APPLICATION OF MODEL-BASED PREDICTIVE AND ROBUST LOOP SHAPING CONTROL TO AUTOMATIC CAR STEERING [1]

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Abstract: The automatic steering assistance system presented in this paper takes advantage of the technique of Model-based predictive control (MPC) of systems with hard input constraints in order to compute the appropriate steering angle of the front tires. In addition the Quantitative Feedback Theory (QFT) is used to find a robust controller that maintains the key properties of the loop for the implementation of the Model-based Predictive Controller input, despite uncertainty, i.e. stability, tracking performance and disturbance rejection. The procedure involves deriving a steering angle rate from the front tire angle and applying it to the steering column. The feasibility of the proposed assistance system is evaluated in a simulation using Matlab/Simulink and a test vehicle. The results demonstrate the effectiveness of the automatic steering system in lane keeping and yaw dynamic improvement.

Keywords: Automotive Control, MPC, Robust Control, Quantitative Feedback Theory (QFT), Vehicle Dynamics, Automatic Steering, Steering Angle Rate Control

1. INTRODUCTION

The interdisciplinary research program ”intelligent traffic and user-friendly technology” (invent) (*invent*, 2004) covers many different research subjects in the field of automotive control. The Congestion-Assistance consortium ”invent-STA” focusses on the development of an automatically acting assistance system for vehicle guidance at low velocity range in order to improve the road’s safety and the driver’s comfort in congestion situations. For this purpose a Model-based Predictive Controller for the longitudinal guidance of vehicles was designed and validated for different test vehicles (Zambou *et al.*, 2004). In this paper an approach for an automatic steering control system based on MPC of the lateral deviation, and a robust control of the steering angle rate (Fig. 1) is presented.

Herein the predictive component integrated into the control-algorithm is based on the General-Predictive-

[1] Within the framework of the interdisciplinary research program ”intelligent traffic and user-friendly technology” (invent), supported by the German Federal Ministry of Education and Research, the Institute of Automatic Control (IRT) of the RWTH Aachen University along with the AUDI AG et al. have developed an automatically acting assistance system for automatic steering of vehicles driven at low velocity range.

Control approach (GPC) (Clarke *et al.*, 1987) and uses a linear state space model (Krauss, 1995) of the process in order to predict the behaviour of relevant process variables in the future. The prediction is performed using both current and past process data as well as the manipulated variable. The control target is hereby to compute the desired control input - the front tire steering angle - at the lowest possible deviation between the future output - the lateral deviation -, and a determined set point within the horizon under consideration (Fig. 4a).

For an automatically acting steering system, correct measures will have to be taken to overcome the obstacles caused by the complete substitution of the driver in the steering control loop and the uncertainty due to parameter variation of the corresponding actuator. In this paper, the problem of designing a robust steering angle rate controller for an electric steering column with structured parametric uncertainty is addressed using Quantitative Feedback Theory (QFT). QFT is a frequency domain method and emphasises the fact that feedback is necessary because of uncertainty and that the amount of feedback should be directly related to the extent of plant uncertainty and unknown external disturbances (Houpis and Rasmussen, 1999). Therefore the method takes into account quantitative information about the plant's variability, robust performance requirement, tracking specification, and disturbance rejection requirement. Furthermore it requires no exact knowledge about the plant, and no online estimation or tuning algorithm is necessary. The controller is designed to compute the desired manipulated variable - the torque as the servo motor's input (Fig. 3) - and to ensure that robustness and disturbance attenuation requirements can be met. This means tracking the reference steering angle rate by rejecting any other disturbances other than driver intervention in critical situations.

2. CONTROL LOOP AND DYNAMIC MODEL

2.1 Control Loop Structure

Figure 1 represents the outline principle of a control loop structure for the automatic steering of vehicles driven at low velocity range. It includes an inner loop for the steering angle rate ($\dot{\delta}_{ss}$) control and an outer loop for the lateral deviation (y_l) control. The inner loop consists of the robust steering angle rate controller and the steering system as the controlled system. The outer control loop includes the model predictive controller for the lateral deviation relative to the center line of the reference path, the vehicle lateral dynamic as the plant, and a transformation block to derive the required steering angle rate ($\dot{\delta}_{ss/r}$) from the front tire steering angle (δ_f). Additional variables such as the curvature (κ_{ref}) of the reference path and the required lateral deviation ($y_{l/r}$) are external inputs to the outer loop of the cascade structure.

Table 1. System parameters and symbols

Symbols	Description	Value
$_{/r}$, $_{/a}$	required, achieved	-
δ_{ss}	steering angle	- $[rad]$
t_{ss}	torque of the servo motor	- $[Nm]$
α_f	front tire slip angle	- $[rad]$
δ_f	steering angle of the front tires	- $[rad]$
y_l	lateral deviation	- $[m]$
κ_{ref}	curvature of the reference path	- $[1/m]$
v	longitudinal velocity	4.17 $[m/s]$
Ψ	yaw angle	- $[rad]$
Ψ_{ref}	reference yaw angle	- $[rad]$
Ψ_{rel}	relative yaw angle	- $[rad]$
β	sideslip angle	- $[rad]$
m	mass of the vehicle	1915 $[kg]$
J	vehicle yaw moment of inertia	4728 $[kgm^2]$
i_{ss}	steering system gear ratio	15.7 $[-]$
l_f	distance from CG to front axle	1.46 $[m]$
l_r	distance from CG to rear axle	1.42 $[m]$
c_f	front cornering stiffness	83000 $[N/rad]$
c_r	rear cornering stiffness	162000 $[N/rad]$
K_1	gain of the servo motor	1.083$\pm$10% $[-]$
T_1	time constant of the servo motor	0.07$\pm$10% $[ms]$
I_{ss}	inertial moment steering system	0.05$\pm$30% $[kgm^2]$

Since this paper concerns itself with the design of inner and outer loop controllers, it must also include some details about the models of the controlled systems, i.e. the models of the vehicle lateral dynamic and the steering system.

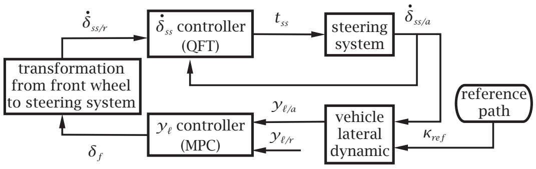

Fig. 1. Block diagram of the control loop

2.2 Vehicle model and reference path

A prerequisite for vehicle lateral dynamic description is the definition of an appropriate model. In this paper a "single-track" model with a ground fixed (inertial) (x_i, y_i) and a vehicle fixed (x, y) coordinate system which is rotated by the yaw angle (Ψ) as shown in figure 2 is used. The essential features to develop a model are described in (Wallentowitz, 2001; Ackermann, 1993). The equations of lateral and yaw motion in the vehicle fixed coordinate system are given as:

$$mv(r-\dot{\beta}) = F_f + F_r \tag{1}$$

$$J\dot{r} = F_f l_f - F_r l_r \tag{2}$$

where, F_f and F_r denote the cornering forces at front and rear tires which are obtained from a linear tire model. For small front and rear sideslip chassis angles (β_f) and (β_r) respectively the corresponding cornering forces are formulated as:

$$F_f = c_f\left(\delta_f + \beta - \frac{l_f r}{v}\right) \tag{3}$$

$$F_r = c_r\left(\beta + \frac{l_f r}{v}\right) \tag{4}$$

where $r := \dot{\Psi}$ is the yaw rate. The tangent to the centerline of the reference path at the intersection with the radial line from the instantaneous center of motion (M), referred to as $\vec{v}_{ref}$ is rotated by a reference yaw angle (Ψ_{ref}) with respect to x_i. The rate of change of the lateral deviation of the vehicle's center of gravity (CG) relative to the reference path (y_{CG}) is given by $\dot{y}_{CG} = v(\beta - \Psi_{rel})$ for small angles where β is the car sideslip angle and $\Psi_{rel} := \Psi - \Psi_{ref}$ as depicted in figure 2. When a vehicle moves on a path composed of circular arcs with constant radius R_{ref}, the rate of change of the relative yaw angle can be defined as:

$$\dot{\Psi}_{rel} = r - v\kappa_{ref} \tag{5}$$

where κ_{ref} is the reference curvature. In the test vehicle a camera system mounted at a distance l_s in front of the CG measures the lateral deviation relative to the centerline of the reference path and converts it to the value of a sensor fixed at the CG. Therefore the measured deviation (y_l) changes with $\dot{y}_{CG}$, and the resulted rate of change is given as:

$$\dot{y}_l = v(\beta - \Psi_{rel}). \tag{6}$$

Note that in all equations it is assumed that the longitudinal velocity $v > 0$ and its rate $\dot{v}$ are constant. The vehicle parameters are described in table 1.

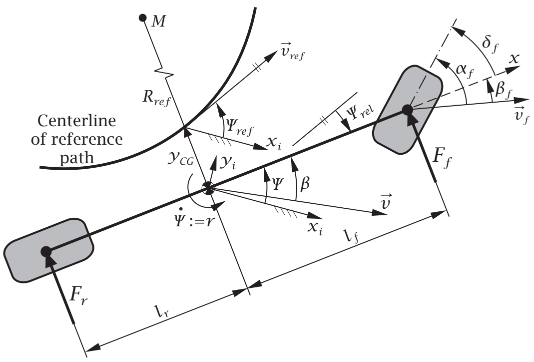

Fig. 2. Single-track model and reference path

2.3 Steering System Model

Figure 3 shows a picture of the steering system in the test vehicle. The system is equipped with a servo motor which provides an interface to the robust steering angle rate controller and a torque sensor. This offers the possibility of data collection with the purpose of identification of the behaviour of this system in mind. The analysis of the prevailing dynamics between input and output data shows that the achieved torque ($t_{ss/a}$) follows the control input like a first-order lag system:

$$T_1\dot{t}_{ss/a} + t_{ss/a} = K_1 t_{ss/r} \tag{7}$$

where, K_1 and T_1 are the servo motor gain and time constant respectively.

Since the driver has been completely substituted in the steering control loop, the concept in this paper does not allow for driver input nor any other assisting torque in the safe operation of the automatic system. Thus the resulting steering angle rate $\dot{\delta}_{ss/a}$ is given by the equation of roll motion of the steering column:

$$I_{ss}\ddot{\delta}_{ss/a} = t_{ss/a} \tag{8}$$

where I_{ss} is the inertial moment of the steering system. Variation in the model parameters in (7) and (8) as indicated in table 1 is first of all due to the dependency on the longitudinal velocity, to non-considered effects from torsional stiffness and torque of the steering gear as well as to model uncertainties.

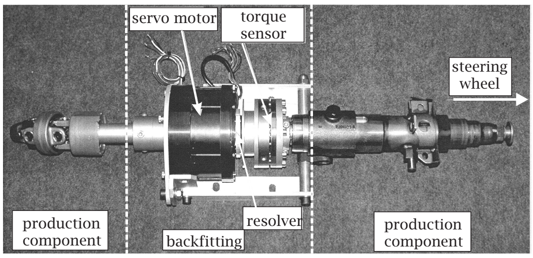

Fig. 3. Steering column in the test vehicle

3. CONTROL CONCEPT

3.1 Model-based Predictive Controller

For the prediction according to the GPC approach, a classical fourth order state space model is used. It is derived from equations (1) to (6) of section 2.2. The state vector is $X = [\beta, r, \Psi_{rel}, y_l]^T$. The input vector is defined as $U = [\delta_f, \kappa_{ref}]$ and the output vector is given as $Y = [\Psi_{rel}, y_l]$. The application of an observer structure (Fig. 4b) is permitted by the state space representation, which assumes that the states of the model mirror the behaviour of those of the plant.

In order to obtain the control law for the MPC method used in this paper a cost function is proposed. The general aim is for the future output (y) on the considered horizon to follow a determined reference signal (w) and at the same time penalise the control effort (Δu) necessary to achieve this. The expression for such an objective function would then be:

$$J = \gamma \sum_{j=N_1}^{N_2} (w_{k+j} - \hat{y}_{k+j})^2 + \lambda \sum_{j=0}^{N_u-1} (\Delta u_{k+j})^2 \tag{9}$$

In the cost function a set of parameters must be specified: N_1 and N_2 (minimum and maximum cost horizons) describing the period (prediction window), in which the estimated controlled variable ($\hat{y}$) is considered. N_u (control horizon) corresponds to the number of manipulated variable steps with which the minimal error signal is to be generated (T_s is the sampling rate) (Fig. 4a). It does not necessarily have to coincide with the maximum cost horizon. The coefficients γ and λ can be used as tuning parameters to cover an ample range of the controller behaviours.

The minimisation of this cost function depending on the control effort (Δu) represents an optimisation problem. Since the controlled system is subject to boundary conditions like maximal/minimal front tire angle ($\delta_f^{max/min}$) or rate of the steering wheel angle ($\dot{\delta}_{ss}^{max/min}$), it is recommended to consider these constraints during the optimisation. All constraints can be expressed as control effort dependant linear inequalities ($N\Delta u \geq g$) where N is the identity or the prediction matrix and g denotes the boundaries minus the free response of the prediction. By the introduction of constraints in the optimisation an analytic solution for this problem cannot be found. Suitable methods for the solution of optimisation problems with linear inequalities for constraints are known in the literature under the name of Quadratic Programming (QP) (Lawson and Hanson, 1974). The predictive method, by the use of a time discrete computation of the control effort (Δu) allows the determination of the manipulated variable $u_k = \delta_{f,k}$ for the time k using the discrete backstep integration. Since the MPC offers the possibility to compute a sequence of the control effort (i.e. the steering angle of the front tires) within the control horizon, it is appropriate to use these values for the computation of the required steering angle rate ($\dot{\delta}_{ss/r}$) (see Fig. 1).

Additional information about the prediction and the formulation of the optimisation problem are provided in (Krauss, 1995).

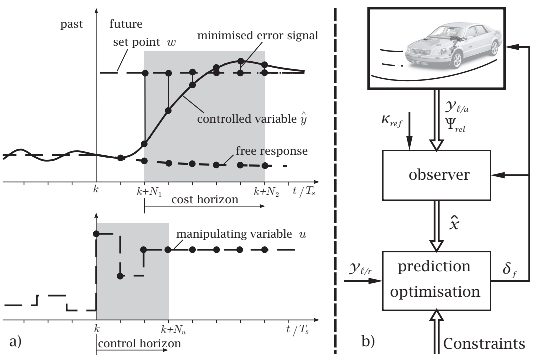

Fig. 4. a) Basic principle of MPC b) Block diagram

3.2 QFT Controller Design

A transfer function of the steering system - plant of the inner control loop - is obtained by transforming equations (7) and (8) in Laplace domain. Thus the set of transfer functions which describe the region of plant parameter uncertainty is defined for different parameter combinations $\{K_1, T_1, I_{ss}\}$ using:

$$P(s) = \frac{K_1/I_{ss}}{(1+T_1 s)s} \tag{10}$$

where, the parameters are subject to variation as given in table 1. A typical two degree-of-freedom (DOF) feedback structure is shown in figure 5 where $G(s)$ is the cascade compensator and $F(s)$ is an input prefilter. A detailed description of QFT design procedure can be found in (Houpis and Rasmussen, 1999). A strictly

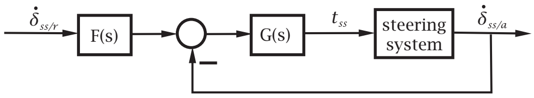

Fig. 5. QFT feedback structure

proper controller $G(s)$ and prefilter $F(s)$ are to be designed such that the following performance and stability specifications are satisfied.

Assume the inner loop desired performance specifications in the time domain - figures of merit for unit step forcing functions - are based on a underdamped second-order system with a maximum peak $M_P = 1.2$ and a settling time $T_{set} = 0.4s$ for the upper bound, as well as on an over-damped system with the same settling time for the lower bound, the control ratio for tracking $T_R(s)$ should satisfy the inequality $|T_{RL}(j\omega)| \leq |T_R(j\omega)| \leq |T_{RU}(j\omega)|\ \forall\omega \in [0,\infty)$. According to figure 5 the open-loop transfer function is denoted as $L(s) := G(s)P(s)$ and the control ratio for tracking is defined as

$$T_R(s) = \frac{F(s)G(s)P(s)}{1+G(s)P(s)} \tag{11}$$

and the transfer functions based on the given performance specifications, are identified by

$$T_{RU}(s) = \frac{60s+1500}{s^2+70s+1500} \tag{12}$$

$$T_{RL}(s) = \frac{1600}{0.0667s^3+4.467s^2+158.7s+1600} \tag{13}$$

for the upper and lower bounds respectively.

Since the control system does not exclude any driver intervention, an extremely rapid disturbance rejection is not required. Therefore if a unit step driver input is considered as a disturbance on the output signal which should be attenuated within $15s$, then equation (14) is appropriate to represent the upper bound imposed on the disturbance as $|T_D(j\omega)| \leq |T_{DU}(j\omega)|$.

$$T_{DU}(s) = \frac{20s^2+2.5s}{20s^2+10.5s+1}; T_D(s) = \frac{1}{1+L(s)} \tag{14}$$

For the closed-loop robust stability an associated QFT robust constraint is given by $|L(j\omega)|/|1+L(j\omega)| \leq M = 1.2$ which corresponds to the $1.6db$ M-contour on the Nichols chart (NC).

The design specifications are translated into some constraints on the nominal open-loop transfer function ($L_0(s) = G(s)P_0(s)$) where $P_0(s)$ is the nominal plant transfer function. These constraints, typically determined on the NC are designated as QFT bounds. A procedure known as loop shaping in QFT is then applied to determine an admissible $L_0(s)$ that meets all the bounds. Figure 6 presents the QFT bounds as well as the results of the final loop shaping. The controller $G(s)$ is then extracted from $L_0(s)$ by dividing it by the nominal plant transfer function.

$$G(s) = \frac{0.753s + 38.19}{s + 25.46} \quad (15)$$

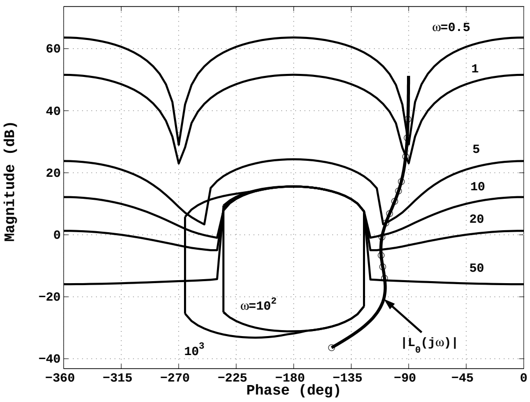

Fig. 6. QFT bounds and nominal open-loop

A prefilter is designed such that the closed-loop frequency responses lie between the tracking boundaries given by the equations (12) and (13). The suitable prefilter is determined using the Bode analysis.

$$F(s) = \frac{30}{s + 30} \quad (16)$$

The closed-loop frequency responses according to the tracking control ratio of equation (11) are computed for the extent of plant uncertainty by means of the proposed controller (15) and prefilter (16). The computation results are presented in figure 7, there it emerges that the proposed system as the steering wheel rate controller achieves the design specifications.

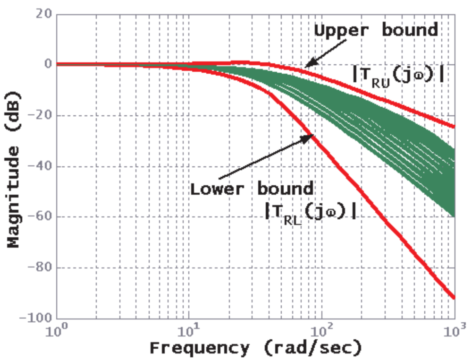

Fig. 7. Closed-loop frequency responses

4. SIMULATION

To demonstrate the effectiveness of the automatic steering system in lane keeping situations and yaw dynamic enhancement, simulations and experiments using a test vehicle are considered. In the simulation a nonlinear single-track vehicle model along with a database for the centerline of the reference path as well as a model of the steering system are used as a substitute of the controlled systems and the guideline in a horizontal plane. The required steering wheel angle rate is derived from the front tire angle by means of differentiation within the control horizon and multiplication with the steering system gear ratio (i_{ss}). Since the automatic acting steering system is developed for vehicles driven at low velocity, a constant velocity ($v = 15km/h$) is applied in the simulation. The reference path consists of sections with different curvatures, i.e. positive and negative for left and right cornering respectively, as well as zero for straight lines. Figure 8 shows the time responses in the simulation using the lateral deviation controller based on the MPC algorithm and the robust steering wheel angle controller designed by means of the QFT technique. For the computation of the MPC law a fixed control horizon (N_u=2) was selected. Since further control parameters are considerably affected by the system dynamics and the available computation performance, correct measures have been taken as recommended in the literature (Lawson and Hanson, 1974), so that the arithmetic complexity of the manipulated variable (δ_f) sequence does not rise arbitrarily. Therefor the boundaries of the prediction window (N_1 and N_2) was set as to maintain the dimensions of the matrices in an adequate range.

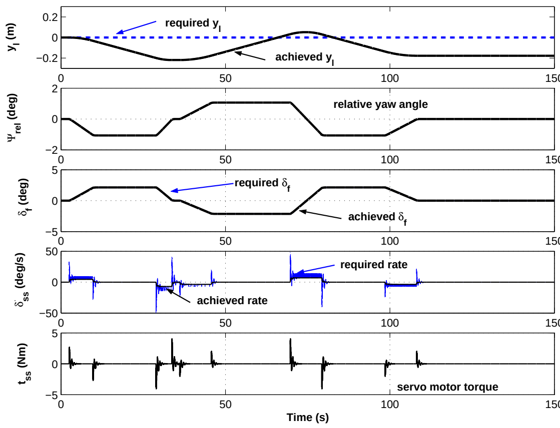

Fig. 8. Time responses in the simulation

The reference path used in the simulation is composed of a sequence of sections with a straight line at the beginning, which is followed by left, right, and left cornering and a straight line at the end. In figure 8 (top) the resulting lateral deviation lies in between -10 and $20cm$ underlining that a good steering behaviour is preserved under different curvatures. The following diagrams confirm this result. There it emerges that the outer loop controller computes an adequate steering angle of the front tires δ_f such as to keep the lateral deviation y_l so close as possible to the required value and thereby improves the relative yaw angle Ψ_{rel}. The last two diagrams present the time responses of the steering wheel angle rate and the servo motor torque. It is recognisable how the robust inner loop control contributes to obtain smoother control with less effort. From the simulation results it follows that the preliminaries for the application of the developed control system to a test vehicle were successfully conducted.

5. APPLICATION

For the implementation into the test vehicle a rapid control prototyping tool is used to link the controller algorithms to a dSPACE AutoBox which provides an interface to the vehicle's standard serial bus system Controller Area Network (CAN). To measure the vehicle dynamics the test vehicle is equipped with a great variety of sensors (see fig. 9). The camera system, the yaw rate and the torque sensors as well as the resolver are important components used in the yaw dynamic determination. In addition the electrical steering system is equipped with a switcher module, which enables the transition between the conventional and automatic car steering and vice versa. The im-

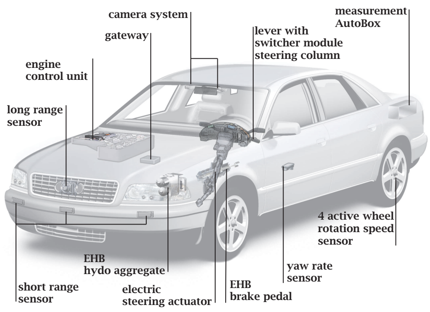

Fig. 9. Test vehicle

plementation and the test of the developed controller algorithms is carried out in several steps. First of all the algorithms of the robust steering angle rate control are implemented into the test vehicle, and a performance check is done with the vehicle at standstill. Successful results lead to further tests with the vehicle driven at low velocity range. Good performance of the inner control loop is a prerequisite for the extension of the implemented algorithms to the complete control structure in oder to perform an automatic steering assistance system for vehicles driven at low velocity. Figure 10 shows the experimental results for the application of the controller algorithms to the test vehicle according to the described procedure. The performance tests of the robust steering rate control are carried out in standstill and for vehicle velocities which lie in between 10 and $30 km/h$. Starting with a lateral deviation from the centerline as a disturbance, the automatic steering system compensates the deviation and maintains the test vehicle on the reference path, which consists of a straight line and a left and right cornering. The last two diagrams confirm good performance of the inner control loop with the vehicle in motion. Further diagrams in figure 10 present the time responses which elucidate good performance of the automatic car steering system when the test vehicle is driven within the given velocity range. This

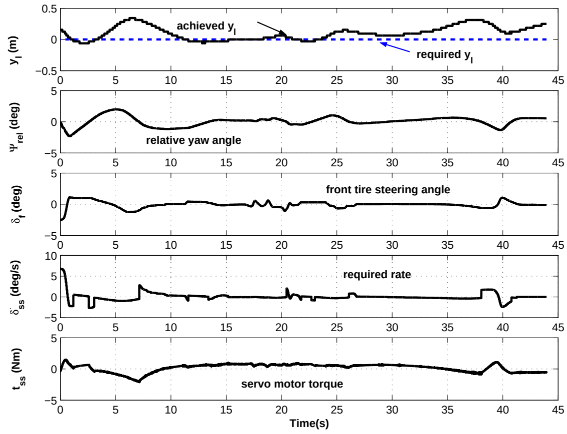

Fig. 10. Time responses in the application

performance leads to similar conclusions as in the simulation.

6. CONCLUSION

This paper is concerned with the MPC and Robust loop shaping control design in order to improve the automatic steering assistance system of vehicles driven at low velocity range. The MPC algorithm is based on a QP-solution of the GPC-approach. The robust inner loop control solves a two-DOF problem by means of the QFT design procedure. The simulated and the applied results demonstrate that the combination of an inner loop robust controller and a MPC algorithm in the outer loop is an appropriate cascade structure to achieve automatic car steering for driver assistance systems.

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