FUNCTIONAL ANALYSIS AND T-S FUZZY SYSTEM DESIGN

D. T. Pham and R. X. Qiu

Manufacturing Engineering Centre, Cardiff University, CF24 0YF, U.K.
Email Address QiuR@cf.ac.uk

Abstract: This paper introduces the idea of viewing fuzzy modelling and fuzzy system design from a functional analysis perspective. Fuzzy transforms, which are based on the generalised Fourier transform in functional analysis, are proposed. It is demonstrated that, mathematically, a T-S fuzzy model is equivalent to a fuzzy transform. Hence the parameters of a T-S fuzzy system can be identified by solving equations constructed using the inner product between membership functions and a given target function. The functional point of view leads to an insight into the behaviour of a fuzzy system. It provides a theoretical basis for exploring improvements to the efficiency of T-S fuzzy modelling.

Keywords: Fuzzy hybrid systems, Fuzzy inference, Fuzzy logic, Fuzzy modelling, Function approximation, Transforms

1. INTRODUCTION

This paper focuses on the possibility of applying functional analysis theory to the design of fuzzy systems. Functional analysis is the mathematician's "black-box diagram" (Curtain and Pritchard 1977). It was developed to deal with general functions instead of specific values. The motivation of applying functional analysis to fuzzy systems is twofold. First, as an exact mathematical analytical method, functional analysis can handle inexact data and knowledge. Second, the simple notions in functional analysis avoid many of the complicated details in design and analysis, highlighting only the essential aspects. Functional analysis is a convenient way to examine the behaviours of various models, including fuzzy models.

Based on functional analysis theory, a functional point of view for fuzzy system design will be developed. From a functional point of view, fuzzy system modelling consists of two parts:

- Searching for an optimum basis (membership functions)
- Measuring the minimal distance between the given basis and the target function

This is an iterative process. If the target function is covered by the space spanned by the given basis, the distance between the basis and the target function is zero. If this is not the case, a minimal distance can be derived from fuzzy transform action. Based on the distance, further optimisation of the basis is performed iteratively until a basis that is close enough to the target function is obtained. Fuzzy system design involves a combination of "soft-computing" and "hard-computing". The first part of fuzzy system design, "searching for an optimum basis", is a soft-computing activity. Any suitable optimisation techniques, such as genetic algorithms and Tabu search can be applied. The second part of fuzzy system design, "measuring the minimal distance", can be regarded as "hard-computing". The task can be completed analytically. The most commonly used method is the least-squares method. Therefore, in order to outperform modern fuzzy system design approaches, next generation fuzzy system design needs to be enhanced in both the "soft-computing" direction and the "hard-computing" direction. Areas for improvement includes:

- Reducing the search space and choosing better initial conditions
- Increasing the efficiency of the minimal distance calculation

Within the last decade, the "soft-computing" aspect of fuzzy system design has received considerable attention, with various neuro-fuzzy approaches having been proposed. On the other hand, since the introduction of the least-squares method for fuzzy system design in the early 1980s, the "hard-computing" aspect of fuzzy modelling has been almost untouched. This paper presents a new "hard-computing" algorithm for fuzzy system design. The algorithm is based on the generalised Fourier transform. It will be demonstrated that instead of using the least-squares method, the minimal distance can be derived by fuzzy transform action. This facilitates the determination of optimum designs. The paper is organised as follows. The functional perspective of fuzzy systems and the analogy between fuzzy system design and functional analysis are explored in section 2. In section 3, the concept of fuzzy transforms is proposed based on the generalised Fourier transform and examples are provided to illustrate the proposed concept.

2. FUNCTIONAL PERSPECTIVE OF FUZZY SYSTEMS

The treatment of imprecision and vagueness can be traced back to the work of Lukasiewicz in the 1930s (Bonissone et al 1999). Zadeh proposed the theory of fuzzy sets (Zadeh, 1965), which provided a systematic way to deal with ambiguous and ill-defined concepts. Based on fuzzy sets theory, fuzzy systems were developed. In the last three decades, considerable attention has been paid to research into fuzzy systems, especially into the theory of fuzzy sets. Researchers believe that fuzzy sets may lead to a better understanding of fuzzy systems and possibly more advanced fuzzy computing approaches. Thousands of papers have been published in this area and fuzzy sets theory has become a respected part of science. As fuzzy sets theory grew in popularity, various fuzzy inference mechanisms were proposed for different application areas. Among them, Mamdani's fuzzy model and the T-S fuzzy model have been widely accepted in fuzzy system design. It is interesting to note that despite the popularity of fuzzy sets theory, only fuzzy models based on simple fuzzy sets theory have been commonly implemented. In particular, for the T-S fuzzy model, little fuzzy sets theory is involved. This indicates that there is a gap between modern fuzzy sets theory and its practice in fuzzy system design. People may argue that since fuzzy systems mimic human thinking, their applications should be straightforward and easy to understand. However, when fuzzy systems are employed for more sophisticated problems, current theory turns out to be inadequate. Innovation in fuzzy theory is required to address challenges such as the construction of fuzzy systems for complicated systems from input/output data. During the last decade, a number of researchers tried to explain the behaviour of fuzzy systems using input-output based mathematical descriptions. They have carried out studies with titles such as "The universal approximation ability of fuzzy system based on fuzzy basis functions", "Functional equivalent between some fuzzy systems and the Radial Basis Function Networks", and "Fuzzy PID controller is equivalent to multilevel relay and a local nonlinear proportional-integral controllers" (Ying, 1993), (Wang and Mendel, 1993), (Jang and Sun 1993), (Kosko, 1997). As opposed to the inference viewpoint commonly accepted in the fuzzy systems community, these studies were based on a functional point of view, regarding the fuzzy system as some kind of function. Their success provides a new understanding of fuzzy system behaviours. However, because a complete input-output based mathematical description of fuzzy system design is not yet available, the results of the above studies have appeared individually, and have not been applied to fuzzy system design directly.

According to Zadeh (1997), there are three basic concepts in human cognition: granulation, causation and organisation. Informally, granulation involves the decomposition of a whole into parts, organisation, the integration of parts into a whole, and causation, the association of a cause with effects. In a fuzzy system, these concepts are implemented in the following way: granulation is achieved by converting crisp values into fuzzy values, organisation is represented by the process of converting local information into global information, and causation is implemented by rules and the inference mechanism. For example, in the Mamdani fuzzy model, granulation and organisation are expressed through fuzzification and defuzzification, respectively. Causation is performed in the so-called compositional rule of inference (CRI). From the logician's point of view, this is a balanced process with all the components in a fuzzy system having their own role. Together, fuzzy values, rules and the inference mechanism, construct a "set-to-set" mapping. They successfully avoid the drawbacks of "point-to-point" mappings when dealing with ambiguity and uncertainty.

If inexact data are regarded as exact signals plus random noise, mathematically, in fuzzy system modelling, the noise is eliminated by the membership functions. This is because the convex-shaped membership functions are perfect low pass filters. The ability of fuzzy systems to deal with uncertainty is fully dependent on the size of the support of membership functions as shown later in the paper. At the same time, membership functions also construct a function space for implementing further approximation. The membership functions become the basis of the space. The role of the remaining parts of the fuzzy system is to approximate the fuzzy system to the signals, which have been extracted from inexact data, as accurately as possible. This process can be regarded as a transform action between the function space constructed by the membership functions and the target system. Since any inexactitude and uncertainty in the input/output data pairs will have been filtered by membership functions, the term "as accurately as possible" has its normal meaning in mathematical transform theory. This viewpoint is somewhat different from the inference viewpoint that is commonly accepted in the fuzzy theory literature. Because it is very close to the concept of transforms in functional analysis, this will be called the functional viewpoint, and the transform will be called the fuzzy transform. It will be demonstrated that this new point of view provides a better explanation of the input-output behaviour of fuzzy systems. From this point of view, the best approximation comes from a combination of rules, inference mechanisms and local information in the output space that gives the closest transform from the space constructed by input membership functions to the target function. The universal approximation ability of a fuzzy system is fully dependent on the completeness of the membership functional space, which is determined by its membership functions. It can be proved that if the number of membership functions is not limited, a complete space that covers any target function can be constructed. This is the principle behind the universal approximation ability of a fuzzy system.

The functional point of view leads to an insight into the behaviours of a fuzzy system. Some important results in fuzzy theory, such as the universal approximation ability of fuzzy systems and the functional equivalence between fuzzy systems and Radial Basis Function networks, become clearer from this point of view. One important reason for the gap between modern fuzzy sets theory and fuzzy modelling practice is that the logician's viewpoint fails to realise the difference between the roles of the components in a fuzzy system. In practice, this may generate great difficulties for fuzzy modelling.

3. FUNCTIONAL ANALYSIS FOR FUZZY SYSTEM DESIGN

3.1 *General Formula of Mathematical Transforms*

In the previous section, the idea of a functional viewpoint for fuzzy system design has been explained. A complete fuzzy system design theory based on functional analysis will be presented in this section. Given $i \in \{0,\cdots,n\}$, a basis $\{\phi_i\}$ and a set of parameters $c_i^* \in C$, a function $S^* = \sum_{i=0}^{n} c_i^* \phi_i$ is the best approximation of f, if and only if $f - S^*$ is orthogonal to ϕ_i. That is $< f - S^*, \phi_i >= 0$ and $i \in \{0,\cdots,n\}$. If $f(x)$ is best approximated by $S^* = \sum_{i=0}^{n} c_i^* \phi_i$, for any $S = \sum_{i=0}^{n} c_i \phi_i$ (formed by unknown parameters $c_i \in C$ and the basis $\{\phi_i\}$), the following holds:

$$\int_a^b \rho(x)[f(x) - S^*]^2 dx \le \int_a^b \rho(x)[f(x) - S]^2 dx \tag{1}$$

This means the length of $f - S$ reaches its minimum value when $S = S^*$. $S^* = \sum_{i=0}^{n} c_i^* \phi_i$ is called the generalised Fourier transform of $f(x)$ in functional analysis. Let:

$$I(c_0, c_1, \cdots, c_n) = \int_a^b \rho(x)[f(x) - S]^2 dx$$
$$= \int_a^b \rho(x)[f(x) - \sum_{i=0}^{n} c_i \phi_i]^2 dx \tag{2}$$

I in Equation (2) is a linear function of $(c_0, c_1, \cdots, c_n)$. To find the best approximation of $f(x)$ in function space constructed by $\{\phi_i\}$, the minimal value for $I(c_0, c_1, \cdots, c_n)$ needs to be calculated, from

$$\frac{\partial I}{\partial c_k} = -2\int_a^b \rho(x)[f(x) - \sum_{i=0}^{n} c_i \phi_i]\phi_k dx = 0$$
$$k \in \{0,\cdots,n\} \tag{3}$$

$$\sum_{i=0}^{n} (\int_a^b \rho(x)\phi_i \phi_k dx) c_i = \int_a^b \rho(x) f(x) \phi_k(x) dx$$
$$k \in \{0,\cdots,n\} \tag{4}$$

Equation (4) can be rewritten in an inner product form

$$\sum_{i=0}^{n} < \phi_k, \phi_i > c_i = < f, \phi_k > \quad k \in \{0,\cdots,n\} \tag{5}$$

For a normalised orthogonal basis, $< \phi_i, \phi_j >= k\delta_{i,j}$ and $< \phi_i, \phi_i >=< \phi_j, \phi_j >$. The solution of Equation (5) is:

$$c_i = < \phi_i, f > / < \phi_i, \phi_i > \tag{6}$$

Equation (6) is the general formula for mathematical transformation in functional analysis. Well-known transforms, such as Fourier Transform, Gobor Transform and some orthogonal wavelet transforms, can all be derived from Equation (6).

3.2 *Fuzzy Modelling and Fuzzy Transform*

Consider a simple zero-order T-S fuzzy system.

If Z is $A_i(Z)$ Then $y_i = a_i$

⋮

and $y = \sum_{i}^{p} y_i \cdot \phi_i(Z) = \sum_{i}^{p} a_i \cdot \phi_i(Z)$

where $i \in \{1,\cdots,p\}$, p is the number of rules, $Z \in R^n$, $y \in R$, $y_i \in R$, a_i is a fuzzy singleton and

$\phi_i(Z) = A_i(Z) \Big/ \sum_j^p A_j(Z)$. From given input/output data pairs $\{Z_j, y_j\}$, $j \in \{1 \cdots m\}$, a fuzzy system can be constructed using the above T-S model by applying a hybrid learning method, for examples ANFIS(Jang, 1993). The parameters $\{a_i\}$ in the fuzzy model are usually identified by using a least-squares estimator (Takagi and Sugeno, 1985) as:

$$\begin{bmatrix} a_1 & \cdots & a_p \end{bmatrix}^T = (A^T A)^{-1} A^T Y \tag{7}$$

where:

$$A = \begin{bmatrix} \phi_1(Z_1) & \cdots & \phi_p(Z_1) \\ \vdots & & \vdots \\ \phi_1(Z_m) & \cdots & \phi_p(Z_m) \end{bmatrix}_{mxp} \tag{8}$$

In order to identify the best parameters for the T-S model, the least-squares method has to calculate $(A^T A)$ and the inverse of $(A^T A)$. When p and m are large, the calculation can be very lengthy. Furthermore, in a hybrid learning process, the calculation needs to be performed repeatedly and this can make the fuzzy modelling process very slow. To solve this problem, alternative approaches are developed in this work. As shown previously, from functional analysis, the best approximation from the basis $\{\phi_i\}$ to target function $f(Z)$ is the generalised Fourier transform, $\hat{y} = \sum_i^p \hat{a}_i \cdot \phi_i(Z)$, which satisfies

$$< f(Z) - \hat{y}, \phi_i >= 0, \qquad i \in \{1 \cdots p\} \tag{9}$$

where $<,>$ denotes an inner product. Equation (9) indicates that the error function $f - \hat{y}$ is orthogonal to all of the basis vectors ϕ_i. Then the distance from basis $\{\phi_i\}$ to target function f is minimised. As designing a fuzzy system is equivalent to deriving a transform from membership functions to target function, the task of fuzzy system design can be equated to that of finding a generalised Fourier transform. To distinguish the generalised Fourier transform $\hat{y} = \sum_i^p \hat{a}_i \cdot \phi_i(Z)$ for fuzzy systems from other types of transforms in computational harmonic analysis, the transform is called the fuzzy transform in this case. Equation (9) can be written as

$$\sum_{i=0}^p < \phi_k, \phi_i > \hat{a}_i = < f(Z), \phi_k > \quad k \in \{1 \cdots p\} \tag{10}$$

which is:

$$A * a = B \tag{11}$$

where $a = \begin{bmatrix} \hat{a}_1 & \cdots & \hat{a}_p \end{bmatrix}^T$

$$A = \begin{bmatrix} <\phi_1, \phi_1> & \cdots & <\phi_1, \phi_p> \\ \vdots & <\phi_i, \phi_j> & \vdots \\ <\phi_p, \phi_1> & \cdots & <\phi_p, \phi_p> \end{bmatrix}$$

$$B = \begin{bmatrix} <\phi_1, f(Z)> & \cdots & <\phi_p, f(Z)> \end{bmatrix}^T$$

$\{\phi_i\}$ denote normalised membership functions.

Since matrix A represents the relationships between membership functions, it is called a membership relational matrix. Matrix B reflects the interaction between the membership functions and the target function; it is called an interaction matrix. The solution of equation (11) can be used to construct the fuzzy transform; $\hat{a}_i$ is called a coefficient of the fuzzy transform. Unlike other transforms in computational harmonic analysis (such as the Fourier transform and Gabor transform), the fuzzy transform cannot be expressed explicitly due to the non-orthogonal basis. These non-orthogonal basis vectors reflect the lack of sharp boundaries between fuzzy sets in fuzzy systems. A unique property of the fuzzy transform is that the basis in the transform is not fixed but varies from one application to another. They need to be identified in some training process or using the experience of the operator. This makes the fuzzy transform a very flexible technique in function approximation. The fuzzy transform provides a mathematical description for the principle of fuzzy system design. A fundamental problem in fuzzy system design is to find the optimum basis (membership functions). When a basis has been chosen, an optimum fuzzy system is constructed by computing the fuzzy transform as the inner product of the basis and the target function. It is easy to verify that $<\phi_i, \phi_j> = <\phi_j, \phi_i>$ and $<\phi_i, \phi_j> = 0$ when ϕ_i and ϕ_j do not have common support. Therefore, the membership relational matrix A is symmetrical and only those members close to the diagonal are not zero. This simplifies the solution of Equation (11).

EXAMPLE 1

For $n+1$ normalised triangular membership functions as illustrated in Figure 1, the membership relational matrix is:

$$A = \begin{bmatrix} <\phi_1, \phi_1> & \cdots & <\phi_1, \phi_p> \\ \vdots & <\phi_i, \phi_j> & \vdots \\ <\phi_p, \phi_1> & \cdots & <\phi_p, \phi_p> \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{3}v_0 & \frac{1}{6}v_0 & 0 & & \cdots & & 0 \\ \frac{1}{6}v_0 & \frac{1}{3}(v_1+v_0) & \frac{1}{6}v_1 & 0 & & & \\ & \ddots & \ddots & \ddots & & & \vdots \\ & 0 & \frac{1}{6}v_{i-1} & \frac{1}{3}(v_i+v_{i-1}) & \frac{1}{6}v_i & 0 & \\ \vdots & & & \ddots & \ddots & \ddots & \\ & & & 0 & \frac{1}{6}v_{n-2} & \frac{1}{3}(v_{n-1}+v_{n-2}) & \frac{1}{6}v_{n-1} \\ 0 & & \cdots & & 0 & \frac{1}{6}(v_{n-1}) & \frac{1}{3}(v_{n-1}) \end{bmatrix} \tag{12}$$

where $v_i = C_{i+1} - C_i$ and C_i is the centre of the i^{th} membership function. This is because

$$<\phi_i, \phi_i> = \int_{C_{i-1}}^{C_i} \left(\frac{x - C_{i-1}}{v_{i-1}} \right)^2 dx + \int_{C_i}^{C_{i+1}} \left(\frac{x - C_{i+1}}{-v_i} \right)^2 dx$$

$$= \frac{1}{3}\left(C_{i+1} - C_{i-1}\right) = \frac{1}{3}\left(v_i + v_{i-1}\right)$$

$$< \phi_i, \phi_{i+1} >= \int_{C_i}^{C_{i+1}} \left(\frac{x - C_{i+1}}{C_i - C_{i+1}} \right)\left(\frac{x - C_i}{C_{i+1} - C_i} \right) dx$$

$$= \frac{1}{6}\left(C_{i+1} - C_i\right) = \frac{1}{6} v_i$$

Example 1 indicates that, given the type of membership functions (triangular, Gaussian etc.), the form of the membership relational matrix can be predetermined.

EXAMPLE 2

Approximate $y = \cos\left(1.5\pi \cdot t\right)$, $t \in [0,1]$, using a zero-order T-S model:

If t is $u_1(t)$ Then y is a_1

If t is $u_2(t)$ Then y is a_2

If t is $u_3(t)$ Then y is a_3

and $y = \sum_i^3 a_i \cdot u_i(t)$

Normalised triangular membership functions are used in this example, which are illustrated in figure 2. Four parameters need to be identified in the fuzzy model. They are a_1, a_2, a_3 and the centre of middle membership function C. When C is given, the combination of a_1, a_2, a_3 that gives the best approximation to the target function can be derived by using the fuzzy transform action Equation (11). Therefore, the approximation problem is converted to an optimisation problem with only one unknown variable, C. Based on equation (12), the membership relational matrix and the interaction matrix are:

$$A = \begin{bmatrix} \frac{1}{3}C & \frac{1}{6}C & 0 \\ \frac{1}{6}C & \frac{1}{3} & \frac{1}{6}(1-C) \\ 0 & \frac{1}{6}(1-C) & \frac{1}{3}(1-C) \end{bmatrix}$$

$$B = \begin{bmatrix} < u_1, \cos(1.5\pi \cdot t) > \\ < u_2, \cos(1.5\pi \cdot t) > \\ < u_3, \cos(1.5\pi \cdot t) > \end{bmatrix}$$

Given the form of the membership relational matrix A and the interaction matrix B, theoretically an optimum value of C can be obtained analytically. However, this is not the aim of fuzzy transform action. The idea behind fuzzy transform action is not to eliminate "soft-computing" activities in fuzzy system design, but to seek an optimal balance between "soft" and "hard" computing.

Iteration	C	*Error*	a_1	a_2	a_3	**Step**
1	0.5	0.0339	1.362	-0.880	-0.451	0.1
2	0.6	0.01	1.280	-1.096	-0.240	0.1
3	0.7	0.095	1.164	-1.232	-0.023	0.1
4	0.65	0.063	1.226	-1.175	-0.132	0.05
Results	0.65	0.063	1.226	-1.175	-0.132	

In order to find the best combination of parameters, the gradient descent method is applied to search for the value of C. For the initial condition C =0.5 and step=0.1, the following result are obtained. The best approximation is achieved when C is close to 0.65. This is illustrated in figure 2.

It should be mentioned that when the support of membership functions is sufficiently wide, the fuzzy transform is not sensitive to noise. Let $g(Z)$ denote the input/output data pairs. The data consists of the target function $f(Z)$ plus white noise $n(Z)$:

$g(Z) = f(Z) + n(Z)$.

Let D represent the support of a membership function $u(Z)$. It is easy to verify that for white noise $n(Z)$ when D is large enough.

$$\int_D \left[u(Z) \cdot n(Z)\right] dZ = 0 \quad \text{Then}$$

$$< u(Z), g(Z) >= \int_D \left[u(Z) \cdot g(Z)\right] dZ$$

$$= \int_D \left[u(Z) \cdot (f(Z) + n(Z))\right] dZ$$

$$= \int_D \left[u(Z) \cdot f(Z)\right] dZ =< u(Z), f(Z) >$$

which shows that the fuzzy transform is not sensitive to noise $n(Z)$. Noise filtering is illustrated in figure 4.

It should also be mentioned that the sum and product used in Equation (11) can be replaced by other triangular operators. When different norms and co-norms are adopted in fuzzy inferencing (Max-Min, Max-Product, Sum-Product etc.), only the way in which the equations are solved is changed.

3.3 *Higher-Order Approximation*

The fuzzy transform can be easily extended to higher-order approximation. Consider a first-order T-S fuzzy system:

If Z is $A_i(Z)$ Then y_i is $q_i Z + r_i$

$\vdots$

and $y = \sum_i^p y_i \cdot u_i(Z) = \sum_i^p (q_i Z + r_i) \cdot u_i(Z)$

where $i \in \{1 \cdots p\}$, p is the number of rules, $Z \in R^n$, $y \in R$, $q_i \in R^n$ $r_i \in R$ and $u_i(Z) = A_i(Z) \Big/ \sum_j^p A_j(Z)$

It can be verified that the coefficients of the fuzzy transform are in the form

$$\begin{bmatrix} V_{11} & \cdots & V_{1p} \\ \vdots & V_{ii} & \vdots \\ V_{p1} & \cdots & V_{pp} \end{bmatrix} \cdot \begin{bmatrix} l_1 \\ l_i \\ l_p \end{bmatrix} = \begin{bmatrix} W_1 \\ W_i \\ W_p \end{bmatrix} \qquad (13)$$

where

$$V_{i,j} = \begin{bmatrix} <u_i, u_j> & <u_i, u_j z> \\ <u_i z, u_j> & <u_i z, u_j z> \end{bmatrix},$$

$$l_i = \begin{bmatrix} q_i \\ r_i \end{bmatrix} \text{ and } W_i = \begin{bmatrix} <f, u_i> \\ <f, u_i z> \end{bmatrix}$$

The advantage of higher-order approximation is that it improves the approximation ability of a fuzzy system without increasing the search space. For the same number of membership functions (the same search space), higher-order approximation adds new basis vectors to the membership functional space. The basis in membership functional space changes from $\{u_i\}$ to $\{u_i, u_i Z\}$, and the coverage of the membership functional space increases accordingly.

The membership relational matrix A of Equation (13) is symmetrical and only those members close to the diagonal of matrixes are not zero. Again, this kind of equation can be solved efficiently by applying Cholesky's algorithm (Press, Flannery 1993).

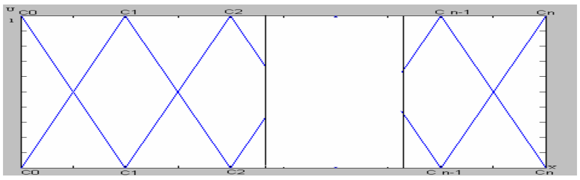

Fig.1. Normalised triangular membership functions

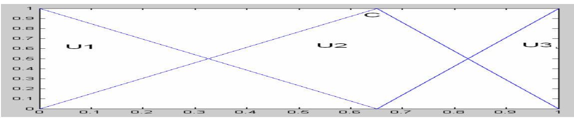

Fig.2. Membership functions in example 2

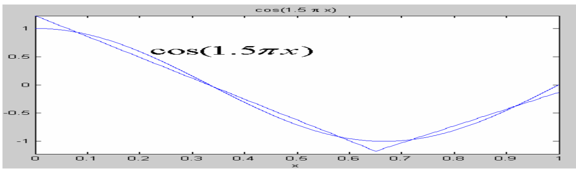

Fig.3.Target function and the approximation result in example 2

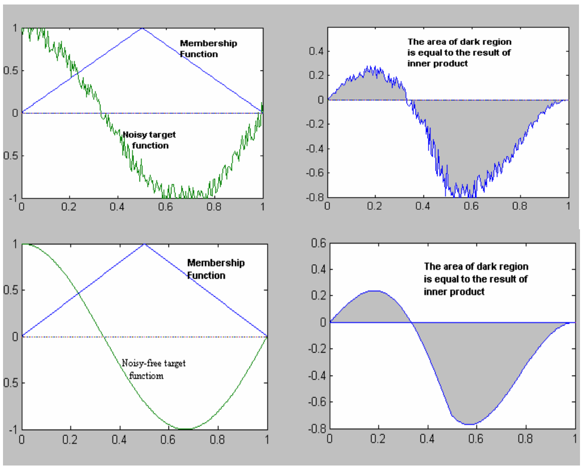

Fig.4. Inner product filters out the noise in the target function. The result of the inner product is equal to the area of the dark region.

4. SUMMARY

This paper has introduced the idea of viewing fuzzy modelling and fuzzy system design from a functional analysis perspective. The paper has described fuzzy transforms, which are based on the generalised Fourier transform in functional analysis, and showed how fuzzy transforms can be applied to improve the efficiency of fuzzy modelling through predetermined membership relational matrices and Cholesky algorithm.

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