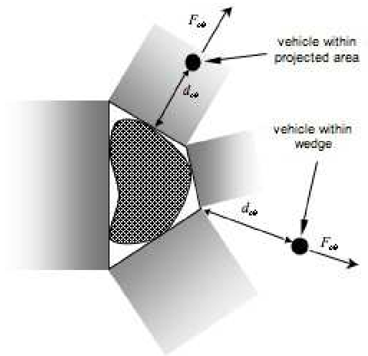

Fig. 2. Obstacle surrounded by convex polygon with rectangular and wedge shaped obstacle fields indicated.

Note that this computation has to be performed for each and every obstacle within range, thus the total obstacle force on a given node is given by:

$$\boldsymbol{F}_{ob} = \sum_{k=1}^{n} K_{ob} / \left(\boldsymbol{d}_k^{ob}\right) \qquad (16)$$

where n is the number of obstacles present, and the distance is either the perpendicular distance to the face or the straight distance from a vertex, as appropriate. As seen in Fig. 2, a vehicle must be determined to lie either in a rectangle in front of a face, or in the wedge between two faces. This is used to compute the virtual force on the node of the swarm in question. In order to determine the location of a node with respect to the face of a polygon, we use a coordinate transformation to determine both the location and the perpendicular distance.

We define the nodes of the convex polygon than encloses the obstacle in sequential order from 1 to n. Thus, the side of the polygon between vertices **1** and **2** is defined as the vector difference between the coordinates of the vertices. In order to determine if a vehicle lies within the projected rectangle emanating outward from the face, we first translate the coordinate frame to have the origin at vertex 1: $\boldsymbol{v}_1 = \boldsymbol{v} - \boldsymbol{\rho}_1$.

Next, we rotate the entire coordinate frame such that the polygon side $1 \rightarrow 2$ is lined up with the y-axis. Note that the angle of rotation is defined by the coordinates of the vertices such that:

$$\phi = \arctan \frac{2_x - 1_x}{2_y - 1_y} \qquad (17)$$

where 1_x is the x coordinate of the **1** vertex.

Using the trigonometric relationships between $\sin\phi$ and $\cos\phi$ and the coordinates, we can write the Rotation matrix as:

$$R_\phi = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (18)$$

where

$$\sin\phi = \frac{2_x - 1_x}{\sqrt{(2_y - 1_y)^2 + (2_x - 1_x)^2}} \quad (19)$$

$$\cos\phi = \frac{2_y - 1_y}{\sqrt{(2_y - 1_y)^2 + (2_x - 1_x)^2}} \quad (20)$$

After further manipulation, we arrive at the following relationship for the coordinates of the vehicle, v, expressed in the rotated and translated frame:

$$\boldsymbol{v}_{1\to2} = \frac{1}{\lambda}\begin{bmatrix}(2_y - 1_y)\,v_{1,x} - (2_x - 1_x)\,v_{1,y}\\(2_x - 1_x)\,v_{1,x} + (2_y - 1_y)\,v_{1,y}\end{bmatrix} \quad (21)$$

where $\lambda = \sqrt{(2_y - 1_y)^2 + (2_x - 1_x)^2}$, and is the length of the polygon face from $1 \to 2$. In order to find if the point is contained within the projection of the polygon face, we simply examine the y-coordinate of $\boldsymbol{v}_{1\to2}$:

$$0 \le v_y^{1\to2} \le \lambda \quad (22)$$

If $v_y^{1\to2}$ satisfies Eq. 22, then the vehicle is within the projected rectangle, and the perpendicular distance is found from the x-coordinate of Eq. 21

If the case of being within the projection of the face, d_k^{ob} is simply the length of the line perpendicular to and extending from the edge to i. In the case of the wedge, d_k^{ob} is the distance between the closest vertex and i. Variations in the functions themselves can have a large effect of the behavior of a flock. Changing Eq. 15 such that the denominator is cubed keeps the sign value while significantly changing the magnitude of repulsion from an obstacle:

$$\boldsymbol{F}_{ob} = K_{ob}/\left(\boldsymbol{d}_{ob}\right)^3 \quad (23)$$

5. VIRTUAL LEADER MOTION AND SWARM CONTROL

Up to this point we have discussed how the artificial potentials are determined, and how the resultant forces are computed. This will achieve a static flock formation, but will not result in useful motion of any kind. Motion of the flock is achieved by moving the virtual leader, which in turn drags the rest of the swarm. This is a great advantage of this methodology, as only a single trajectory need be planned: that of the virtual leader. The flocking methodology will take care of resolving the collision free paths for each member of the flock.

Movement of the group is achieved by advancing the virtual leader along a path in discreet steps. Every movement of the virtual leader causes a change in d_{VL} for each vehicle i, in turn changing F_{VL}. It is the total force acting on any given vehicle which dictates the nature of its movements.

$$F_{tot} = F_{i-1}^{ij} + F_{i+1}^{ij} + F_{VL} + F_1^{ob} + ... + F_n^{ob} \quad (24)$$

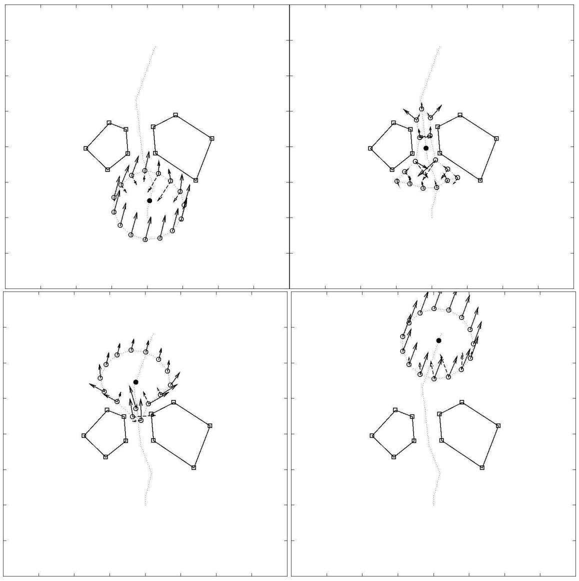

Fig. 3. A group of vehicles moving between 2 obstacles. Total force is shown as solid arrows, net obstacle force as dotted arrows.

Control of the flock is performed by proportional feedback, and is very suitable to distributed implementation. At each step, the net forces on each node are computed, and a displacement is calculated that is proportional to that displacement. The displacement is limited to a maximum at each step, which is important when dealing with obstacle fields that rise very quickly. Note that each node needs only the information about its own location, the location of the obstacles (defined by vertices), the location of its nearest neighbors, and the location of the virtual leader. This data, which presents very modest requirements on the intra-vehicle communications channel, represents a complete model required to compute the forces in this lightweight methodology, and thus the control.

Note also that the constants K_{ij}, K_{VL} and K_{ob} need not be time invariant; it may be desirable to give a certain obstacle a higher value of K_{ob} keeping vehicles further at bay. Within the structure of this approach there exists tremendous room for variation. In the artificial potential functions described in this paper, the ratio K_{ij} to K_{VL} to K_{ob} can have a massive effect on the behavior of the flock and its performance.

6. SIMULATION STUDIES

In order to test this lightweight flocking methodology, a simulation study was used to determine the effectiveness of this method for navigating a flock through an obstacle field. A MATLAB simulation was designed to move flocks of several sizes, ranging from 5 to 100 vehicles, through various numbers of obstacles. No attempt was made to simulate vehicle dynamics, nor were any simulations of obstacle detection capability included.

Various combinations of the spring constants, K_{ij}, K_{VL}, and K_{ob}, and proportionality gains for the feedback control were used in the simulations. Fig. 3 and Fig. 4 show the flock going through a set of obstacles. The base formation of the flock used in these simulations is the circular formation described in Section 3. The virtual leader is at the center of the circle.

Visual analysis of trial runs shows promising results. In all cases tested, the flock is able to navigate in such a way as to avoid collisions between the obstacles and the vehicles with each other. Indeed, visually, the metaphor of liquid flowing is shown to be very apt. Computationally, this methodology is very simple to implement, and requires very little in the way of intra-vehicle communication. These computational requirements are well within the capability of the embedded microcontrollers typical of a small autonomous vehicle.

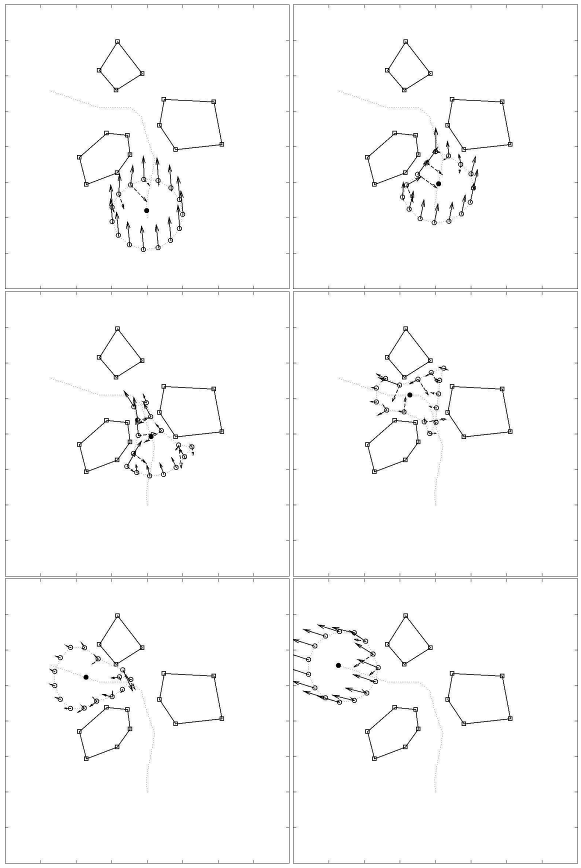

Fig. 4. A group of vehicles moving between 3 obstacles. Total force is shown as solid arrows, net obstacle force as dotted arrows.

7. CONCLUSIONS AND FUTURE WORK

This paper presents a computationally lightweight method of planning the path of multiple autonomous vehicles moving in a flock formation, using artificial potentials, virtual leaders, geometrical object modeling and proportional feedback in position computation. Control is distributed such that each vehicle determines its behavior based on low bandwidth information from the other vehicles. Because of the modest requirements for individual knowledge and simple rules governing motion, this approach is suitable for implementation in the embedded systems typical of small autonomous robots. Simulation studies, without dynamics taken into account, show it to be robust, even in complex environments. Current limitations in the framework include certain obstacle types that split the flock into sub-flocks. Group cohesion remains an issue of further exploration. Dynamics must be included in further simulations, as the vehicles cannot be made to jump instantaneously from one position to the next. Future work includes experimental demonstration of this framework a number of small autonomous vehicles. The challenge in this would be accurate obstacle modeling and real time communication of the position of the virtual leader.

REFERENCES

C. M. Breder. Equations descriptive of fish schools and other animal aggregations. *Ecology*, 35:361–370, 1954.

S. Levin G. Flierl, D. Grunbaum and D. Olson. From individuals to aggregations: The interplay between behavior and physics. *J. Theoret. Biol*, 196:397–454, 1999.

O. Khatib. Real-time obstacle avoidance for manipulators and mobile robots. *International Journal of Robotics Research*, 5:90–98, 1986.

J.C. Latombe. *Robot Motion Planning.* Kluwer Academic Publishers, Boston, USA, 1991.

N. Leonard and E. Friorelli. Virtual leaders, artificial potentials and coordinated control of groups. In *IEEE Conf. Decision Control*, Orlando, FL, 2001.

A. Okubo. aspects of animal grouping: Swarms, schools, flocks, and herds. *Adv. Biophys*, 22: 1–94, 1986.

R. Olfati-Saber and R. M. Murray. Distributed cooperative control of multiple vehicle formations using structural potential functions. In *IFAC World Congress*, Barcelona, Spain, July 2002.

C. Reynolds. Flocks, birds, and schools: A distributed behavioral model. *Comput. Graph.*, 21: 25–34, 1987.

K. Warburton and J. Lazarus. Tendency-distance models of social cohesion in animal groups. *J. Theoret. Biol*, 150:473–488, 1991.