ON THE EQUIVALENCE OF A MINIMAL ORDER *MPC* AND A *GTDOF* CONTROL OF TIME-DELAY PLANTS

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Abstract: It is shown that for open-loop stable plants the task of an *MPC* can be solved by identifying the plant model only instead of the redundant prediction model. Interesting structural considerations can be derived from comparison of the topologies and structural forms of regulators obtained by pole-placement design, the *GTDOF* scheme and the *MPC*. A new simple method is also introduced for adaptive and combined iterative identification and control for time-delay plants.

Keywords: time-delay, pole-placement, model predictive control, two-degree of freedom regulator

1. INTRODUCTION

One of the first systematic pole-placement design was discussed in (Åström and Wittenmark (1984)). The described method is quite general, includes both stable and unstable, inverse stable (*IS*) and inverse unstable (*IU*) processes. These approaches are based on a *Diophantine-equation* (*DE*) technique for finding the numerator and denominator of the pole-placement regulator using an almost standard scheme given in Fig. 1, where y_r, u, y are the reference, process input, output signals respectively. The polynomial triple $\mathcal{R}, \mathcal{S}, \mathcal{T}$ mean a *two-degree of freedom* (*TDOF*) regulator connected to the plant S.

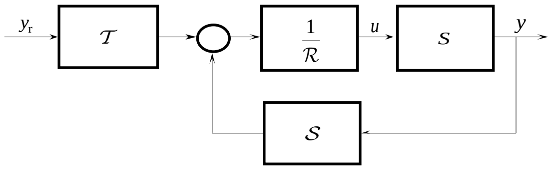

Figure 1. Standard scheme for pole-placement design

Another approach the *Model Predictive Control* (*MPC*) subject area (Clarke and Gawthrop (1975, 1979) became a success story (Soeterboek, 1992; Camacho and Bordons (1999); Maciejowski, 2002) for control theory and application in the past decades, where the algorithms are relatively simple and robust for even industrial applications.

2. SHORT SUMMARY OF PREDICTOR BASED CONTROLLERS

The derivation of the classical *minimum variance* (*MV*) regulator is based on the *d*-step ahead prediction of the output in case of an additive colored output noise. If the process model is *IS*

$$y(k) = \frac{\mathcal{B}}{\mathcal{A}} z^{-d} u(k) + \frac{\mathcal{C}}{\mathcal{D}} e(k) = \frac{\mathcal{B}}{\mathcal{A}} u(k-d) + \frac{\mathcal{C}}{\mathcal{D}} e(k) \quad (1)$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ are the polynomials of the process and noise model and $e(k)$ is the so-called independent white source noise. $\mathcal{C}$ and $\mathcal{D}$ must be stable to have a unique spectral factorization. Introducing the *DE*

$$\mathcal{C} = \mathcal{D}\mathcal{F} + z^{-d}\mathcal{G} \quad (2)$$

the *MV* predictor (nonlinear in the parameters and separates the past and future) of the process output is

$$\hat{y}(k+d \mid k) = \frac{\mathcal{B}\mathcal{D}\mathcal{F}}{\mathcal{A}\mathcal{C}} u(k) + \frac{\mathcal{G}}{\mathcal{C}} y(k) \quad (3)$$

providing an independent additive prediction error

$$y(k+d) = \hat{y}(k+d \mid k) + \varepsilon(k+d) =$$

$$= \hat{y}(k+d \mid k) + \mathcal{F}\, e(k+d) \tag{4}$$

The well-known MV regulator minimizing $E\{y(k) - y_r(k)\}$ can be obtained by equating the MV prediction to the desired reference signal

$$y_r(k+d) = \hat{y}(k+d \mid k) \tag{5}$$

Simple calculation gives that the obtained regulator triple (*Predictive Control*=*PC*) in this case is

$$\mathcal{R} = \mathcal{B}\mathcal{D}\mathcal{F} \quad ; \quad \mathcal{S} = \mathcal{A}\mathcal{G} \quad ; \quad \mathcal{T} = \mathcal{A}\mathcal{C} \tag{6}$$

providing a characteristic equation (CE) $\mathcal{C} = 0$ and regulator

$$R = \frac{\mathcal{S}}{\mathcal{R}} = \frac{\mathcal{A}\mathcal{G}}{\mathcal{B}\mathcal{D}\mathcal{F}} \tag{7}$$

The obtained regulator is a pole/zero cancellation regulator, therefore it is applicable for stable, IS processes only. If the tracking task is to follow the output of a reference model P_r and not a signal then

$$y_r'(k+d) = P_r\, y_r(k+d) = \hat{y}(k+d \mid k) \tag{8}$$

formally means the change of $\mathcal{T}$ to $T' = P_r \mathcal{T}$. Several simpler forms of (1) can also be found in the classical references for the special cases of $\mathcal{D} = \mathcal{A}$ and/or $\mathcal{C} = 1$. It is interesting to observe that the noise free d-step ahead prediction of y

$$\hat{y}(k+d \mid k) = \mathcal{B}\mathcal{F}\, u(k) + \mathcal{G}\, y(k) \tag{9}$$

is based on the special DE

$$1 = \mathcal{A}\mathcal{F} + z^{-d}\, \mathcal{G} \tag{10}$$

which is an MV predictor for the case $\mathcal{C} = 1$ and $\mathcal{D} = \mathcal{A}$, when the additive "equation error" is independent. The DE (10) corresponds to a special reparametrization of an IS process

$$S = \frac{\mathcal{B}\mathcal{F}\, z^{-d}}{1 - \mathcal{G} z^{-d}} = S_+ z^{-d} \tag{11}$$

The first "tuning" applications of the predictor based regulators started by the observation that the closed-loop transfer characteristics under the MV regulator condition (8)

$$\begin{aligned} &\mathcal{T}\, y_r'(k+d) = \mathcal{T}\, P_r\, y_r(k+d) = \\ &= \mathcal{R}\, u(k) + \mathcal{S}\, y(k) + \tilde{\mathcal{F}}\, e(k+d) + e(k+d) \\ &[\mathcal{A}\mathcal{C}] y_r'(k+d) = \\ &= [\mathcal{B}\mathcal{D}\mathcal{F}] u(k) + [\mathcal{A}\mathcal{G}] y(k) + \tilde{\mathcal{F}}\, e(k+d) + e(k+d) \end{aligned} \tag{12}$$

is "quasi-linear" in the parameters of the polynomials $\mathcal{R}, \mathcal{S}, \mathcal{T}, \tilde{\mathcal{F}}$. (Here $\mathcal{F} = 1 + \tilde{\mathcal{F}}$ is used.) So it is relatively easy to construct an identification algorithm to estimate these polynomials, which are practically the same as the regulator polynomials. Observe that the coefficients of the "quasi-linear" form are redundant in the real process parameters. The predictor form (12) has 4*n*+*d*-2 parameters, which is considerably more than the number of process parameters: 2*n*. (Here *n* is the order of $\mathcal{A}$ and $\mathcal{D}$ for the sake of simplicity.) This large number is not surprising, because the predictor requires to estimate (indirectly) the noise model, too.

Since the applicability of the *MV* regulator is limited for IS processes only, the applied original MV criterion was generalized to include weighting filters in the form of filtering the output and also penalizing the variance of the regulator output as

$$E\left\{ \left(Q_y \left[y(k+d) - y_r(k+d) \right] \right)^2 + \left(\mathcal{Q}_u \left[u(k) \right] \right)^2 \right\} \tag{13}$$

The main influence of this change was that the CE of the closed-loop also changed to $\left[Q_y\, \mathcal{B} + \mathcal{Q}_u\, \mathcal{A} \right] \mathcal{C} = 0$. It is easy to see that selecting relatively "large" $\mathcal{Q}_u$ comparing to Q_y the unstable zeros of $\mathcal{B}$ move closer to the stable $\mathcal{A}$. The obtained *Generalized* PC=GPC only slightly differs from the PC form. Instead of the original output y the filtered output $y^F = Q_y\, y = \left(\mathcal{Q}_{y1} / \mathcal{Q}_{y2} \right) y$ should be used and the regulator polynomial $\mathcal{R}$ is changed to $\mathcal{R}' = [\mathcal{B}\mathcal{D}\mathcal{F}' + .\ \ \mathcal{Q}_u \mathcal{A}\mathcal{C}] \mathcal{Q}_{y2}$. The influence of the generalized criterion sometimes was called "de-tuning". Although the effect of $\mathcal{Q}_u$ on the closed-loop system is clear it is hard to choose $\mathcal{Q}_u$ and $\mathcal{Q}_y$ such that the system behaves as desired. One way of the optimal selection could be the combination of simulation with the well-known and often used trial-and-error method.

3. SHORT SUMMARY OF *GTDOF* CONTROLLER SCHEME

A *generic two-degree of freedom* (*GTDOF*) scheme was introduced in (Keviczky, 1995) for open-loop stable processes. This framework and topology is based on the *Youla-parametrization* (Maciejowski, 1989) providing all realizable stabilizing regulators (*ARS*) for open-loop stable plants and also capable to handle the plant time-delay.

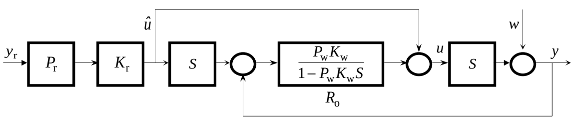

Figure 2. The *generic TDOF* (*GTDOF*) control system

A *GTDOF* control system is shown in Fig. 2, where w is the output disturbance signal. The optimal *ARS* regulator of the *GTDOF* scheme can be given by an explicit form

$$R_o = \frac{P_w K_w}{1 - P_w K_w S} = \frac{Q_o}{1 - Q_o S} = \frac{P_w G_w S_+^{-1}}{1 - P_w G_w S\_z^{-d}} \tag{14}$$

where

$$Q_o = Q_w = P_w K_w = P_w G_w S_+^{-1} \tag{15}$$

is the associated optimal *Y-parameter* furthermore

$$Q_{\mathrm{r}} = P_{\mathrm{r}} K_{\mathrm{r}} = P_{\mathrm{r}} G_{\mathrm{r}} S_+^{-1} \; ; \; K_{\mathrm{w}} = G_{\mathrm{w}} S_+^{-1} \; ; \; K_{\mathrm{r}} = G_{\mathrm{r}} S_+^{-1} \tag{16}$$

assuming that the process is factorable as

$$S = S_+ \overline{S}_- = S_+ S_- z^{-d} = \frac{\mathcal{B}}{\mathcal{A}} z^{-d} \tag{17}$$

where S_+ means the inverse stable (*IS*) and S_- the inverse unstable (*IU*) factors, respectively. z^{-d} corresponds to the discrete time-delay, where d is the integer multiple of the sampling time. (In a practical case the factor S_- can incorporate the underdamped zeros and neglected poles providing realizability, too). It is interesting to see how the transfer characteristics of the closed-loop looks like:

$$\begin{aligned} y &= P_{\mathrm{r}} K_{\mathrm{r}} S\, y_{\mathrm{r}} + \left(1 - P_{\mathrm{w}} K_{\mathrm{w}} S\right) w = \\ &= P_{\mathrm{r}} G_{\mathrm{r}} S_- z^{-d} y_{\mathrm{r}} + \left(1 - P_{\mathrm{w}} G_{\mathrm{w}} S_- z^{-d}\right) w = y_{\mathrm{t}} + y_{\mathrm{d}} \end{aligned} \tag{18}$$

where y_{t} is the tracking (servo) and y_{d} is the regulating (or disturbance rejection) independent behaviors of the closed-loop response, respectively. So the delay z^{-d} and S_- can not be eliminated, consequently the ideal P_{r} and P_{w} design goals are biased by the $G_{\mathrm{r}} S_-$ and $G_{\mathrm{w}} S_-$. Here P_{r} and P_{w} are assumed stable and usually strictly proper transfer functions, that are capable to place desired poles in the tracking and the regulatory transfer functions, furthermore they are usually referred as reference signal and output disturbance predictors. They can even be called as reference models, so reasonably $P_{\mathrm{r}}(\omega = 0) = 1$ and $P_{\mathrm{w}}(\omega = 0) = 1$ are selected. The unity gain of P_{w} ensures integral action in the regulator, which is maintained if the applied optimization provides the constraint $G_{\mathrm{w}} S_-(\omega = 0) = 1$. It is easy to check that the *ARS* optimal *GTDOF* regulator in (13) gives the *MV* regulator if the reference model $P_{\mathrm{w}} = \mathcal{G}/\mathcal{C}$ is selected and $S_+^{-1} = \mathcal{A}/\mathcal{B}$ is used.

An interesting result was found in (Keviczky and Bányász (1999)) that the optimization of the *GTDOF* scheme can be performed in $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norm spaces by the proper selection of the serial G_{r} and G_{w} embedded filters attenuating the influence of the invariant process factor S_-. Using $\mathcal{H}_2$ norm a *DE* should be solved to optimize only these filters and not the whole regulator itself. If the optimality requires a $\mathcal{H}_\infty$ norm, then the Nevanlinna-Pick (*NP*) approximation is applied. The order of the *DE* (and the *NP* approximation) for this task is usually considerably lower than in case of the original formulation (Åström and Wittenmark (1984)).

It is important to note that the general pole-placement using the *DE* technique mentioned in the Introduction gives an explicit algebraic solution for stable processes. This solution corresponds to the regulator (14) obtained in the *GTDOF* scheme.

4. COMPARISON OF *MPC* AND *GTDOF* CONTROLLERS

Let us find the *GPC* form of the optimal *ARS* regulator in the *GTDOF* scheme. After some straightforward block manipulations the *GTDOF* control system can be transformed to a much simpler form shown in Fig. 3.

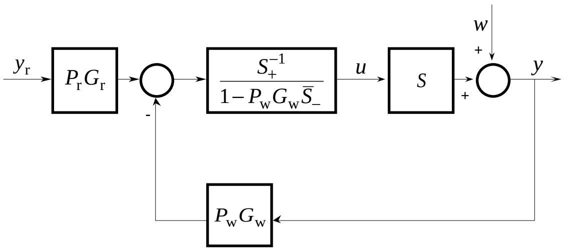

Figure 3. Simplified form of the *GTDOF* control system

Introduce the following notations

$$P_{\mathrm{r}} = \frac{\mathcal{B}_{\mathrm{r}}}{\mathcal{A}_{\mathrm{r}}} \; ; \; G_{\mathrm{r}} = \frac{1}{\mathcal{G}_{\mathrm{r}}} \; ; \; P_{\mathrm{w}} = \frac{\mathcal{B}_{\mathrm{w}}}{\mathcal{A}_{\mathrm{w}}} \; ; \; G_{\mathrm{w}} = \frac{1}{\mathcal{G}_{\mathrm{w}}} \tag{19}$$

$$S = \frac{\mathcal{B}_+ \mathcal{B}_-}{\mathcal{A}} z^{-d} = S_+ \mathcal{B}_- z^{-d}$$

Here we assume that $S_- = \mathcal{B}_-$ (it contains only the invariant unstable zeros) and in this case G_{r} and G_{w} have only denominators. The equivalent form comparable to Fig. 1 can be seen in Fig. 4 and it is assumed that the same optimality criterion is used to determine G_{r} and G_{w}, therefore they are equal.

The one-to-one comparison gives the following results

$$\mathcal{R} = \boldsymbol{\mathcal{B}}_+ \left(\mathcal{A}_{\mathrm{w}} \mathcal{G}_{\mathrm{w}} - \mathcal{B}_{\mathrm{w}} \boldsymbol{\mathcal{B}}_- z^{-d}\right); \; \mathcal{S} = \mathcal{B}_{\mathrm{w}} \boldsymbol{\mathcal{A}} \; ; \; \mathcal{T} = \mathcal{B}_{\mathrm{w}} \boldsymbol{\mathcal{A}} \tag{20}$$

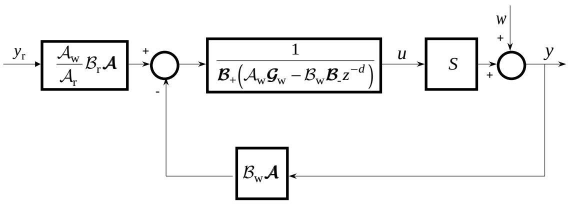

Figure 4. Equivalent form of the *GTDOF* control system corresponding to the $\mathcal{R},\mathcal{S},\mathcal{T}$ pole-placement and *GPC*

One of the advantages of the previous analysis is that it makes possible to calculate the necessary orders of the unknown *GPC* regulator polynomials to be estimated. It is also easy to observe the high redundancy in these parameters and the relatively high percentage of those parameters which are apriori known from our design goals (bold letters are used to indicate the unknown parameters). It is also clear that the linearity in the parameters is lost if only the unknown parameters are to be estimated. On the other hand it is also clear that the unknown $2n$ parameters are the process parameters $\boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{B}}_+, \boldsymbol{\mathcal{B}}_-$ and $\boldsymbol{\mathcal{G}}_{\mathrm{w}}$.

(However, $\mathcal{G}_{\mathrm{w}}$ depends on the selected optimality criterion and $\mathcal{B}_{-}$.)

The controller in Fig 3 can be slightly redrawn into the form given in Fig. 5. This form is special because the controller is splited into two parts. The first part depends only on the design parameters and the invariant factors with their optimal attenuation while the second only on the model of the plant. (Please note that this scheme is for interpretation and not for implementation, because $1/S$ is usually not realizable.)

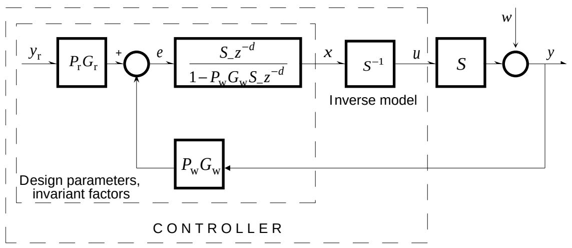

Figure 5. Special form of the *GTDOF* controller

The generally applied $\mathcal{R},\mathcal{S},\mathcal{T}$ forms in the MPC algorithms, are easy to be used for parameter estimation and direct formulation of self-tuning regulators. However, the above analysis showed that the pole-placement *MPC* and *GTDOF* controllers are very redundant in the unknown parameters. The polynomials strongly depend on the design and optimization parameters. If we want to take this internal structure into consideration the "quasi-linearity" is lost and a difficult nonlinear parameter estimation task remains, because the really unknown parameters are the process parameters $\mathcal{A},\mathcal{B}_{+},\mathcal{B}_{-}$ only. Therefore new possibilities are investigated to help the direct identification of the plant under the constraint of a special pole-placement controller in the framework of the *GTDOF* scheme.

5. REFORMULATION OF THE *MPC* TO IDENTIFY THE PROCESS MODEL ONLY

Be M the model of the process. Assume that the discrete-time model is factorizable as the true process in (17)

$$M = M_{+}\overline{M}_{-} = M_{+}M_{-}z^{-d_{\mathrm{m}}} = \frac{\hat{\mathcal{B}}}{\hat{\mathcal{A}}}z^{-d_{\mathrm{m}}} \tag{21}$$

where M_{+} means the inverse stable (IS), M_{-} the inverse unstable (IU) factors, respectively. z^{-d} and $z^{-d_{\mathrm{m}}}$ correspond to discrete time delays, which are the integer multiple of the sampling time, usually $z^{-d} = z^{-d_{\mathrm{m}}}$ is assumed. (To get a unique factorization it is reasonable to ensure that S_{-} and M_{-} are monic, i.e., $S_{-}(1) = M_{-}(1) = 1$, having unity gain.) It is important that the inverse of the term z^{-d} is not realizable, because it would mean an ideal predictor z^{d}. These assumptions mean that $\overline{M}_{-} = M_{-}z^{-d_{\mathrm{m}}}$ is the uncancelable invariant factor for any design procedure.

Consider the model based controller equation of the *GTDOF* scheme shown in Fig. 3

$$u = \frac{M_{+}^{-1}}{1-P_{\mathrm{w}}G_{\mathrm{w}}M_{-}z^{-d}}\left(P_{\mathrm{r}}G_{\mathrm{r}}y_{\mathrm{r}} - P_{\mathrm{w}}G_{\mathrm{w}}y\right) \tag{22}$$

which can be rearranged according to (12) as

$$M_{+}\left(1-P_{\mathrm{w}}G_{\mathrm{w}}M_{-}z^{-d}\right)u = P_{\mathrm{r}}G_{\mathrm{r}}y_{\mathrm{r}} - P_{\mathrm{w}}G_{\mathrm{w}}y \tag{23}$$

Having formulated a completely model-based parallel closed-loop the controller equation is

$$\hat{u} = \frac{M_{+}^{-1}}{1-P_{\mathrm{w}}G_{\mathrm{w}}M_{-}z^{-d}}\left(P_{\mathrm{r}}G_{\mathrm{r}}y_{\mathrm{r}} - P_{\mathrm{w}}G_{\mathrm{w}}\hat{y}\right) \tag{24}$$

where $\hat{u}$ and $\hat{y} = M\,\hat{u}$ are the regulator and closed-loop output, respectively, in the parallel loop. The equivalent form of (23) is now

$$M_{+}\left(1-P_{\mathrm{w}}G_{\mathrm{w}}M_{-}z^{-d}\right)\hat{u} = P_{\mathrm{r}}G_{\mathrm{r}}y_{\mathrm{r}} - P_{\mathrm{w}}G_{\mathrm{w}}\hat{y} \tag{25}$$

Forming the difference of (23) and (25)

$$\begin{aligned} M\left(u-\hat{u}\right) &= M\,\Delta\hat{u} - \frac{P_{\mathrm{w}}G_{\mathrm{w}}M_{-}z^{-d}}{1-P_{\mathrm{w}}G_{\mathrm{w}}M_{-}z^{-d}}\left(y-\hat{y}\right) = \\ &= F_{\mathrm{w}}\left(y-\hat{y}\right) = \Delta\hat{\varepsilon}^{\mathrm{F}} = F_{\mathrm{w}}\,\varepsilon \end{aligned} \tag{26}$$

can be obtained after some not very sophisticated manipulations, where $\Delta\hat{u} = u-\hat{u}$, $\Delta\hat{y} = y-\hat{y} = \varepsilon$. It is interesting to note that F_{w} depends on the regulatory, $\hat{u}$ and $\hat{y}$ depend on the tracking properties of the design requirements. It is easy to check that (26) is "quasi-linear" in the parameters

$$\begin{aligned} \Delta\hat{\varepsilon}^{\mathrm{F}}(k) &= M\,\Delta\hat{u}(k) = \frac{\hat{\mathcal{B}}z^{-d}}{\hat{\mathcal{A}}}\Delta\hat{u}(k) = \\ &= \hat{\mathcal{B}}\,\Delta\hat{u}(k-d) - \hat{\bar{\mathcal{A}}}\,\Delta\hat{\varepsilon}^{\mathrm{F}}(k-1) \end{aligned} \tag{27}$$

introducing

$$M = \frac{\hat{\mathcal{B}}z^{-d}}{\hat{\mathcal{A}}} = \frac{\hat{\mathcal{B}}z^{-d}}{1+\hat{\bar{\mathcal{A}}}} \tag{28}$$

Here we used the auxiliary signal

$$\hat{u} = P_{\mathrm{r}}K_{\mathrm{r}}y_{\mathrm{r}} = P_{\mathrm{r}}G_{\mathrm{r}}S_{+}^{-1}\,y_{\mathrm{r}} \tag{29}$$

This analysis can be interpreted as a direct reformulation of the classical adaptive MPC providing minimal number of parameters to be estimated to a *GTDOF* control design problem.

The scheme of *GTDOF* control system in Fig. 2 suggests a special way for combined ID and control. Observe that it is possible to use $\hat{u}(k)$ as an input signal and $y(k)$ as output signal generated by the apriori part of the controller in a closed-loop to the identification procedure.

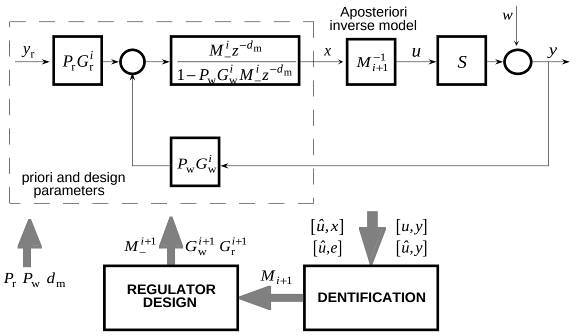

Figure 6. Special scheme formulating combined ID and control strategy

Besides the above two: $\left[\hat{u},\Delta\varepsilon^{\mathrm{F}}\right]$ and $[\hat{u},y]$ signal pairs it is possible to find further pairs: $[u,y]$, $[\hat{u},x]$ or $[\hat{u},e]$, which can also be used for closed-loop identification of the model M (see Fig. 6).

6. CLOSED-LOOP *ID* ERROR COMPARISONS

Introduce the additive

$$\Delta = S - M \quad ; \quad \Delta_+ = S_+ - M_+ \quad ; \quad \Delta_- = \overline{S}_- - \overline{M}_- \tag{30}$$

and relative model errors

$$\ell = \frac{\Delta}{M} = \frac{S-M}{M} \quad ; \quad \ell_+ = \frac{\Delta_+}{M_+} \quad ; \quad \ell_- = \frac{\Delta_-}{\overline{\overline{M}}_-} \tag{31}$$

In the sequel it is shown how the modeling errors of different *ID* methods depend on the relative model error ℓ.

Open-loop ID

$$\varepsilon_{\mathrm{ol}} = y - Mu' = M\,\ell\,u'\big|_{\ell\to 0;\,u'=y_r} \approx S\,\ell\,y_r \tag{32}$$

where u' used in open-loop is assumed equal to y_r.

Parallel-in-loop ID

$$\varepsilon_{\mathrm{pil}} = y - Mu = \left.\frac{P_r G_r M_- z^{-d}}{1+P_w G_w M_- z^{-d}\,\ell}\,\ell\,y_r\right|_{\ell\to 0} \approx \approx P_r G_r S_- z^{-d}\,\ell\,y_r\big|_{\overline{S}_-=1} = P_r\,\ell\,y_r \tag{33}$$

where the *ID* is performed in the closed-loop between u and y. It is interesting to note that in this case u depends also on the output noise w, which makes the input correlated (caused by the so-called "circulating noise"), therefore special further conditions are to be fulfilled.

ID based on KB parametrization

There is a natural possibility to perform *ID* avoiding the above "circulating noise" issue, namely to perform the *ID* between $\hat{u}$ (see Fig. 2) and y. In this approach (called *KB-parametrization* (Keviczky and Bányász (1999))) $\hat{u}$ depends on the apriori model estimate M_i, so only iterative scheme can be constructed.

$$\begin{aligned}\varepsilon_{\mathrm{KB}} &= y - M\hat{u}\big|_{\ell\approx 0} \approx \\ &\approx \left(P_r G_r S_- z^{-d}\right)\left(1-P_w G_w M_- z^{-d}\right)\ell\,y_r\big|_{\overline{S}_-=1} = \\ &= P_r\left(1-P_w\right)\ell\,y_r\end{aligned} \tag{34}$$

where

$$\hat{u} = P_r K_r\,y_r = P_r G_r M_+^{-1}\,y_r \tag{35}$$

ID using internal signal x

In this case theoretically the *ID* should be performed between u and x. Unfortunately this is a tautology, because $x = Mu$, however the idea can be used in an iterative scheme, where x_{i+1} is calculated by the apriori model M_i and the aposteriori model M_{i+1} is obtained between x_{i+1} and u_i. Instead another approach is suggested here to use $\hat{u}$ similarly to the previous case.

$$\begin{aligned}\varepsilon_x &= x - M\hat{u} = x - M\hat{u}\big|_{\ell\approx 0} \approx \\ &\approx -\left(P_r G_r S_- z^{-d}\right)\left(P_w G_w M_- z^{-d}\right)\ell\,y_r\big|_{\overline{S}_-=1} = \\ &= -P_r\,P_w\,\ell\,y_r\end{aligned} \tag{36}$$

ID using internal signal e

The modeling error is special in this case

$$\begin{aligned}\varepsilon_e &= e(u) - e(\hat{u}) = e(u) - e(\hat{u})\big|_{\ell\approx 0} \approx \\ &\approx -\left(P_r G_r S_- z^{-d}\right)\left(1-P_w G_w M_- z^{-d}\right)\left(P_w G_w M_- z^{-d}\right)\ell\,y_r\big|_{\overline{S}_-=1} = \\ &= -P_r\left(1-P_w\right)P_w\,\ell\,y_r\end{aligned} \tag{37}$$

This case - more or less - corresponds to the original *MPC* using a "quasi-linear" parameter estimation *ID* (12). This approach is based on the observation that the pole-placement results in an equivalent system equation

$$e^{\mathrm{F}} = S_-\left(P_r G_r y_r - P_w G_w y\right) = S_- e = S_+ S_- u^* \tag{38}$$

which also provides possibility for closed-loop *ID*.

The five cases are summarized in Table 1, where the different weighting factors H_j are shown for the different cases. Note that the accuracy of the estimated model at a given frequency is inverse proportional to the weight in the modeling error at that frequency. Observe that H_1, H_2 and H_4 are low-pass filters, where the attenuation of H_4 is the highest at the major medium frequency domain. So "good" model estimation can not be expected around the vital cross-over frequency ω_c using these cases. H_3 gives the best weighting, because its maximum is the geometrical mean of the tracking and regulating bandwidths. H_5 can also be used, because in this case its maximum is at a frequency little bit smaller than ω_c. ($H_3 = \left|(1-P_w)P_w\right|$ has its maximum at ω_c.)

Table 1 Error weighting functions

j	Type	ε	H_j
1	$\varepsilon_{\mathrm{ol}}$	$y - Mu'$	$\lvert S \rvert$
2	$\varepsilon_{\mathrm{pil}}$	$y - Mu$	$\lvert P_{\mathrm{r}} \rvert$
3	$\varepsilon_{\mathrm{KB}}$	$y - M\hat{u}$	$\lvert P_{\mathrm{r}}(1 - P_{\mathrm{w}}) \rvert$
4	ε_x	$x - M\hat{u}$	$\lvert P_{\mathrm{r}} P_{\mathrm{w}} \rvert$
5	ε_e	$e(u) - e(\hat{u})$	$\lvert P_{\mathrm{r}}(1 - P_{\mathrm{w}}) P_{\mathrm{w}} \rvert$

7. ITERATIVE IDENTIFICATION AND CONTROL

Only the method using the pair $[\hat{u}, y]$ is formulated here as an iterative algorithm. Since $\hat{u}$ depends on the model M^i, only an iterative control refinement procedure can be performed. It's simplest - so-called relaxation type - iteration can be built in the following way for an off-line case using n samples based on $\varepsilon_{\mathrm{KB}}$ (i-th iteration is shown):

1. Start from an initial model M^i. Calculate the invariant part M_-^i. Solve the *DE* (using $\mathcal{H}_2$ norm) or the *NP* problem (using $\mathcal{H}_\infty$ norm) to obtain the optimal filters $G_{\mathrm{r}}^i\left(M_-^i\right)$ and $G_{\mathrm{w}}^i\left(M_-^i\right)$ then using a reference signal series $\boldsymbol{y}_{\mathrm{r}}^i = \left\{ y_{\mathrm{r}}^i(k); k = 1, \ldots, n \right\}$ compute the process input as

$$u_i(k) = \frac{\left[M_+^i\right]^{-1}}{1 - P_{\mathrm{w}}\, G_{\mathrm{w}}^i M_-^i z^{-d_{\mathrm{m}}}} \left[P_{\mathrm{r}}\, G_{\mathrm{r}}^i\; y_{\mathrm{r}}^i(k) - P_{\mathrm{w}}\, G_{\mathrm{w}}^i\; y^i(k) \right]$$
$$k = 1, \ldots, n \qquad (39)$$

and apply to the process in closed-loop. Collect the measured signal series

$$\boldsymbol{u}^i = \left\{ u^i(k); k = 1, \ldots, n \right\} \;\; ; \;\; \boldsymbol{y}^i = \left\{ y^i(k); k = 1, \ldots, n \right\} \qquad (40)$$

2. Calculate the auxiliary variable series

$$\hat{u}_i(k) = P_{\mathrm{r}}\, G_{\mathrm{r}}^i \left[M_+^i\right]^{-1} y_{\mathrm{r}}^i(k) \quad ; \quad k = 1, \ldots, n \qquad (41)$$

and form $\hat{\boldsymbol{u}}^i = \left\{ \hat{u}_i(k); k = 1, \ldots, n \right\}$.

3. Using $\hat{\boldsymbol{u}}^i$ as input and $\boldsymbol{y}^i$ as output signal series, identify a model based on the modeling step

$$M^{i+1} = \arg \min_{M \in \mathcal{M}} J_{\mathrm{ic}}\left(\mathcal{M}, \boldsymbol{u}_i, \boldsymbol{y}_i, M^i, P_{\mathrm{r}}, P_{\mathrm{w}}\right) \qquad (42)$$

to get the next iterative model estimate, where J_{ic} is a closed-loop identification criterion with model class $\mathcal{M}$.

4. Note that $\boldsymbol{y}_{\mathrm{r}}^i$ does not necessarily change by iteration. However, there is a possibility to optimize the applied reference signal series $\boldsymbol{y}_{\mathrm{r}}^{i+1}$ in this step by a proper input design procedure or it is possible to use the same excitation $\boldsymbol{y}_{\mathrm{r}}^{i+1} = \boldsymbol{y}_{\mathrm{r}}^i$.

5. The iterative process is continued from step 1, while a stop condition is not fulfilled.

8. CONCLUSIONS

The usually applied $\mathcal{R}, \mathcal{S}, \mathcal{T}$ forms, generally applied in *MPC* algorithms, can be easily used for parameter estimation and direct formulation of self-tuning regulators. However, the detailed comparison of the pole-placement and predictor based controllers showed that the *d*-step ahead predictors "quasi-linear" in the parameters are very redundant considering the unknown parameters. Therefore a new structure was constructed to help the direct identification of the plant under the constraint of a special pole-placement controller in the framework of the *GTDOF* scheme. This new topology clearly shows the special part of the controller, which depends only on our design goals (for *IS* processes) and on the invariant process factor, too (for *IU* processes).

The modeling error of five possibilities were investigated how the frequency weighting factor changes by methods shaping the model error.

Then an iterative controller refinement scheme was formulated, which can be used for off-line combined identification and control solution.

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