SELF-TUNING NEURO-FUZZY GENERALIZED MINIMUM VARIANCE CONTROLLER

Sergio E. Pinto Castillo[1], Mike J. Grimble[2] and Reza Katebi[2]

[1,2] Industrial Control Centre, University of Strathclyde,
Graham Hills Building, 50 George Street,
Glasgow, G1 1QE, UK
[1] Engineering Department, Autonomous University of Tlaxcala,
Calzada Apizaquito S/N, Apizaco, Tlaxcala, MX

Abstract: The development of a Self-Tuning Neuro-Fuzzy Generalized Minimum Variance (GMV) controller is described. It uses fuzzy expert knowledge of the dynamic weightings to meet desired closed-loop stability and performance requirements. The controller is formulated in a polynomial system approach mixed with a Neuro-Fuzzy model and Fuzzy Self-Tuning mechanism. The proposed method is applied to a model of the Continuous Stirred Tank Reactor with Cooling Jacket and is compared with a PI controller, GMV controller with the correct model and a Fuzzy-PI controller. Simulation results are presented to demonstrate the performance of the proposed method.

Keywords: Self-Tuning Control, Neuro-Fuzzy Modeling, Nonlinear Control.

1. INTRODUCTION

The polynomial approach was used to develop the Minimum Variance (MV) controller in the 60's. This included a colored noise disturbance signal and was suitable in different forms for minimum and non-minimum phase systems (Åström , 1979). The GMV controller including an additional costing term of the control signal was developed by Clarke and Hastings-James (Hastings-James, 1970; Clark and Hastings-James, 1971). Clark and Gawthrop introduced the GMV Self-Tuning controller which took advantage of the characteristics above mentioned (Clark and Gawthrop, 1975). Grimble used a GMV structure in order to control nonminimum phase systems in (Grimble, 1981). Additionally, Grimble designed linear systems using GMV control laws (Grimble, 1988). This was followed by the Generalized H_∞ controller obtaining a dynamic costing solution using the dynamic cost weightings (Grimble, 1994).

Up to this point, all the algorithms were applied to stochastic linear systems in the discrete time domain. Founded on these ideas, Grimble developed the GMV controller for nonlinear multivariable processes with time varying properties (Grimble, 2003). This had the structure of the Smith Predictor and therefore could be called a Nonlinear Smith Predictor.

Fuzzy Logic has been applied in many areas with success. It is a qualitative representation of the natural process using fuzzy rules (Pinto, 2001b). Fuzzy identification uses also fuzzy rules to identify the objective model. This model has been used to control plants using adaptive Neuro-Fuzzy (NF) controllers and models (Jang , et. all., 1997; Babuška, 1998). The Tuning of controllers always represent a challenge because it is usually heuristic and trail-error based. The fuzzy tuning mechanism has been reported before in some previous works in order to tune several types of controllers (He et all. , 1993; Molengraft, 1995; Wang , 1997; Mudi and Nikhil ,1999; Pinto ,2001a; Babuška et. all., 2002). The advantages of these structures are the capacity to adapt the parameters of the controller to the changes of the parameters of the plant and also save time in the process of tuning the controller.

The main contribution of this article is to design and apply the Self-Tuning Neuro-Fuzzy GMV (STNFGMV), which uses the fuzzy expert knowledge of dynamical weightings (Error Weighting and Control Weighting) to tune the controllers. Also, other important feature of this controller is that we only need input-ouput data of the plant in order to identify the nonlinear model. This controller is based on a polynomial system approach mixed with the Neuro-Fuzzy (NF) model and the Fuzzy Self-Tuning Mechanism. The STNFGMV is applied to a model of the Continuous Stirred Tank Reactor with Cooling Jacket and is compared with a PI controller, GMV controller with the correct model and the Fuzzy-PI controller heuristically designed and used in (Mudi and Pal Nikhil ,1999; Pinto 2001a, 2001b). Also this controller gives us the possibility to control nonlinear systems with delay. Simulation results are presented to demonstrate the performance of the proposed method.

2. STNFGMV CONTROLLER

The STNFGMV controller has a structure formed by the Neuro-Fuzzy GMV controller and the Fuzzy Self Tuning Mechanism. The general structure of the STNFGMV is shown in Figure 1. This NFGMV is a hybrid controller that mixes the stochastic control (controller colored by white noise zero-mean) with the NF modeling.

In addition, the NF modeling is a hybrid intelligent system, it mixes two concepts of the artificial intelligent as Artificial Neural Networks (ANN) and Fuzzy Logic (Jang , et. all., 1997; Babuška, 1998). The hybrid algorithm takes advantage of the individual characteristics of the ANN and FL in order to make the physical meaning clear, using the fuzzy rules and the acquisition of the expert knowledge by the training as an ANN. To tune the NFGMV controller by trial and error, much time was spent in order to find appropriate values for the control weighting P_c.and error weighting F_c.

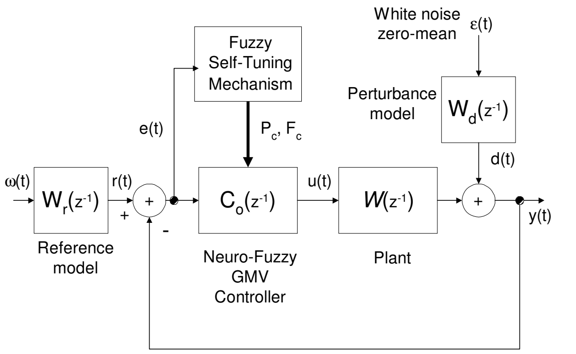

Fig. 1. General Structure of the Self-Tuning Neuro-Fuzzy Generalized Minimum Variance Controller

3. MODELS AND SIGNALS PRESENT IN GMV

The polynomial models of system are defined by the next equation:

$$\left[W_d, W_r, W_{ok}\right] = A^{-1}\left[C_d, E_r, B_{ok}\right] \tag{1}$$

where W_d, W_r and W_{ok} are the disturbance, reference models and linear part of nonlinear plant respectively. In addition A, C_d, E_r, B_{ok} are polynomials in the forward shift operator $\left(z^{-1}\right)$ without any common factors. All the analysis and mathematical development are included in (Grimble, 2003). The nonlinear time-varying plant model:

$$y(t) = (W\, u)(t) = D_k\left(W_k\, u\right)(t) \tag{2}$$

and

$$W_k = W_{ok} = A^{-1}\, B_{ok}\, W_{1k} \tag{3}$$

where D_k is delay of the nonlinear plant and W_{ok} is a stable/unstable linear time invariant block with any unstable modes of the nonlinear plants.

The pseudo output function $\phi_o(t)$ is:

$$\phi_o(t) = P_c e(t) + F_c\, u(t) \tag{4}$$

where P_c and F_c are the control weighting and error weighting. They are defined with the polynomial structure as:

$$P_c\left(z^{-1}\right) = P_{cd}^{-1}\left(z^{-1}\right)\; P_{cn}\left(z^{-1}\right) \tag{5}$$

and

$$F_c\left(z^{-1}\right) = D_k F_{ck}\left(z^{-1}\right) = D_k F_{cd}^{-1}\left(z^{-1}\right) F_{cn}\left(z^{-1}\right) \tag{6}$$

where F_{ck} is the control weighting without delay and D_k is the plant time delay.

The combined white noise signal is defined by:

$$Y_f\left(z^{-1}\right) = A_f^{-1}\left(z^{-1}\right)\; D_f\left(z^{-1}\right) \tag{7}$$

where Y_f is strictly minimum phase and D_f is a strictly Schur polynomial.

The spectral factor is derived from the power spectrum of the combined noise signal and it is expressed as:

$$Y_f\, Y_f^{\,*} = W_r W_r^{\,*} + W_d W_d^{\,*} \tag{8}$$

In order to design the NFGMV it is necessary to cover the following conditions:

$$A_f P_{cd} F_0 + z^{-k} G_0 = P_{cn} D_f \tag{9}$$

and

$$A_f^{-1} P_{cf} = P_{cn} A^{-1} \tag{10}$$

Equation (9) is called the Diophantine equation with a solution (G_0, F_0) with $(\deg F_0 < k)$.

All the assumptions and mathematical development can be found in (Grimble, 2003) giving as a result the next pseudo output function:

$$\phi_o(t) = F_0 \varepsilon(t) + z^{-k}\left(F_{ck} u(t) - F_0 Y_f^{-1}(W_k u)(t) + \left(A_f P_{cd}\right)^{-1} G_0 Y_f^{-1} e(t)\right) \tag{11}$$

and the control law defined for the minimum variance of the input is defined as

$$u(t) = F_{ck}^{-1}\left(F_0 Y_f^{-1}(W_k u)(t) - \left(A_f P_{cd}\right)^{-1} Y_f^{-1} e(t)\right) \tag{12}$$

Finally, applying the GMV controller is possible to predict k step ahead the pseudo-output and find the correct output for following the reference signal.

4. NEURO-FUZZY MODELLING USING

The Takagi-Sugeno-Kang (TSK) model uses the principle "divide and conquer" by using overlapping local linear models to approximate the behaviour in the operating range of nonlinear plants. These local linear models are linear models or piece-wise linear models that are locally stable and so all TSK model are stable (Jang , et. all., 1997; Babuška, 1998).

The TSK model uses an NARX (nonlinear autoregressive with exogenous input) input-output model. It defines the *predicted output* $y(t)$ at a future time instant and is part of the function of the *regressor vector* $z(t)$, consisting of a finite number of past inputs and outputs (Babuška, 1998):

$$\mathbf{z}(t) = \left[y(t-1), \cdots, y(t-n_y), u(t), u(t-1), \cdots, u(t-n_u)\right] \tag{13}$$

where n_u and n_y define the dynamic order of the inputs and outputs delayed in order to cover the complete requirement for the adequate approximation of the nonlinear system.

The NARX model is defined by:

$$y(t) = f(\mathbf{z}(t)) \tag{14}$$

This model is defined by means of the qualitative characteristic of the nonlinear system using Fuzzy rules. The Fuzzy rules are given as:

$$R_i : \text{If } \mathbf{z}(t) \text{ is } A_i \text{ then } y_i = a_i^T \mathbf{z}(t) + b_i \; ; i = 1, 2, \cdots, N \tag{15}$$

where R_i, A_i, $z(t)$, y_i, $\mathbf{a}_i$ and b_i are the i-th fuzzy rule, the i-th fuzzy set, the regressor vector, the i-th linear model, the i-th consequent parameter vector and the i-th scalar offset, respectively. In addition, N is the numbers of the rules.

The first-order TSK model of the nonlinear plant is expressed by:

$$y(t) = \sum_{i=1}^{N} \gamma_i(\mathbf{z}(t))\; y_i \tag{16}$$

$$y_i = \mathbf{a}_i^T\; \mathbf{z}(t) + b_i \tag{17}$$

where $\gamma_i(\mathbf{z}(t))$, y_i are the i-th normalized degree of fulfillment function (nonlinear function) and the i-th piece-wise linear model, respectively. By means of a mathematical manipulation it is possible to define the nonlinear model without delay as:

$$y_k(t) = (W_k\, u)(t) = W_{ok}\,(W_{1k}\, u)(t) = \sum_{i=1}^{N} \gamma_i(\mathbf{z}(t))\; y_i \tag{18}$$

For instance the equation (2) could be expressed as:

$$y(t) = D_k \sum_{i=1}^{N} \gamma_i(\mathbf{z}(t))\; y_i \tag{19}$$

The Fuzzy model is found by training the NF system (Jang, et. all., 1997).

5. THE ERROR AND CONTROL WEIGHTING

The Error Weighting P_c and Control Weighting F_c were defined initially in the equations (5-6). The Error Weighting is expressed by:

$$P_c = \frac{P_{cn}}{P_{cd}} = \frac{(k_{P0} + k_P)(k_{I0} + k_I) - (k_{P0} + k_P) z^{-1}}{1 - z^{-1}} \tag{30}$$

where k_{P0} and k_{I0}, are the proportional and integral gain of the PI controller that can initially stabilize the nonlinear plant, respectively. The variables k_P and k_I are the gains computed by the Fuzzy Self-Tuning Mechanism and they take the initial value of zero.

The Control Weighting is expressed by:

$$F_{ck} = \frac{F_{cn}}{F_{cd}} = \left(\rho_0 + \rho\right) * \frac{\left(1 - a\,z^{-1}\right)}{\left(1 - b\,z^{-1}\right)} \qquad (31)$$

where ρ_0, a and b are constants for tuning the Control Weighting without delay F_{ck}. The variable ρ is a gain introduced by the Self-Tuning Mechanism. The Error Weighting P_c and Control Weighting F_{ck} are nonlinear filters due to their adaptation on-line in each sample time.

6. SELF-TUNING MECHANISM

The Fuzzy Self-Tuning Mechanism of the controller has been founded in the expert knowledge to tune the gains in the PI controller (heuristically designed) for the Error Weighting P_C and in the dynamic effect over the response of ρ for the Control Weighting F_c. The expert knowledge was defined using Fuzzy rules with a structure of Mamdani's Fuzzy System [Jang , et. all., 1997; Babuška, 1998; Pinto, 2001a, 2001b]. The rules definition was obtained using the error signal $e(k)$ and increment of the error signal $\Delta e(k)$ as indicators of the dynamic behaviour of the output. The fuzzy rules for the Fuzzy Self-Tuning Mechanism have the following structure:

$$\begin{aligned} &IF\ e(k)\ is\ Z\ and\ \Delta e(k) is\ Z \\ &Then\ k_p\ is\ Z and\ k_i\ is Z\ and\ \rho\ is\ Z \end{aligned} \qquad (32)$$

The Fuzzy Rules Base for the variables k_p and ρ are the same and are defined in the Table 1. The Fuzzy Rules Base for the variable k_I is defined in the Table 2.

Table 1. Fuzzy Rule of the Variables e(k), Δe(k), k_P and ρ

Δe / e	***MN***	***N***	***Z***	***P***	***MP***
MN	MN	MN	MN	N	MN
N	MN	N	N	N	MP
Z	MN	N	Z	P	MP
P	P	P	P	P	MP
MP	MP	MP	MP	MP	MP

Table 2. Fuzzy Rule of the Variable k_I

Δe / e	***MN***	***N***	***Z***	***P***	***MP***
MN	MP	MP	MP	MP	MP
N	MP	P	P	P	P
Z	P	Z	Z	Z	P
P	P	P	P	P	MP
MP	MP	MP	MP	MP	MP

The variables k_p, k_i and ρ are nonlinear functions of the error and the difference of the error, expressed by the next equations:

$$k_p(k) = f\left(e(k), \Delta e(k)\right) \qquad (33)$$

$$k_I(k) = g\left(e(k), \Delta e(k)\right) \qquad (34)$$

$$\rho(k) = h\left(e(k), \Delta e(k)\right) \qquad (35)$$

The comment above mentioned defines the equations (30) and (31) as nonlinear digital filters. The membership functions for the error signal $e(k)$, increment of the error $\Delta e(k)$, proportional gain k_P and control weighting ρ are shown in the Figure 2. The membership function for the integral gain k_I is shown in the Figure 3.

There are 25 fuzzy rules for each variable (k_p, k_i and ρ). All the expert knowledge of the variables was obtained with the NFGMV developed in [19] and in order to tune the dynamic effect was observed over the output of the Nonlinear System.

For the variable k_i positives values were selected because is the variable can only be positive.

In the Tables 1 and 2 names of the Fuzzy set are More Negative (MN), Negative (N), Zero (Z), Positive (P) and More Positive (MP). The fuzzy sets are defined in the Figure 2 and 3.

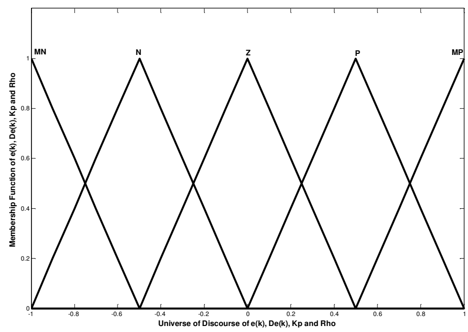

Figure 2. Membership Function of the Variables $e(k), \Delta e(k)$, k_P and ρ

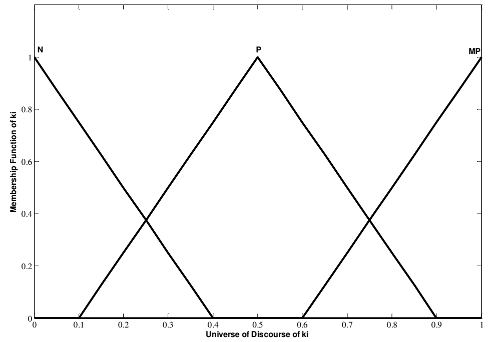

Fig 3. Membership Function of the Variable k_I

General Structure of the Self-Tuning Generalized Minimum Variance is shown in the Figure 4.

8. SIMULATION EXAMPLE

An irreversible exothermic reaction that occurs in the Continuous Stirred Tank Reactor with Cooling Jacket (CSTRCJ) model used to test the performance of the STNFGMV. The Nonlinear model of the Reactor is expressed by:

$$\frac{dC_a(t+kd)}{dt}=\frac{q(t)}{V}\left[C_a^0(t)-C_a(t+k_d)\right] - k_0 C_a(t+k_d)\exp\left(-\frac{E}{RT(t)}\right) \tag{35}$$

and

$$\frac{dT(t)}{dt}=\frac{q(t)}{V}[T_0(t)-T(t)]-\frac{k_0\Delta H}{\rho C_p}C_a(t+k_d)\exp\left(-\frac{E}{RT(t)}\right) +\frac{\rho_c C_{pc}}{\rho C_p V}q_c(t)\left[1-\exp\left(-\frac{hA}{q_c(t)\rho_c C_{pc}}\right)\right][T_{C0}(t)-T(t)] \tag{36}$$

The energy and mass balance equations are defined in (35) and (36), respectively. The nominal parameter values in the model of the Reactor are: $C_a = 0.1$ mol L^{-1}, $T = 438.5$ K, $q_c = 103.41$ L min^{-1}, $q = 100$ L min^{-1}, $C_{a0} = 1$ mol L^{-1}, $T_0 = T_{c0} =$ 350 K, $V = 100$ L, $hA = 7.0 \times 10^5$ cal min^{-1} K^{-1}, $k_0 = 7.2 \times 10^{10}$ min^{-1}, $E/R = 1.0 \times 10^4$ K^{-1}, $\Delta H =$ -2.0 x 10^5 cal mol^{-1} and $\rho = \rho_c = 1.0 \times 10^3$ g. The time delay of the concentration is $k = 0.5$ min. The control objective is the concentration $C_a(t)$. The digital filter (PI controller) is defined by: $k_p = 10$ and $k_i = 10$.

The initial parameters for the STNFGMV controller are:

$k = 3$; $A_f = 1 - 0.8z^{-1}$; $C_d = 0.005$; $E=0$; $k_p = 9$; $k_{i0} = 20$; $k_{d0} = 0$; $\rho_0 = 6.5$; $a = 0.785$; $b = 1$.

8.1 The Neuro-Fuzzy Model of the Nonlinear Plant

In order to find the adequate approximation of the free delay of the CSTRCJ was generated a random signal that covered all its operating range. After that, were compared the output of the correct model of the CSTRCJ and the Neuro-Fuzzy (NF) model. The NF model was training off-line and was expressed by the following fuzzy model (16 fuzzy rules):

Rule1: If $q_c(t)$ is in1mf1 and $C_a(t-1)$ is in2mf1 and $C_a(t-2)$ is in3mf1 and $q_c(t-1)$ is in4mf1 then $C_{a1} = -0.6326q_c(t)+0.05138C_a(t-1)-0.04837C_a(t-2)+0.0099882q_c(t-1)+0.03116$

⋮

Rule16: If $q_c(t)$ is in1mf2 and $C_a(t-1)$ is in2mf2 and $C_a(t-2)$ is in3mf2 and $q_c(t-1)$ is in4mf2 then $C_{a1} = 0.00000921q_c(t)+2.371C_a(t-1)-1.344 C_a(t-2)-0.0003219 q_c(t-1)+0.04551$

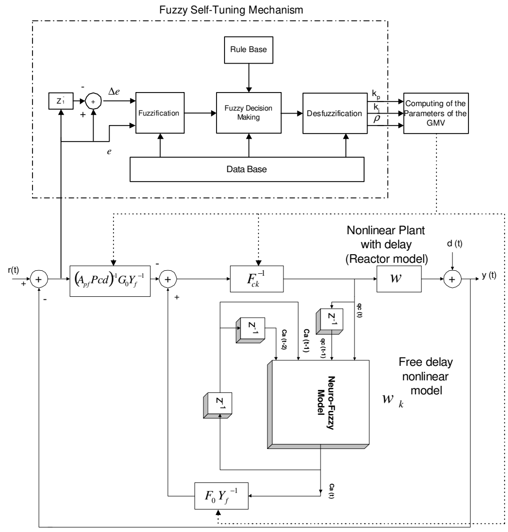

Fig. 4. Final Structure of the Self-Tuning Neuro-Fuzzy Generalized Minimum Variance

The Figure 4 shown the Final Structure of the Self-Tuning Neuro-Fuzzy Generalized Minimum Variance formed by the Fuzzy Self-Tuning Mechanism and the Neuro-Fuzzy Generalized Minimum Variance.

8.2 Tracking Reference Test

In this test the reference signal was changed in the extremes of operation range of the CSTRCJ. The reference signal Fc_{ref} varies from to 0.1 mol L^{-1} to 0.125 mol L^{-1}, from 0.125 mol L^{-1} to 0.1 mol L^{-1} , and from 0.1 mol L^{-1} to 0.055 mol L^{-1} in intervals of 10 min. The results of this test are shown in the figures 6.

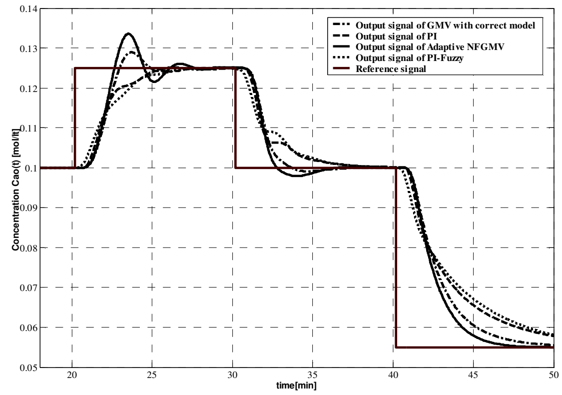

Figure 5. Output Signals of the Digital Filters and STNFGMV in the Tracking Reference Test

8.3 Robustness Test

In this test, the reference signal Fc_{ref} was varied from to 0.1 mol L^{-1} to 0.125 mol L^{-1}, from 0.125 mol L^{-1} to 0.1 mol L^{-1} , and from 0.1 mol $L^{-1}$to 0.055 mol L^{-1} in intervals of 10 min. The results of this test are shown in the figures 7.

The variation of the nominal CSTR parameter are:

V=95 L, hA=6.95e5 cal min^{-1} K^{-1}, k_0=7.25 x 10^{10} min^{-1}, E/R=1.0 x 10^4 K^{-1}, ΔH = -2.2 x 10^5 cal mol^{-1}, $\rho = \rho_c$ = 1.04 x 10^3 g L^{-1} and $C = C_{pc}$ = 1.0 x 10^3 cal K^{-1}.

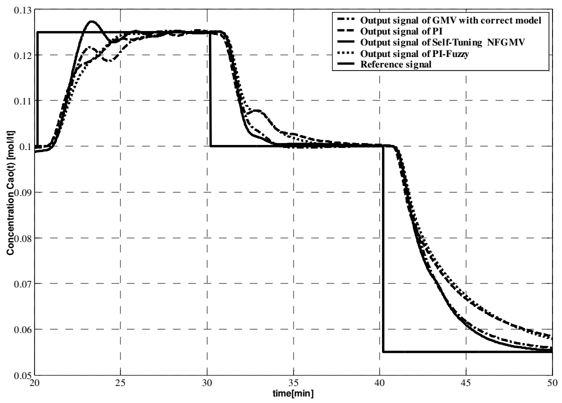

Fig. 6. Output Signals of the Digital Filters and STNFGMV in the Robustness Test

CONCLUSIONS

The STNFGMV simplifies the tuning of the NFGMV controller, because it is automatic. The dynamic behavior of the STNFGMV is better than the GMV controller with the correct model and the PI controller and the PI fuzzy controller. The proposed algorithm saves time in the tuning process of the controller.

ACKNOWLEDGMENTS

The first author is grateful to the financial support by the PROMEP scholarship in the Engineering Department at the Autonomous University of Tlaxcala in Mexico and the EPSRC.

REFERENCES

Åström K.J. (1979), *Introduction to stochastic control theory*, Academic Press, London.

Babuška, R. (1998), *Fuzzy Modeling for Control*, Kluwer Academic Publisher, International Series in Intelligent Technologies.

Babuška, R. and J. Oosterhoff and A. Oudshoorn and P. M. Bruijn (2002), *Fuzzy self-tuning PI control of pH in fermentation*, , .

Clark D.W. and R. Hastings-James (1971), *Design of digital controllers for randomly disturbed systems*, Proc. IEE, Vol. 118, No. 10, pp.1502-1506

Hastings-James R. (1970), *A linear stochastic controller for regulation of systems with pure delay*, University of Cambribge, Department of Engineering, Research Report No. CN/70/3.

Grimble M. J. (1988), *Generalized minimum variance control law revisited*, Optimal Control Applications and Methods, Vol. 9, pp.63-77.

Grimble M. J. (1993), *H_∞ Multivariable control law synthesis*, IEE Proceedings, Control Theory and Applications, Vol. 140, No. 5, pp. 353-363.

Grimble M. J. (1994), *Robust Industrial Control*, Prentice Hall, Hemel Hempstead.

Grimble, M.J. (2003), *Generalized Minimum Variance Control of Nonlinear Multivariable Systems*, Internal report ICC-210, Industrial Control Centre, Univ. of Strathclyde.

Wang L.-X. (1997), *A course in Fuzzy Systems and Control*, ISBN:0-13-540882-2, Prentice Hall PTR.

Mudi Rajani K. [and] Pal Nikhil R. (1999), *A Robust Self-Tuning Scheme for PI- and PD-Type Fuzzy Controllers*, IEEE TRANSACTIONS ON FUZZY SYSTEMS, Vol. 7, No.1, February.

Pinto Castillo ,S. E. (2001), *Control System with Adaptive Fuzzy Decoupling for the Regulation of Load of a Fossil Power Plant*, Thesis of Master in Science in Electronic Engineering, CENIDET, Cuernavaca - Morelos, Mexico

Pinto Castillo, S. E. and G. Madrigal (2001), *Adaptive Fuzzy Control System for the Load Regulation of a Fossil Power Plant*, CICINDI, Mexico.

Jang J.-S. R. and C.-T. Sun and E. Mizutani (1997), *Neuro-Fuzzy and Soft Computing- A Computational Approach to Learning and Machine Intelligence*, ISBN 0-13-261066-3, Pearson Education editorial.