REAL-TIME PATH PLANNING IN UNKNOWN ENVIRONMENTS USING A VIRTUAL HILL

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Abstract: The artificial potential field based path planning has been most wisely used for local path planning because it provides simple and efficient motion planners for practical purposes. However, this approach has a local minimum problem which can trap a robot before reaching its goal. The local minimum problem is sometimes inevitable when a mobile robot moves in unknown environments, because the robot cannot predict local minima before it detects obstacles forming the local minima. The avoidance of local minima has been an active research topic in the potential field based path planning. In this study, we propose a new concept using a virtual hill to escape local minima that occur in local path planning for a mobile robot. A virtual hill is located around local minimum to repel a robot from local minimum.

Keywords: mobile robot, path planning, artificial potential field, virtual hill, extra potential, navigation.

1. ITRODUCTION

Artificial potential field methods provide simple and effective motion planners for practical purposes (Lee and Park, 1991). These approaches have been widely applied to path planning of a mobile robot and a manipulator (Borenstein and Koren, 1989; Chuang, 1998; Chuang and Ahuja, 1998; Chuang et al., 2000; Guldner and Utkin, 1995; Haddad et al., 1998; Hwang and Ahuja, 1992; Tsai et al, 2001; Vadakkepat, 2001; Veelaert and Bogaerts, 1999). The applications of artificial potential field for obstacle avoidance was first developed by Khatib (Khatib, 1985; Khatib, 1986). This approach uses two types of potential, which are a repulsive potential field to force a robot away from obstacles or forbidden regions and an attractive potential field to drive the robot to its goal. The robot moves under the action of the artificial force which is proportional to the negative gradient of artificial potential. The robot is driven from the positions with the higher potential to that with the lower potential.

However, the path planning by the artificial potential field approach has a major problem, a local minimum problem, which can trap a robot before reaching its goal. The local minimum problem is sometimes unavoidable in local path planning, because the robot can detect only local information on obstacles. In other words, the robot cannot predict local minima before experiencing the environment. An avoidance of a local minimum has been an active research topic in potential field based path planning (Chang, 1996; Cho and Kwon, 1996; Connolly et al., 1990; Connolly, 1992; Janabi-Sharifi and Vinke, 1993; Kim and Khosla, 1992; Lee and Park, 1991; McFetridge and Yousef-Ibrahim, 1998; Park and Lee, 2003; Rimom and Koditschek, 1992; Volpe and Khosla, 1990). However, the previous solutions were limited to simple formations of obstacles or available for path planning in known environments.

In this research, a virtual hill concept is proposed as an idea to escape local minima. The virtual hill is located around the local minimum point to repel the robot from this point. This technique is useful for local path planning in unknown environments.

2. POTENTIAL THEORY

In this section, we review the attributes of the attractive potential function and the repulsive one adopted in this study. The attractive potential function used in this study is the conical well proposed by Andrews (Andrews, 1983). This function is quadratic within a given range and the value of the function increases linearly in the outer range. Therefore, it is adaptable for path planning in wide environments. The conical well is described by

$$U_{att} = \begin{cases} k_a d^2 & \text{if } d < d_0 \\ k_a (2 d_0 d - d_0^2) & \text{if } d > d_0 \end{cases} \quad (1)$$

where d_0 is the radius of the quadratic range, k_a is the proportional gain of the function and $d = \|\mathbf{p} - \mathbf{p}_{goal}\|$, where $\mathbf{p}$ is the position vector of the robot and $\mathbf{p}_{goal}$ is the position vector of the goal. The attractive force $\mathbf{F}_{att}$ is obtained by the negative gradient of the attractive potential:

$$\mathbf{F}_{att} = -\nabla U_{att} = \begin{cases} -2k_a (\mathbf{p} - \mathbf{p}_{goal}) & \text{if } d \le d_0 \\ -2k_a d_0 \dfrac{\mathbf{p} - \mathbf{p}_{goal}}{d} & \text{if } d > d_0 \end{cases} \quad (2)$$

The conical well provides a force with constant magnitude for distances larger than d_0 .

The second category of potential, the repulsive potential, is necessary to repel the robot away from obstacles that obstruct the robot's path of motion in the global attractive potential field. The repulsive potential functions have a limited range of influence to prevent an obstacle from affecting the motion of a robot when it is far away from the obstacle. The following repulsive potential function is the FIRAS function proposed by Khatib. This function uses the shortest distance to an obstacle as

$$U_{rep} = \begin{cases} \dfrac{1}{2} k_r \left(\dfrac{1}{r} - \dfrac{1}{r_0} \right)^2 & \text{if } r \le r_0 \\ 0 & \text{if } r > r_0 \end{cases} \quad (3)$$

where r_0 represents a potential field's distance limit of influence and is the shortest distance to an obstacle. The selection of the distance r_0 depends on the maximum speed of the robot and the control period. The repulsive force is driven as

$$\mathbf{F}_{rep} = -\nabla U_{rep} = \begin{cases} k_r \left(\dfrac{1}{r} - \dfrac{1}{r_0} \right) \dfrac{1}{r^2} \dfrac{\partial r}{\partial \mathbf{p}} & \text{if } r \le r_0 \\ 0 & \text{if } r > r_0 \end{cases} \quad (4)$$

where $\partial r / \partial \mathbf{p}$ can be represented as

$$\frac{\partial r}{\partial \mathbf{p}} = \left(\frac{\partial r}{\partial x} \; \frac{\partial r}{\partial y} \right)^T = \frac{\mathbf{p} - \mathbf{p}_{co}}{r} \quad (5)$$

where $\mathbf{p}_{co}$ is the position vector of the closest obstacle in the xy-coordinate system.
The total potential can be obtained by adding together the sum of the attractive potential and repulsive potential. The total force is obtained by the negative gradient of a global potential

$$\begin{aligned} \mathbf{F} &= -\nabla U \\ &= -\nabla U_{att} - \nabla U_{rep} \\ &= \mathbf{F}_{att} + \mathbf{F}_{rep} \, . \end{aligned} \quad (6)$$

3. NEW APPROACH TO ESCAPE LOCAL MINIMUM

3.1 Virtual Hill Concept

The local minimum problem is sometimes inevitable because the local minima cannot be predictive in unknown environment. A virtual hill is a new concept to escape local minima. In the conventional artificial potential field approach, a local minimum is formed when an attractive force is equal or similar to a repulsive force. A virtual hill has the role of repelling the robot from a local minimum. The virtual hill generates extra force instead of attractive force to repel the robot from a local minimum. The extra potential is designed not to generate new local minima and it makes possible that the robot escape a local minimum in complex environments.

The virtual hill approach is applied when the robot is trapped by a local minimum. The judgment whether a robot is trapped by a local minimum or not should be preceded before an application of a virtual hill method. In real-time path planning, the obstacles are detected by range sensors of a robot. The analytic searching of a local minimum requires intensive computational time and that is inadequate for real-time path planning. For fast judgment whether the robot is trapped by a local minimum or not, the following criterion is defined :
Local-minimum-criterion
When $t \ge T_a$, if $\|\mathbf{p}(t) - \mathbf{p}(t - T_a)\| \le S_a$ then the robot is trapped in a local minimum, where $\mathbf{p}$ represents the position vector of the robot, T_a is the time interval, and S_a is set to the minimum distance that the robot moves for T_a in the non-local minimum

condition. S_a is set to a very small value because the distance between $\mathbf{p}(t)$ and $\mathbf{p}(t-T_a)$ has a very small value when the robot is trapped in a local minimum.

When a robot is trapped by a local minimum, a virtual hill is generated and a robot moves by repulsive potential and extra potential until escaping local minimum area. As position of the virtual hill, the trapping obstacle is defined as the closest obstacle from the robot when the robot is just trapped by local minimum. The position vector of the trapping obstacle is denoted as $\mathbf{p}_{to}$. The trapping obstacle may have major influence on trapping a robot because the closest obstacle generates the largest repulsive force which is opposites to an attractive force.

3.2 Extra Potential

The extra potential is to overcome local minimum problems. That is defined as

$$U_{ext} = -k_{e1} y + k_{e2} r^2 \qquad (7)$$

where k_{e1} and k_{e2} are proportional gains and r is the distance between a robot and its closest obstacle. y is defined as the path integral :

$$y = \int_g dy \qquad (8)$$

where g is defined as a path-of-the-closest-obstacles which represents the trajectory of the closest obstacle when a robot moves in a local minimum area. g can be expressed as

$$g : \left\{ \mathbf{Q}\left(t^*\right) : t_0 \le t^* \le t \text{ and } t_0 \le t \le t_k \right\} \qquad (9)$$

where t_0 is the time at that a robot is just trapped by a local minimum, t_i is arbitrary time, and t_k represent the time at that the robot just escapes a local minimum area. $\mathbf{Q}$ represents the position vector of the closest obstacle. The closest obstacle $\mathbf{p}_{co}$ is discretely detected by range sensors and the continuous function $\mathbf{Q}(t)$ is interpolated from the detected obstacles as shown in Fig. 1. $\mathbf{Q}$ satisfies following equation :

$$\mathbf{Q}(t_i) = \mathbf{p}_{co}(t_i) \text{ where } i = 0,1,\cdots,K \qquad (10)$$

Then Ψ can be expressed as

$$y\left(\mathbf{p}(t)\right) = \int_g dy = \int_{t_0}^{t} \frac{dy}{dt} dt = \int_{t_0}^{t} \left\|\dot{\mathbf{Q}}\right\| dt \qquad (11)$$

where $\dot{\mathbf{Q}} = d\mathbf{Q}/dt$. $y(t)$ means the displacement between $\mathbf{Q}(t_0)$ and $\mathbf{Q}(t)$, which increases along

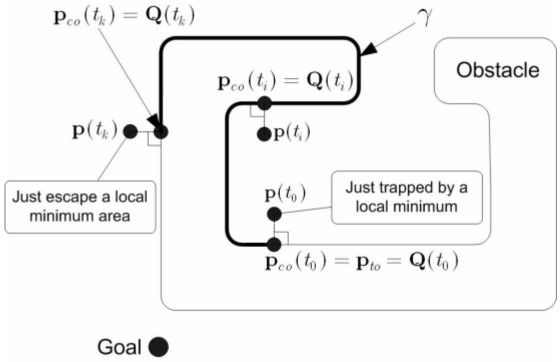

Fig. 1. Concept of g

g as shown in Fig. 1. y satisfies the following expressions :

1. $y(t_0) = 0$
2. if $t_j < t_i$ then $y(t_j) \le y(t_i)$.

$\mathbf{e}_t$ is defined as the tangent vector of g and it is expressed as

$$\mathbf{e}_t = \frac{\dot{\mathbf{Q}}}{\left\|\dot{\mathbf{Q}}\right\|} \qquad (12)$$

$\mathbf{e}_t$ satisfies (13)

$$\mathbf{e}_t = \mathbf{e}_n \times \mathbf{e}_b \qquad (13)$$

where $\mathbf{e}_n$ is the normal vector and $\mathbf{e}_b$ is binormal vector of g as shown in Fig 1.

In this study, it is assumed that the outline of obstacles is differentiable because the outline of real obstacles is continuous and a critical edge of obstacle may be smooth in a micro world. Consequently, we can assume that $\mathbf{Q}$ is differentiable except the start point and the final point because it is interpolated obstacle-line. If the path g is continuous and smooth at an arbitrary point B on contour g and a point A is an arbitrary point in the space, and a line $\overline{AB}$ is the closest line from A to g, $\overline{AB}$ is always a vertical line to g. Therefore, the line $\overline{\mathbf{pp}_{co}}$ may be always vertical from $\mathbf{Q}$ because $\mathbf{p}_{co}$ is the closest point on $\mathbf{Q}$. The normal vector can be expressed as

$$\mathbf{e}_n = \frac{\mathbf{p} - \mathbf{p}_{co}}{\left\|\mathbf{p} - \mathbf{p}_{co}\right\|} \qquad (14)$$

where $\mathbf{p}_{co}$ is detected by range sensors. In the path planning on a two-dimensional plane, $\mathbf{e}_n$ and $\mathbf{e}_t$ are always on the xy-plane, and then a binormal vector $\mathbf{e}_b$ is always on the k-axis. That means $\mathbf{e}_b = \mathbf{k}$ or $\mathbf{e}_b = -\mathbf{k}$. In Fig. 1, $\mathbf{e}_t$ is tangent vector of g and $\mathbf{e}_n$ is able to be obtained by (14) and then the direction of $\mathbf{e}_t$ is determined by $\mathbf{e}_b$. Therefore the direction of g is determined by $\mathbf{e}_b$ because $\mathbf{e}_t$ is the tangent vector of g. $\mathbf{e}_b$ is defined as

$$\mathbf{e}_b = \begin{cases} \mathbf{B}/\|\mathbf{B}\| & \text{if } \mathbf{B}/\|\mathbf{B}\| \neq 0 \\ \mathbf{k} & \text{if } \mathbf{B}/\|\mathbf{B}\| = 0 \end{cases} \quad (15)$$

where $\mathbf{B} \equiv (\mathbf{p}_{goal} - \mathbf{p}_{to}) \times (\mathbf{p}(t_0) - \mathbf{p}_{to})$. In the extra potential function, $-k_{e1}y$ term may drive a robot to direction of g because the robot moves from a high energy-position to a low energy-position in artificial potential approach. The conditional expression (15) is to set the direction of g to goal side.

The extra force is the negative gradient of extra potential. By the principle of superposition, the extra force can be expressed as

$$\mathbf{F}_{ext} = -\nabla U_{ext} = k_{e1} \nabla y - k_{e2} \nabla (r^2) \quad (16)$$

A robot can detect obstacles by range sensor but they cannot detect absolute outlines of obstacles. Therefore, the outline of obstacles should be estimated by discrete obstacle-points. In Fig. 1, $\mathbf{p}(t_0)$ is the position vector of a robot when the robot just trapped by local minimum, and then the position vector of the closest obstacle is expressed as $\mathbf{p}_{co}(t_0)$ or $\mathbf{p}_{to}$. Consequently, $\mathbf{p}_{co}(t_i)$ is the position vector of the closest obstacle from $\mathbf{p}(t_i)$. Then $y(\mathbf{p}(t_i))$ can be approximately expressed as

$$y(\mathbf{p}(t_i)) = \int_{t_0}^{t_i} \left\| \frac{d\mathbf{Q}}{dt} \right\| dt \approx \sum_{j=0}^{i} \|\Delta \mathbf{Q}_j\| \quad (17)$$

where $\|\Delta \mathbf{Q}_j\|$ is a displacement of g shown in Fig. 1. Then $\nabla y(\mathbf{p}(t_i))$ can be expressed as

$$\nabla y(\mathbf{p}(t_i)) \approx \nabla \left(\sum_{j=0}^{i} \|\Delta \mathbf{Q}_j\| \right) \quad (18)$$

By the definition of gradient and the principle of superposition, $\nabla \left(\sum_{j=0}^{i} \|\Delta \mathbf{Q}_j\| \right)$ can be expressed as

$$\begin{aligned} \nabla \left(\sum_{j=0}^{i} \|\Delta \mathbf{Q}_j\| \right) &= \frac{\partial}{\partial x} \left(\sum_{j=0}^{i} \|\Delta \mathbf{Q}_j\| \right) \mathbf{i} + \frac{\partial}{\partial y} \left(\sum_{j=0}^{i} \|\Delta \mathbf{Q}_j\| \right) \mathbf{j} \\ &= \frac{\partial}{\partial x} \left(\sum_{j=0}^{i-1} \|\Delta \mathbf{Q}_j\| \right) \mathbf{i} + \frac{\partial}{\partial x} (\|\Delta \mathbf{Q}_i\|) \mathbf{i} \\ &\quad + \frac{\partial}{\partial y} \left(\sum_{j=0}^{i-1} \|\Delta \mathbf{Q}_j\| \right) \mathbf{j} + \frac{\partial}{\partial y} (\|\Delta \mathbf{Q}_i\|) \mathbf{j} \end{aligned} \quad (19)$$

where $\frac{\partial}{\partial x} \left(\sum_{j=0}^{i-1} \|\Delta \mathbf{Q}_j\| \right) = 0$ and $\frac{\partial}{\partial y} \left(\sum_{j=0}^{i-1} \|\Delta \mathbf{Q}_j\| \right) = 0$ because $\Delta \mathbf{Q}_j$ is independent term with respect to the displacement of $\mathbf{Q}$ at ti, and then we obtain

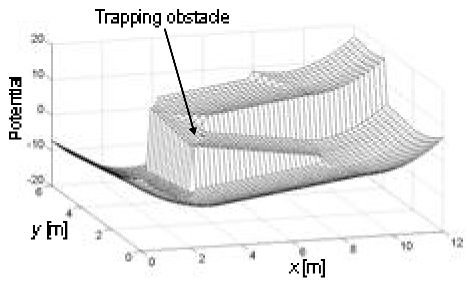

Fig. 2. Extra potential in closed aisle

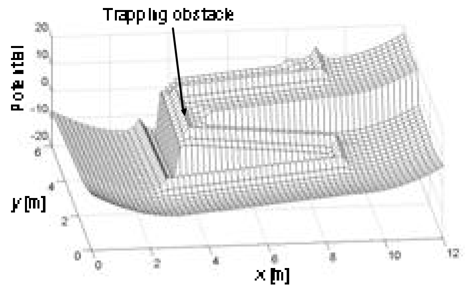

Fig. 3. Total potential with virtual hill

$$\nabla \left(\sum_{j=0}^{i} \|\Delta \mathbf{Q}_j\| \right) = \frac{\partial}{\partial x} (\|\Delta \mathbf{Q}_i\|) \mathbf{i} + \frac{\partial}{\partial y} (\|\Delta \mathbf{Q}_i\|) \mathbf{j} = \mathbf{e}_t \quad (20)$$

(16) can be obtained from Eqs. (16)-(20)

$$\nabla y = \mathbf{e}_t = \mathbf{e}_n \times \mathbf{e}_b \quad (21)$$

where, $\mathbf{e}_n$ and $\mathbf{e}_b$ are obtained by (14) and (15). Then we can simply get ∇y by (21) and that is very available for a real-time path planning. The $\nabla (r^2)$ term of the extra force is derived as

$$\nabla (r^2) = 2(\mathbf{p} - \mathbf{p}_{co}) = 2r\,\mathbf{e}_n \quad (22)$$

By (21) and (22), $\mathbf{F}_{ext}$ can be expressed as

$$\mathbf{F}_{ext} = -\nabla U_{ext} = k_{e1}\mathbf{e}_t - 2k_{e2} r\,\mathbf{e}_n \quad (23)$$

In (23), the extra force is composed of two parts which are $k_{e1}\mathbf{e}_t$ and $-2k_{e2} r\,\mathbf{e}_n$. First part is the force in direction of the tangent vector of g. Second part is in direction of the negative normal vector of g and it attract the robot to the closest obstacle. The second component of the force is to prevent the robot diverging too far from an effective path. The most strong point of the proposed extra potential is that it does not generate new local minimum. We proof that as follows :

After a robot is trapped by a local minimum, the robot moves under the action of the force generalized by a repulsive force and an extra force until the robot escapes the local minimum area. The total force is expressed as

$$\mathbf{F} = \mathbf{F}_{rep} + \mathbf{F}_{ext} \quad (24)$$

If $\mathbf{e}_n$ of (14) is substituted into a repulsive force, a repulsive force can be expressed as

$$\mathbf{F}_{rep} = \begin{cases} k_r \left(\dfrac{1}{r} - \dfrac{1}{r_0} \right) \dfrac{1}{r^2} \mathbf{e}_n & r \le r_0 \\ 0 & r > r_0 \end{cases} \quad (25)$$

Then the total force is given by

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_{rep} + \mathbf{F}_{ext} \\ &= \begin{cases} \left(k_{e1}\mathbf{e}_t + k_r \left(\dfrac{1}{r} - \dfrac{1}{r_0} \right) \dfrac{1}{r^2} - 2k_{e2} r \right) \mathbf{e}_n & r \le r_0 \\ k_{e1}\mathbf{e}_t - 2k_{e2} r \mathbf{e}_n & r > r_0 \end{cases} \quad (26) \end{aligned}$$

The local minimum is defined as the point at which the negative gradient of the artificial potential is zero. In other words, the artificial force is zero at a local minimum. On the other hand, in the total force of (26), $\mathbf{e}_t$ and $\mathbf{e}_n$ are independent each other and $k_{e1}\mathbf{e}_t \neq 0$ because $\mathbf{e}_t$ is unit vector and k_{e1} is not zero. Therefore, the totoal force $\mathbf{F}$ is always not zero. A local minimum is the point which satisfy $\mathbf{F} = 0$. Hence, the generalized potential always has not local minima. Fig. 2 shows the formation of the extra potential field with respect to a closed aisle and Fig. 3 shows the total potential.

4. EXPERIMENTS

We performed various experiments to evaluate the virtual hill approach. It is assumed that a robot initially does not have any information on the environment, and it can detect obstacles up to 1.5m from itself. The values of parameters are adequately set by a trial and error method. In Fig. 4, the obstacle has the shape of a closed aisle. Therefore, the robot is trapped in a local minimum. In the same environment, Fig. 5 shows that the robot can escape the local minimum by the virtual hill approach, and it can successfully reach its goal. These results of the experiments show that this technique is useful for concave obstacles and for deep aisle-shaped obstacles. The robot can successfully reach its goal by the virtual hill approach, as shown in Figs. 6-8. To evaluate the generality of the proposed algorithm, experiments are done in several environments. The results of the experiments show that the proposed path planner has good generality.

5. CONCLUSIONS

In this study, we proposed the virtual hill concept to escape local minimums in local path planning based on the artificial potential field approach. The virtual hill with the extra potential is located at the trapping obstacle when robot is trapped in a local minimum. The extra potential is added to the global potential, and it repels the robot from the local minimum.

The extra potential is designed not to have any new local minima. The concept of a new path g is newly proposed. g represents the path of the detected closest obstacles after a robot is trapped by a local minimum. The extra potential function has the term

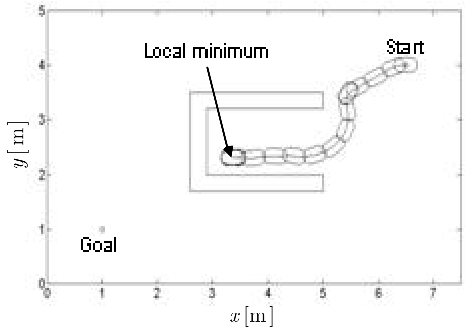

Fig. 4. Experiment 1

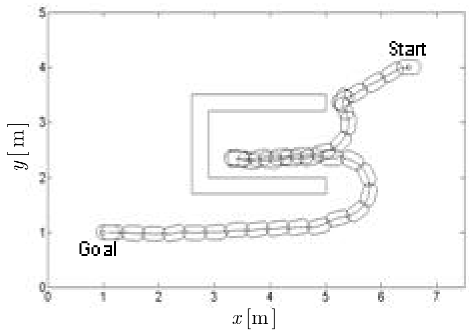

Fig. 5. Experiment 2

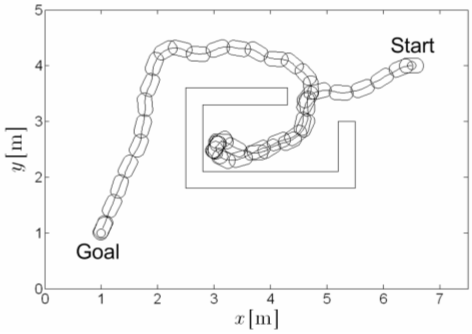

Fig. 6. Experiment 3

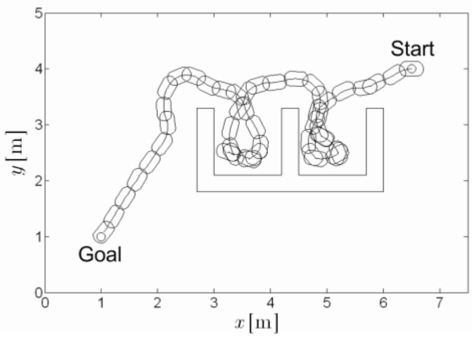

Fig. 7. Experiment 4

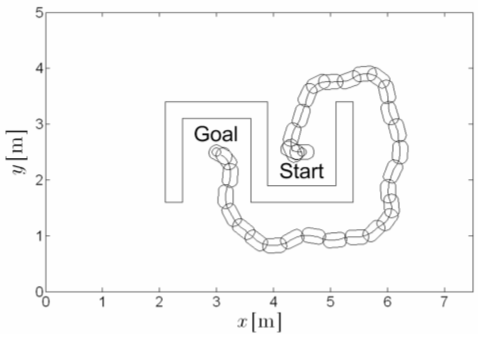

Fig. 8. Experiment 5

of y which is defined as the path integral of g. In real-time navigation, obstacles may be continuously detected by a robot and then y will continuously increase with respect to time. The extra potential is designed to be inversely proportional to y. Therefore, this potential field dose not have a local minimum because the potential continuously changes with respect to time. The results of the experiments and experiments in the various environments showed that virtual hill approach did not generate a local minimum and the robot could successfully escape a local minimum area.

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