SIMPLIFIED ADAPTIVE NONLINEAR OBSERVER USING B-SPLINE BASED APPROXIMATORS

Dane Baang*, Julian Stoev, Jin Young Choi* and Jaehong Park***

**School of Electrical Engineering and Computer Science, Seoul National University, Korea.*
email:dibang@neuro.snu.ac.kr, jychoi@snu.ac.kr, blue@ee.snu.ac.kr
*** Samsung Advanced Institute of Technology, Korea. email: julian.stoev@samsung.com*

Abstract: It is well known that universal approximators can be used for adaptive control and estimation. In this paper, the problem of adaptive state observation of a large class of nonlinear uncertain systems is considered and it is shown that splines have some special properties, which can lead to simplified observer structure. In particular, the observer filter has fixed dynamical order, independent of the number of parameters to be estimated. This simplification is possible because of the local support property, which is specific to splines.

Keywords: Uncertain dynamic systems, Adaptive systems, Observers, B-splines, Stability

1. INTRODUCTION

It is well known that universal approximators can be used for adaptive control and estimation. Splines are universal approximators with well established properties, but there is a relative lack of results related specifically to splines and, in particular, for splines used in nonlinear system identification.

In this paper, we consider the problem of adaptive state observation of a large class of nonlinear uncertain systems and we show that splines have some special properties, which can lead to simplified observer structure. In particular, the number of integrators in adaptive observer filter is constant, independent of the number of parameters to be estimated. This appears to bring a significant benefit for the observer design, especially compared to other kinds of universal approximators.

As a result of this important property, increasing the number of B-splines can be used to decrease the approximation error, without increasing the computational complexity related to the observer filters. The simplification is possible because of the local support property, which is specific to splines.

In section 2, we pose the problem of adaptive observer design by using universal approximators. In section 3, we give some basic properties of splines, which are necessary for the understanding of the proposed design. In section 4, we show our main result, which is a particular form of plant parametrization using splines. Using this main result, the observer design is performed in a straightforward way. We design our adaptive observer and analyze it in section 5. In section 6, we finish with a numerical simulation demonstrating the method.

2. ADAPTIVE OBSERVER PROBLEM FORMULATION

Adaptive observers for large classes of nonlinear systems are discussed in (Marino et al., 1995; Besancon, 2000; Bastin, 1988). A large class of SISO systems can be represented in the output feedback form, where the dynamics is linear and the nonlinear terms on the right hand side of the differential equations depend on measurable signals.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + F(y(t), u(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $y(t), u(t) \in R$ are the measurable system outputs and inputs, respectively. The functions $F(\cdot) = [f_1(\cdot), ..., f_n(\cdot)]^T$ are often partially known. The information about $F(\cdot)$ may be from the physical knowledge of the system or some identification procedure. In this paper, it is assumed that functional relation of $F(\cdot)$ is unknown, or the functional relation available is nonlinear in the unknown constant parameters. Universal approximators can be used to model these terms. Several classes of universal approximators, which are linear in parameters can be used.

3. SPLINE-BASED SYSTEM APPROXIMATION

We give some of the B-spline definitions and properties(Boor, 1978). All functions here are chosen to be right-continuous.

Definition 1: For a nondecreasing knot sequence $\Upsilon := \{y_i, i = 1...m, y_i \le y_{i+1}\}$, the B-spline of order 1 over the domain $y_i \le y \le y_{i+1}$ is defined as

$$B_{i,1}(y) \equiv X_i(y) := \begin{cases} 1, \ if \ y_i \le y \le y_{i+1} \\ 0, \quad otherwise \end{cases} \tag{2}$$

Higher-order B-splines are defined recursively:

$$B_{i,k}(y) := \omega_{i,k}(y) B_{i,k-1}(y) + (1 - \omega_{i+1,k}(y)) B_{i+1,k-1}(y)$$
$$\omega_{i,k}(y) := \begin{cases} \dfrac{y - y_i}{y_{i+k-1} - y_i}, \ if \ y_i \ne y_{i+k-1} \\ 0, \quad otherwise \end{cases} \tag{3}$$

Remark 1: From the above definitions, it is clear that $B_{i,k}(y)$ is defined only on $k+1$ knots and it has non-zero value only on $y_i \le y < y_{i+k}$.

Corollary 1: $B_{i,k}(y)$ can be represented as a polynomial in y of degree $k-1$ with switching parameters. For some particular y, (3) is equivalent to

$$\begin{aligned} B_{i,k}(y) &= [\beta_{i,j,0} \dots \beta_{i,j,k-1}] \phi_k(y) \\ \phi_k(y) &= [y^0 \ y^1 \ \dots \ y^{k-1}]^T, \end{aligned} \tag{4}$$

where $\beta_{i,j,k}, j = 1...k$, are the polynomial coefficients for the piecewise segments of $B_{i,k}(y)$, which can be calculated recursively using (2) and (3). Consider the sub-class of the system (1) with the structure

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(y), 1 \le i \le n-m-1, 0 \le m, m < n \\ \dot{x}_i &= x_{i+1} + f_i(y) + g_i(y)u(t), n-m \le i \le n-1 \\ \dot{x}_n &= f_n(y) + g_n(y)u(t), y = x_1. \end{aligned} \tag{5}$$

Assumption 1 The $f_i(y), 1 \le i \le n$ are partially known smooth functions. In such case, $f_i(y)$ can be decomposed into $f_i(y) = v_i(y) + \overline{f_i(y)}$, where $v_i(y)$ contains the known terms in $f_i(y)$, and $\overline{f_i(y)}$ are uncertain. This design formulation permits to use the available prior physical or expert information about $f_i(y)$, and $\overline{f_i(y)}$ can be approximated by neural networks, or more generally, by any kind of universal approximator. Since the functions $\overline{f_i(y)}, 1 \le i \le n$ use the same input variable y, a single network with multiple outputs may be used. In this paper, we use B-splines. Define

$$\overline{F}(y) = [\overline{f_1}(y), \dots, \overline{f_n}(y)]^T. \tag{6}$$

Assumption 2: All unknown functions in the vector (6) will be approximated using B-splines defined on the same knot sequence Υ. As a result of Assumption 2, the same B-spline basis can be used to represent each uncertain function in $\overline{F}(y)$ as a linear combination of B-splines. Let $a \in R^{qn}$ be a vector containing the unknown spline weights. Then

$$\begin{aligned} \overline{F}(y) &\approx \Psi_a(y) a \\ \Psi_a(y) &= [B_{1,k}(y) I \ \dots \ B_{q,k}(y) I] \end{aligned} \tag{7}$$

where I is $n \times n$ identity matrix. As mentioned in Remark 1, for some particular value of y, where $y_i \le y < y_{i+1}$ only k number of $B_{i,k}(y), 1 \le i \le q$ are active (non-zero) and they can be calculated using (4).

Assumption 3: $g_{k+n-m}(y), 0 \le k \le m$ are partially known functions. It is assumed, that $g_{k+n-m}(y)$ can be represented as $g_{k+n-m}(y) = b_{m-k}\sigma(y), 0 \le k \le m$ with b_{m-k} unknown constants and $\sigma(y): R \mapsto R$ a known function. Let $b = [b_m, \dots, b_0]^T \in R^{m+1}$ be vectors of unknown constant optimal parameters related to $g_i(y)$. The general class of systems (5) can be represented in the form (1).

$$\dot{x} = Ax + v(y) + \tilde{f}(y,u) + \Psi_a(y) a + \begin{bmatrix} 0 \\ b \end{bmatrix} \sigma(y) u, \ y = Cx$$
$$A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix}$$
$$v(y) = [v_1(y) \dots v_n(y)]^T, \ \tilde{f}(y,u) = [\tilde{f}_1(y,u) \dots \tilde{f}_n(y,u)]^T$$
$$C = [1 \ 0 \ \dots \ 0], \tag{8}$$

where $\tilde{f}_i(y,u)$ are uncertain terms containing the approximating and modelling errors. The functions $\tilde{f}(y,u)$ in (8) can be interpreted as minimum approximation error, which represents the minimum possible deviation between the uncertain functions, $f_i(y,u)$ and $g_{k+n-m}(y)$, and their on-line approximations. Let $\theta := \begin{bmatrix} b \\ a \end{bmatrix} \in R^{qn+m+1}$ and construct the matrix

$$\Psi(y,u) = \left[\begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} \sigma(y) u \quad \Psi_a(y) \right]. \tag{9}$$

Definition 2: Define the approximation error $\tilde{f}(y,u,\hat{\theta})$ for some estimated values $\hat{\theta}$ as

$$\tilde{f}(y,u,\hat{\theta}) = \begin{bmatrix} f_1 \\ . \\ . \\ . \\ f_n(y) \end{bmatrix} + \begin{bmatrix} 0 \\ . \\ . \\ . \\ 0 \\ g_0(y)u \\ . \\ . \\ . \\ g_m(y)u \end{bmatrix} - v(y) - \Psi(y,u)\hat{\theta}. \quad (10)$$

Remark 2: The optimal weights θ are an artificial constant quantities introduced for analytical purposes. They minimise the approximation error

$$\theta := \arg_{\hat{\theta}\in R^{q+m+1}} \min\left\{ \sup_{y\in R^n, u\in R} \| \tilde{f}(y,u,\hat{\theta}) \| \right\}.$$

The values of θ are not needed for the implementation.

Definition 3: Define the minimum approximation error $\tilde{f}(y,u) = \tilde{f}(y,u,\theta)$ as the approximation error (10) obtained when optimal weights θ are used.

Assumption 4: For $\forall y \in R^n$ and $\forall u \in R$, when using the optimal weights θ, the minimum approximation error is bounded.

4. SPLINE-BASED SYSTEM APPROXIMATION

The core property of the spline-based approximator is that the active polynomial parameters are changing abruptly when the measured plant output is moving across different areas. This occurs, for example, when y goes from $y_i \le y < y_{i+1}$ to $y_{i+1} \le y < y_{i+2}$, or to $y_{i-1} \le y < y_i$. These switches occur at some time moments t_j and we assume that it is possible to detect the occurrence of such events and the time at which they occur. These times are a strictly increasing sequence, $\{t_j\}, \lim_{j\to\infty} t_j = \infty$.

Remark 3: Because of the spline structure of $\Psi_a(y)$ in (7) as

$$\Psi_a(y) = \left[0_{n\times n(i-k)} \;\; M_{i\mapsto t_j} \mathrm{P}(y) \;\; 0_{n\times n(q-i)} \right], \quad (11)$$

where

$$P(y) = \left[y^0 I \;\; y^1 I \;\; \ldots \;\; y^{k-1} I \right]$$

and the matrix $M_{i\mapsto t_j}$ depends on the piecewise index i, such that $y_i \le y < y_{i+1}$. This matrix is performing the necessary switching and is constant for $t_j \le t < t_{j+1}$ (when the index i is not changing). The values in this matrix $M_{i\mapsto t_j}$ are easy to be determined using the definition of B-splines and (4). This computation can be performed only once off-line because $M_{i\mapsto t_j}$ depends only on the index i and can be calculated using i and the spline node sequence Υ. We can now formulate the following Lemma, which assumes zero approximation errors for simplicity. These errors, however, will be included later in the adaptive observer stability analysis.

Lemma 1: Assuming that there are no approximation errors, if the vectors a and b are known, the plant (8) can be represented as

$$x(t) = x_v(t) + x_b(t) + x_a(t) \quad (12)$$

$$\dot{x}_v = (A-KC)x_v(t) + v(y) + Ky(t) \quad (13)$$

$$x_b(t) = \Omega_b(t)b \quad (14)$$

$$\dot{\Omega}_b(t) = (A-KC)\Omega_b(t) + \begin{bmatrix} 0_{(\rho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} \sigma(y)u \quad (15)$$

$$x_a(t) = \Omega_a(t)a \quad (16)$$

$$\Omega_a(t) = \Omega_{a,0}(t) + \Omega_{a,t_j}(t) \quad (17)$$

$$\Omega_{a,0}(t) = \Phi(t)\Omega_{a,0}(t_j) \quad (18)$$

$$\dot{\Phi}(t) = (A-KC)\Phi(t) \quad (19)$$

$$\Phi(t_j) = I_{n\times n} \quad (20)$$

$$\Omega_{a,0}(t_j) = \Omega_a(t_j^-) \quad (21)$$

$$\Omega_{a,t_j}(t) = \begin{bmatrix} 0^T_{n\times n(i-k)} \\ (M_{i\mapsto t_j}\Xi(t))^T \\ 0^T_{n\times n(q-i)} \end{bmatrix}^T \quad (22)$$

$$\dot{\Xi}(t) = (A-KC)\Xi(t) + \mathrm{P}(y) \quad (23)$$

$$\Xi(t_j) = 0_{n\times nk}, \quad (24)$$

where K is a feedback gain matrix such that $(A-KC)$ is stable, and t^- denotes the limit $t^- = \lim_{\varepsilon\to 0}(t-\varepsilon)$. The equation (13) represents the part of the system, which is independent of uncertainties. The equation (23) and (19) represents the part of the system, which is approximated by B-splines, because (17) holds only when splines are used. The specific form of (22) is due to the B-spline approximation. The large zero-filled blocks are due to the local support property of the B-splines, as noted in Remark 1. The equation (15) represents the part of the system, which exactly depends on some linear parameters without switching.

Remark 4: The dynamic order of the representation (12-24) is fixed. The only differential equations in this representation are in (13),(15), (19), and (23). The dynamic order is fixed because of the switching performed in (22) and (21) resets of the initial conditions of (19) and (23), performed in (20) and (24), respectively.

Proof: [of Lemma1]
The proof will be given in the form of constructive derivation of (12-24). The original system (8) can be rewritten as

$$\dot{x} = (A-KC)x(t) + v(t) + Ky(t) + \Psi(y,u)\theta, \quad (25)$$

where K is some feedback matrix such that $(A-KC)$ is stable. There are two types of "external" signals in (25):
parameter independent signal: $v(y)+Ky(t)$, with the corresponding

$$\dot{x}_v(t)=(A-KC)x_v(t)+v(y)+Ky(t).$$

This corresponds directly to the equation (13).
parameter dependent signal: $\Psi(t)\theta$,with the corresponding

$$\dot{x}_\theta=(A-KC)x_\theta(t)+\Psi(y,u)\theta\,. \tag{26}$$

Assume that there exists a matrix $\Omega(t)$, such that

$$x_\theta(t)=\Omega(t)\theta. \tag{27}$$

If $\Omega(t)$ is generated using

$$\dot{\Omega}(t)=(A-KC)\Omega(t)+\Psi(y,u), \tag{28}$$

then the equivalence of (26) and (27) is obtained. In this way, $x_\theta(t)$ can be represented as a linear combination of the signals in the filter (28). This approach is known as K-filter representation (Kreisselmeier, 1977). The next step is to use the special properties of $\Psi(y,u)$, which are present in the case of spline-based approximation.

$$\Omega(t)=[\Omega_b(t)\ \mid\ \Omega_a(t)] \tag{29}$$

The first part, $\Omega_b(t)$, is represented in (15).
The second part, $\Omega_a(t)$, can be expressed as

$$\dot{\Omega}_a(t)=(A-KC)\Omega_a(t)+\Psi_a(y). \tag{30}$$

This approach has several important drawbacks with universal approximators. A very important property of the universal approximators is that the minimum approximation error can be made arbitrary small by increasing the complexity of the approximator. With the complexity of the universal approximator increasing, the number of elements in the corresponding parameter vector and the related columns in $\Omega_a(t)$ also incearses very quickly, which increases the dynamic order of the filters and makes the application of such methods problematic in the real world. We can see that $\Psi_a(y)$, which is part of $\Psi(y,u)$ in (9), has a special spline structure (7). As mentioned before, a large part of $\Psi_a(y)$ is guaranteed to contain zero elements. An equivalent expression for $\overline{F}(y)$ containing only the non-zero part $\Psi_a(y)$ can be used as (11). It is natural to use this large sparsity in $\Psi_a(y)$. When applied to to (30), this can be rewritten as

$$\Omega_a(t)=\Omega_{a,0}(t)+\Omega_{a,t_j}(t) \tag{31}$$

$$\Omega_{a,0}(t)=\Phi_{A-KC}(t)\Omega_a(t_j)$$

$$\Omega_{a,t_j}(t)=M_{i\mapsto t_j}\int_{t_j}^{t}\Phi(-\tau)\mathrm{P}(y)d\tau$$

$$\dot{\Phi}_{A-KC}(t)=(A-KC)\Phi_{A-KC}(t)$$

$$\Phi_{A-KC}(t_j)=I_{n\times n}\,,$$

where $\Phi_{A-KC}(t)$ is the transition matrix of $A-KC$, which is resent at t_j and $\Omega_a(t_j)$ is the state of the filter at time t_j . These equations correspond to (17-21). It is possible to implement $\Omega_{a,t_j}(t)$ as

$$\Omega_{a,t_j}(t)=[0_{n\times n(i-k)}\ \ M_{i\mapsto t_j}\Xi(t)\ \ 0_{n\times n(q-i)}]$$

$$\dot{\Xi}(t)=(A-KC)\Xi(t)+\mathrm{P}(y)$$

$$\Xi(t_j)=0_{n\times nk}\,,$$

which corresponds to (22-24). □
Using Lemma 1 and assuming that the values of the parameters a and b are known, it is possible to construct a non-adaptive observer for the system (8) in the form

$$\hat{x}(t)=\hat{x}_v(t)+\hat{x}_b(t)+\hat{x}_a(t)$$

$$\dot{\hat{x}}_v(t)=(A-KC)\hat{x}_v(t)+v(t)+Ky(t)$$

$$\hat{x}_b(t)=\Omega_b(t)b$$

$$\hat{x}_a(t)=\Omega_a(t)a\,.$$

5. ADAPTIVE OBSERVER DESIGN AND STABILITY PROPERTIES

5.1 Observer Stability with Persistently excited plant.

Observe that $\Omega(t)$ in (29) is equivalently generated by the filter (28), where the input of this filter is given by

$$\Psi(y,u)=[\Psi_b(y,u)\ \ \Psi_a(y)]$$

$$\Psi_b(y,u)=\begin{bmatrix}0_{(\rho-1)\times(m+1)}\\ I_{m+1}\end{bmatrix}\sigma(y)u$$

where $\Psi_a(y)$ is defined in (7).

Assumption 5:
Assume that $\Psi(y,u)$ is persistently excitating so that

$$\int_t^{t+T}\Omega^T(\tau)C^TC\Omega(\tau)d\tau\geq\delta I\,, \tag{32}$$

where δ and T are some positive constants. In the next theorem, we formulate the structure of the adaptive observer and analyze its stability properties, taking into account the approximation errors resulting from the use of B-splines.

Theorem 1: Under the Assumption 5, the following adaptive observer for the plant (5) represented in the form (8), has arbitrary small parameter and state estimation errors.

$$\hat{x}(t)=\hat{x}_v(t)+\Omega(t)\hat{\theta}(t)$$

$$\dot{\hat{x}}_v(t)=(A-KC)\hat{x}_v(t)+v(y)+Ky(t)$$

$$\dot{\Omega}(t)=(A-KC)\Omega(t)+\Psi(y,u)$$

$$\dot{\hat{\theta}}(t)=P(t)\varepsilon(t)\Omega^T(t)C^T$$

$$\dot{P}(t)=\begin{cases}\overline{P}(t), & if \ \ \|P(t)\|\le R_0 \\ 0, & otherwise\end{cases}, \quad (33)$$

where $\overline{P}(t)=\beta P(t)-\dfrac{P(t)\Omega^T(t)C^T C\Omega(t)P(t)}{1+\alpha C\Omega(t)\Omega^T(t)C^T}$, α is some positive scalar value and $P(0)=P_0$, $\|P(t)\|\le R_0$ for some symmetric positive definite matrix P_0 and some positive scalar R_0.

Remark 5: From the main properties of modified least-squares update law with forgetting factor (33), $P(t)$ in (33) is guaranteed to be bounded and positive definite for $\forall t\ge 0$, as shown in (Ioannou, 1989, pp. 199). The proof of this theorem requires the following two Lemmas, which can easily be obtained from the work (Zhang,2001) with slight modification.

Lemma 2: Let $\chi(t)\in R^{1\times p}$ be a bounded and piecewise continuous matrix and $P(t)$ be bounded and positive definite matrix, for example, such as (33). If there exist positive constants T,α,β such that $\forall t$

$$\alpha I\le\int_t^{t+T}\chi^T(\tau)\chi(\tau)d\tau\le\beta I, \quad (34)$$

then the system

$$\dot{z}(t)=-P(t)\chi^T(t)\chi(t)z(t) \quad (35)$$

is globally exponetially stable.

Lemma 3: If the autonomous linear time varying system

$$\dot{\zeta}(t)=F(t)\zeta(t) \quad (36)$$

is globally exponentially stable and $u(t)$ is bounded by some $M>0$, then $z(t)$, driven by $u(t)$ of the following system

$$\dot{z}(t)=F(t)z(t)+u(t) \quad (37)$$

is also bounded. Moreover, if M can be designed arbitrary small, then $z(t)$ can be driven to arbitrary small region around the origin. By using lemma 2 and lemma 3, theorem 1 is proved as follows.

Proof:[of Theorem 1]

Let $\tilde{x}=\hat{x}-x$, $\tilde{\theta}=\hat{\theta}-\theta$, $\tilde{x}_v=\hat{x}_v-x_v$ and notice that $\dot{\theta}=0$. Then using the matrix form (8), we obtain

$$\dot{\tilde{x}}(t)=(A-KC)\tilde{x}(t)+\Psi(y,u)\tilde{\theta}(t)$$
$$+\Omega(t)\dot{\tilde{\theta}}(t)+\tilde{f}(y,u,\theta).$$

Consider the following linear combination of $\tilde{x}(t)$ and $\tilde{\theta}(t)$.

$$\tilde{x}_v(t)=\tilde{x}(t)-\Omega(t)\tilde{\theta}(t).$$

Then, considering $\Omega(t)$ is generated by (28), we have

$$\dot{\tilde{x}}_v(t)=(A-KC)\tilde{x}_v(t)+\tilde{f}(y,u,\theta), \quad (38)$$

where $\tilde{f}(y,u,\theta)$ is bounded and can be designed arbitrary small according to properties of universal approximators. Therefore, by using Lemma 3, $\tilde{x}_v(t)$ is guaranteed to be bounded and arbitrary small.

Now we study the behavior of $\tilde{\theta}(t)$. As $\dot{\theta}=0$, we have

$$\begin{aligned}\dot{\tilde{\theta}}(t) &= P(t)\Omega^T(t)C^T(y-C\hat{x}(t))\\ &=-P(t)\Omega^T(t)C^T C(\tilde{x}_v(t)+\Omega(t)\tilde{\theta}(t))\\ &=-P(t)\Omega^T(t)C^T C\Omega(t)\tilde{\theta}(t)\\ &\quad -P(t)\Omega^T(t)C^T C\tilde{x}_v(t).\end{aligned}$$

According to Lemma 2 and Assumption 5, setting $\chi(t)=C\Omega(t)$ makes the homogeneous part of this system globally exponentially stable. Now from Lemma 3 and the fact that $\Omega(t),C,P(t),\tilde{x}_v(t)$ are bounded and especially that $\tilde{x}_v(t)$ can be designed to be arbitrary small, we conclude that $\tilde{\theta}(t)$ is bounded and can be designed arbitrary small. As a result, $\tilde{x}(t)=\tilde{x}_v(t)+\Omega(t)\tilde{\theta}(t)$ is also bounded and can be designed arbitrary small. □

5.2 Observer Stability without persistent excitation

The Assumption 5 is restrictive, but necessary to obtain bounded parameter estimates in the presence of approximation errors. Several approaches are known to modify the observer and obtain bounded parameter estimates even without persistently exciting plant input/output. These methods are discussed in depth in (Ioannou, 1989). The ε-modification can be used, but other choices are also possible.

6. SIMULATION

A numerical simulation was performed to verify the proposed design. The system we consider is a single-link robot arm coupled to a DC motor with a flexible joint.

$$\begin{aligned}&\frac{d\phi_1(t)}{dt}=\omega_1(t),\frac{d\phi_2(t)}{dt}=\omega_2(t)\\ &\frac{d\omega_1(t)}{dt}=\frac{-mgd}{J_1}\sin(\phi_1(t))-\frac{F_1}{J}\omega_1(t)-\frac{K}{J_1}(\phi_1(t)-\frac{\phi_2(t)}{N})\\ &\frac{d\omega_2(t)}{dt}=\frac{-F_2}{J_2}\omega_2(t)-\frac{K_t}{J_2}i(t)-\frac{K}{J_2N}(\phi_1(t)-\frac{\phi_2(t)}{N})\\ &\frac{di(t)}{dt}=-\frac{R}{L}i(t)-\frac{K_b}{L}\omega_2(t)+\frac{1}{L}u(t)\end{aligned} \quad (39)$$

where ϕ_1,ω_1, and ϕ_2,ω_2 are the angular positions and velocities of the arm and the motor shaft, i and

u are the motor armature current and voltage, J_1, J_2, F_1, F_2 are the inertia and viscous friction coefficients, K is a spring constant, K_t and K_b are the torque and e.m.f. constants related with the DC motor, R is the armature resistance and L is the armature inductance, m is the arm mass, d is the position of the arm's center of gravity, N is thegear ratio and g is acceleration of gravity.

Based on the range of the output obtained, the B-spline knot sequence needed for the observer is selected as $\Upsilon = [-\frac{3\pi}{2} + k\frac{\pi}{2}, k = 0, ..., 20]$ .This sequence contains 21 knots equally spaced from $-\frac{3\pi}{2}$ to $\frac{7\pi}{2}$ with step $\frac{\pi}{2}$. On this knot sequence 16 B-splines of order 5 can be defined. The output feedback gains for the K-filter both for the physical observer and for the B-spline observer are selected so that the poles are at [-5,-5,-5,-5,-5]. The simulation was performed for 1000 seconds.

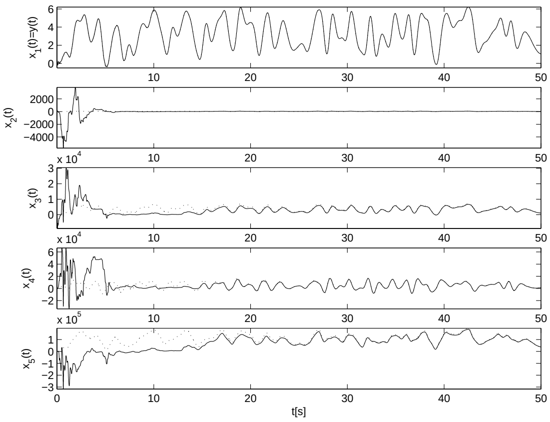

Fig. 1. States $x_1 = y, x_2, x_3, x_4, x_5$ from the B-spline observer in solid lines and actual states in dotted lines

Actual and observed states produced by the adaptive B-spline based observer are shown on Fig. 1. The states converge close to the real ones quite fast. The output of the plant, which is the first state, converges almost immediately. After 30 seconds, all the states from the adaptive observer are very close to the real states of the plant in output feedback form and its convergence improves very little.

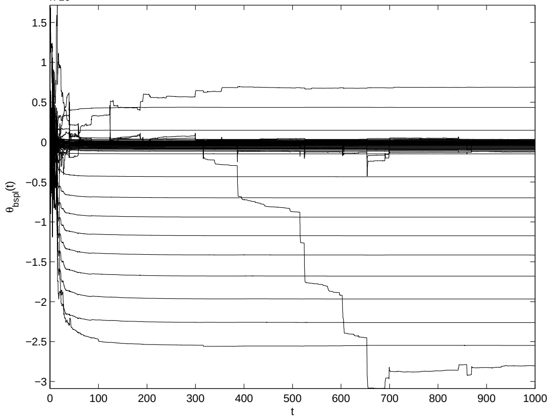

Fig. 2. B-spline weights

Estimated parameters representing the B-spline weights of the adaptive B-spline based observer are shown on Fig. 2. It is interesting to note that the the convergence of these parameters is much slower compared to that of the observer states. A simulation for at least 1000 seconds is necessary to obtain parameter estimates with values close to constants. This observation can be explained using the local support property of the B-splines. Because of the local support property of B-splines, the observed states are very close to the real ones, even when the adaptive parameters are far from converging.

7. CONCLUSION

Using B-splines as universal approximators, we have obtained a plant parametrization, which permits the construction of an adaptive observer. The particular property of this parametrization is that the dynamic order of the filters in this design does not depend on the number of parameters in the plant parameterization. This appears to be a beneficial property especially because the number of such parameters tends to be very high for the approximator based designs. Possible direction for future work include the exploration of approximation errors influence on the observer. Another possible direction is an on-line selection of the B-spline structure, by inserting and/or removing some knots in the knot sequence.

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