

# DRIVER SUPPORT SYSTEM BASED ON A NON-LINEAR SLIP OBSERVER FOR OFF ROAD VEHICLES

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**Abstract:** Off road ground vehicles have many potential applications, including space, defence, agriculture, mining and construction. Increased autonomy of ground vehicles will not only improve the safety of the operators but also assist in trajectory tracking. Accurate estimation of slip is essential in developing autonomous navigation strategies for mobile off road vehicles operating in unstructured terrain. In this paper, a driver assistance and vehicle control capability to increase safety and efficiency by means of non-linear slip estimation is presented. A sliding mode observer and an Extended Kalman Filter are constructed to estimate slip parameters based on the kinematics model of a tracked vehicle and trajectory measurement. Autonomous driver support system for the rural road environment (off road) using estimated slip parameters is investigated. *Copyright © 2005 IFAC*

**Keywords:** Sliding mode observer; Extended Kalman Filter; Slip parameter estimation; Driver assistance; Tracked Vehicles.

## 1. INTRODUCTION

Unmanned ground vehicles (UGVs) have many potential applications, including space, defence, agriculture, mining and construction. Most unmanned ground vehicles are currently controlled by tele-operation. Tele-operation requires continuous and repetitive human intervention, which hampers the speed of the vehicle and the range of potential applications (Zweiri, et al., 2004). Further, they have problems due to bandwidth limitations and communication time delays of the transmission link. Increased autonomy of ground vehicles will not only improve the safety of the operators and but also increase the range of potential applications.

With tracked vehicles, track slips play an important role in determining vehicle tractive efforts and in the estimation of the vehicle position. Track slip includes left track slip, right track slip and the slip angle. The vehicle tractive forces are a function of track slip and soil properties (Wong, 1989). Slips are used to compute tractive forces based on a specific soil model. The drawbar pull (i.e. tractive forces minus resistance force)

is an indicator of the vehicle's ability to move through terrain. So slips are very important in the prediction of traversability.

It is difficult to directly measure slips of a ground vehicle (Le, 1999). Thus indirect methods are essential to be used for the estimation of slip parameters based on available sensor measurements.

Sliding mode observer has been used for many applications. In (Lumsdaine and Lang, 1990) a state observer is used to estimate the position and speed in switched reluctance motors. However, the linear model used leads to saturation. In (Islam, et al., 2003; McCann, et al., 2001), a sliding mode observer (SMO) is employed to overcome the problem associated with (Lumsdaine and Lang, 1990). (Zweiri, 2000) presented a sliding mode observer to estimate both the engine indicated and load torques. Extended Kalman Filter (EKF) is proposed to estimate slip parameters using kinematics model and simulation results given in (Le, 1997). The error in estimated track slips increments rapidly during a turn as it is a linear estimator for a nonlinear system.

(Le, 1999) handles the modelling the control of tracked vehicles, however, trajectory planning and autonomous control problems are ignored. (Ahmadi, et al., 2000) propose a tracking control method using time-varying states as feedback based on the back stepping technique. Its 'non-holonomic' velocity constraints assumption limited the practical applications. A robust control system has been developed in (Wang, 1988) using linearised soil models on the basis of quantitative feedback theory.

This paper presents a sliding mode observer for the estimation of track slips according to the kinematics equations and cheap sensor measurements. The validation of the observer is shown by comparing the measured track slips with the estimated track slips. A comparative study between sliding mode observer, Extended Kalman Filter has been investigated. The simulation and experimental results show the advantages of SMO over other two methods. A trajectory tracking control algorithm is proposed for the driver assistance and improvements on the autonomy. The controller drives the vehicle to follow the trajectory prescribed by the driver, thus improving the autonomy of off road vehicles.

The paper is arranged as follows: In the next Section, the kinematics equations of the vehicle are presented. In Section 3, SMO is presented. In Section 4, the performance of the SMO is compared with an EKF. A trajectory control algorithm is proposed for the driver assistance and improvements on the autonomy. Finally, conclusions are drawn.

## 2. VEHICLE KINEMATICS MODEL

Track slips change abruptly during a status transition from a straight line motion into steering with the variance of tractive force. Sliding Mode observer is designed to react properly to the situation where a small variance of slip is expected when the vehicle moves in straight line and a large variance of slips happens during a turn motion.

The kinematics equations for a tracked vehicle during skid-steering shown in Fig. 1 are given by (Le, 1999):

$$\dot{x} = \frac{r}{2} [\omega_o(1-i_o) + \omega_i(1-i_i)] [\cos \phi(t) - \sin \phi(t) \tan \alpha(t)] \quad (1.a)$$

$$\dot{y} = \frac{r}{2} [\omega_o(1-i_o) + \omega_i(1-i_i)] [\sin \phi(t) + \cos \phi(t) \tan \alpha(t)] \quad (1.b)$$

$$\dot{\phi} = \frac{\Delta V}{B} = \frac{r[\omega_i(1-i_i) - \omega_o(1-i_o)]}{B} \quad (1.c)$$

In Figs.1 and 2, (X,Y) is a fixed global frame, (x,y) is a local frame fixed to the vehicle,  $\phi$  is heading angle,  $\omega_o, \omega_i$  are the angular speeds of the outer and inner sprockets respectively,  $i_o, i_i$  are the slips of the outer and

inner tracks respectively,  $\alpha$  is the slip angle,  $r$  is the turning radius, and  $B$  is the tread of tracks,  $v_c$  is the velocity of the vehicle.

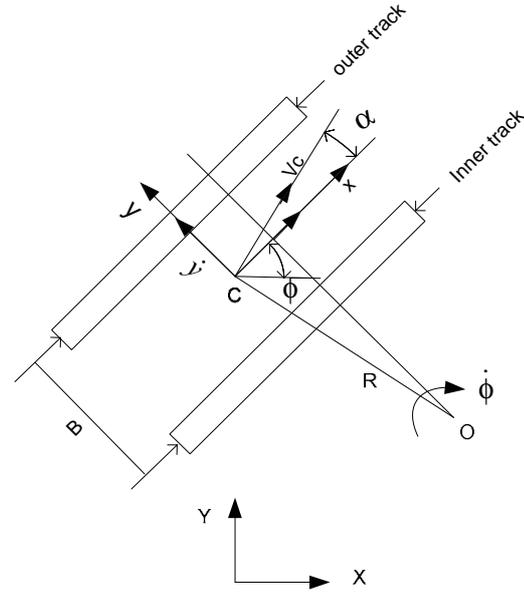


Fig. 1 Free body diagram of a tracked vehicle during steering

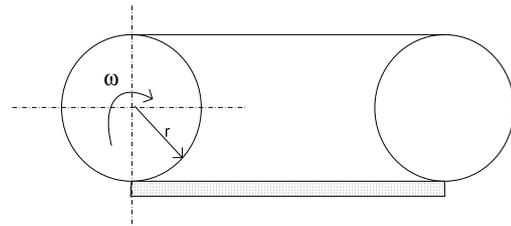


Fig. 2 Motion of a single track during steering

## 3. TRACK SLIP ESTIMATION SCHEMES

### 3.1 Nonlinear sliding mode observer design

An Utkin Sliding Mode Observer (Utkin and Shi, 1999) is designed to estimate slip parameters given the kinematics model of the vehicle and sensor feedback of the vehicle trajectory and the drive sprockets speed. Due to the non-linear nature of the vehicle kinematics equations, it is a complex problem to obtain estimates of the vehicle slip parameters. An Utkin observer is selected in this study since it is proven to be robust and stable for state reconstruction of non-linear systems (Le, 1999). The observer estimates the slip parameters such that the difference between the predicted and measured vehicle speed is minimized. With a sliding mode observer, the control action switches from one value to another in finite time, and this may cause chattering problems; to avoid this effect, the sliding mode gain of the slip estimator is designed as a nonlinear function of the trajectory. The sliding mode observer designed in this study constructs a supplementary system with a discontinuous switching component, and then intentionally creates a sliding motion in the supplementary system (McCann, et al., 2001). The

sliding mode observer is constructed to estimate slip parameters based on kinematics model. Optimization of the structure of the observer is used to minimize the error in velocity. So the observer takes the form of

$$\dot{\hat{x}} = \frac{r}{2} [\omega_o m_1 \text{sign}(e_1 + e_2) + \omega_i m_2 \text{sign}(e_1 + e_2)] \quad (2.a)$$

$$[\cos \phi - \sin \phi \tan(m_3 \text{sign}(e_3))] \quad (2.a)$$

$$\dot{\hat{y}} = \frac{r}{2} [\omega_o m_1 \text{sign}(e_1 + e_2) + \omega_i m_2 \text{sign}(e_1 + e_2)] \quad (2.b)$$

$$[\sin \phi + \cos \phi \tan(m_3 \text{sign}(e_3))] \quad (2.b)$$

$$\dot{\hat{\phi}} = \frac{r}{B} [-\omega_o m_1 \text{sign}(e_1 + e_2) + \omega_i m_2 \text{sign}(e_1 + e_2)] \quad (2.c)$$

Where,  $m_1, m_2, m_3$  are sliding mode gains,  $\hat{x}, \hat{y}$  are estimated velocities of the vehicle,  $\hat{\phi}$  is the estimated directional speed of the vehicle.  $e_1, e_2, e_3$  are the errors in velocity. The estimated slip parameters are as follows (MsCann, 2001)

$$\hat{i}_o = m_1 \text{sign}(e_1 + e_2),$$

$$\hat{i}_i = m_2 \text{sign}(e_1 + e_2), \hat{\alpha} = m_3 \text{sign}(e_3)$$

The observer errors are written as follows:

$$\dot{e}_1 = \dot{\hat{x}} - \dot{x}, \quad \dot{e}_2 = \dot{\hat{y}} - \dot{y}, \quad \dot{e}_3 = \dot{\hat{\phi}} - \dot{\phi}$$

Then the following error system can be obtained by subtracting Equations (1.a-1.c) from Equations (2.a-2.c).

$$\dot{e}_1 = \frac{r}{2} [\omega_o m_1 \text{sign}(e_1 + e_2) + \omega_i m_2 \text{sign}(e_1 + e_2)]$$

$$[\cos \phi - \sin \phi \tan(m_3 \text{sign}(e_3))] - \frac{r}{2} [\omega_o (1 - i_o)$$

$$+ \omega_i (1 - i_i)] [\cos \phi - \sin \phi \tan \alpha]$$

$$\dot{e}_2 = \frac{r}{2} [\omega_o m_1 \text{sign}(e_1 + e_2) + \omega_i m_2 \text{sign}(e_1 + e_2)]$$

$$[\sin \phi + \cos \phi \tan(m_3 \text{sign}(e_3))] - \frac{r}{2} [\omega_o (1 - i_o)$$

$$+ \omega_i (1 - i_i)] [\sin \phi + \cos \phi \tan \alpha]$$

$$\dot{e}_3 = \frac{r}{B} [-\omega_o (m_1 \text{sign}(e_1 + e_2) - i_o) +$$

$$\omega_i (m_2 \text{sign}(e_1 + e_2) - i_i)] \quad (3)$$

In order to use the measured position, error system Equations (3.a-3.c) has to be discretized. The discrete-time formulation of the vehicle is developed from the continuous-time kinematics equations. State space equation is obtained by integrating the continuous-time equations over the time interval from  $t_k$  to  $t_{k+1}$ . The motion of the vehicle is, however, a non-holonomic process: the motion is described according to velocity constraints of the two tracks which can not be integrated to yield an analytical close form solution. The trajectory of the vehicle changes whenever one or both track velocities change. The discrete-time formulation is therefore just an approximation of the continuous-time model assuming a zero-order hold. The first-order Euler approximation is then adequate for the integrator:

$$e_1(k+1) = e_1(k) + \frac{r}{2} \Delta T [\omega_o m_1 \text{sign}(e_1(k) + e_2(k))$$

$$+ \omega_i m_2 \text{sign}(e_1(k) + e_2(k))] [\cos \phi - \sin \phi$$

$$\tan(m_3 \text{sign}(e_3(k)))] - \Delta T \frac{r}{2} [\omega_o (1 - i_o)$$

$$+ \omega_i (1 - i_i)] [\cos \phi(t) - \sin \phi(t) \tan \alpha(t)]$$

$$e_2(k+1) = e_2(k) + \Delta T \frac{r}{2} [\omega_o m_1 \text{sign}(e_1(k) + e_2(k))$$

$$+ \omega_i m_2 \text{sign}(e_1(k) + e_2(k))] [\sin \phi + \cos \phi$$

$$\tan(m_3 \text{sign}(e_3(k)))] - \Delta T \frac{r}{2} [\omega_o (1 - i_o)$$

$$+ \omega_i (1 - i_i)] [\sin \phi(t) + \cos \phi(t) \tan \alpha(t)]$$

$$e_3(k+1) = e_3(k) + \Delta T \frac{r}{B} [-\omega_o (m_1 \text{sign}(e_1 + e_2) - i_o)$$

$$+ \omega_i (m_2 \text{sign}(e_1 + e_2) - i_i)] \quad (4)$$

where,  $\Delta T$  -sampling time interval.

It is stated that the slip angle is zero when the vehicle tracks straight line, and has some signed value when the vehicle is steering. That is, the slip angle appears only when the vehicle is turning. (Le, 1999) relates the slip angle with the directional speed of the vehicle from the experiments, thus the observer gain has a relationship with directional speed. It is possible to choose sliding mode gain as the function of directional speed of the vehicle.

The estimation of slip angle equals the measured value when  $e_3=0$ , as the slip angle is a strong function of directional speed of the vehicle.

Substituting  $e_3=0$  into Equation (3.c) yields

$$\frac{r}{B} [-\omega_o (m_1 \text{sign}(e_1 + e_2) - i_o) + \omega_i (m_2 \text{sign}(e_1 +$$

$$e_2) - i_i)] = 0$$

Equations (3.a) and (3.b) will be transformed into

$$\frac{r}{2} [\omega_o (\text{sign}(e_1 + e_2) - i_o) + \omega_i (\text{sign}(e_1 + e_2) - i_i)]$$

$$[\cos \phi - \sin \phi \tan \hat{\alpha}] = 0$$

$$\frac{r}{2} [\omega_o (\text{sign}(e_1 + e_2) - i_o) + \omega_i (\text{sign}(e_1 + e_2) - i_i)] \quad (5)$$

$$[\sin \phi + \cos \phi \tan \hat{\alpha}] = 0$$

From Equations (5.a-5.c), we obtain

$$\hat{i}_o = m_1 \text{sign}(e_1 + e_2) \approx i_o \quad (6.a)$$

$$\hat{i}_i = m_2 \text{sign}(e_1 + e_2) \approx i_i \quad (6.b)$$

Substituting Equations (6.a) and (6.b) into Equation (3), we get

$$\hat{\alpha} = m_3 \text{sign}(e_3) \approx \alpha$$

### 3.2 Stability of the Sliding Mode Observer

If the observer gains satisfy the sign condition, then as

$$\hat{i}_o \approx i_o$$

$$\hat{i}_i \approx i_i$$

$$\hat{\alpha} \approx \alpha \quad (7)$$

Substituting Equation (7) into Equation (3) yields

$$\dot{e}_3 = 0 \quad (8)$$

In order to derive the sliding mode condition, the Lyapunov function V is defined as:

$$V = \beta e_3^2$$

where,  $\beta$  -any positive number

$$\dot{V} = 2\beta e_3 \dot{e}_3$$

The system is stable only if  $e_3 \dot{e}_3 \leq 0$ .

From (8), we know that

$$\dot{V} = 0$$

Hence, the system is globally stable.

### 3.3 Extended Kalman Filter observer

The Extended Kalman Filter is a recursive and minimum mean-square-error estimator. It estimates the state of a dynamic system by combing observations from sensors with the state of prediction. This procedure is recursive; it means there is no need for reprocessing of previous observations to add a new set of observations. Noise is assumed to be zero-mean, Gaussian distribution. A slip estimator is developed for the estimation of slip parameters according to the kinematics model. The following details the design of estimator for the tracked vehicle moving over a terrain. Track slips change slowly when the vehicle tracks a straight line, while track slip change abruptly when the vehicle is turning because larger tractive efforts are developed to overcome the resistance caused by the moment of turning resistance. "jump theory" is employed to deal with such case. The variance is big when the vehicle turns left or right and small variance while tracking straight line. Thus this allows the Extended Kalman Filter to react properly when the vehicle switches from straight line motion into steering. Equations (1.a-1.c) can be converted into the following discrete-time representation using first-order Euler method.

$$X_1(K) = f(X_1(k-1), u(k-1), v(k-1), k-1)$$

$$= \begin{bmatrix} x(k-1) + 0.5\Delta Tr[\bar{i}_o(k-1)w_o(k) + \bar{i}_i(k-1)w_i(k)] \\ [\cos \phi(k-1) - \sin \phi(k-1) \tan \alpha(k-1)] \\ y(k-1) + 0.5\Delta Tr[\bar{i}_o(k-1)w_o(k) + \bar{i}_i(k-1)w_i(k)] \\ [\sin \phi(k-1) - \sin \phi(k-1) \tan \alpha(k-1)] \\ \phi(k-1) + \frac{\Delta Tr}{B}[\bar{i}_o(k-1)w_o(k) - \bar{i}_i(k-1)w_i(k)] \\ \bar{i}_o(k-1) \\ \bar{i}_i(k-1) \\ \alpha(k-1) \end{bmatrix} \quad (9)$$

where,  $\bar{i}_o(k-1) = 1 - i_o(k-1)$ ,  $\bar{i}_i(k-1) = 1 - i_i(k-1)$

$$u(k-1) = [\omega_o(k-1), \omega_i(k-1)]^T$$

$$X_1(k-1) = [x(k-1), y(k-1), \phi(k-1), \bar{i}_o(k-1), \bar{i}_i(k-1), \alpha(k-1)]^T$$

From Equations (1.a-1.c), the state transition equation is nonlinear so that Extended Kalman Filter is required for the estimation. Process and observation models are as follows

$$x(k) = f[x(k-1), u(k), k] + v(k)$$

$$z(k) = h[x(k)] + w(k)$$

where,  $v(k)$  is the process noise,  $z(k)$  is the observation,  $w(k)$  is the measurement noise.

The Extended Kalman Filter (EKF) is used to estimate slip parameters while a tracked vehicle traversing over unprepared terrain. During moving, the EKF receives the control inputs  $u$  and position from sensors as observations. The EKF produces the estimation of slip parameters and trajectory of the vehicle.

## 4. Trajectory Tracking Control of Tracked Vehicles

The main difficulty arising in the control of tracked vehicles is due to the high resistance forces. Nonlinear velocity dependant or force dependant lateral resistance terms are included to the kinematics model to make trajectory planning and control complicated.

PID controller is widely used in mining industry, construction and military applications due to its simplicity and easy implementation. Fig.3 shows the structure of the controller proposed. The control system is comprised of controller, observers (Kalman filter and sliding mode), plant and measurements from on-board sensors. Measurements include trajectory  $(x, y, \phi)$ , angular velocities of drive sprockets  $(\omega_o, \omega_i)$ . Control inputs  $u$  are angular velocities of drive sprockets  $\omega_o, \omega_i$ . Plant is mainly based on the kinematics model of the vehicle. Outputs are vehicle trajectory  $(x, y)$ .

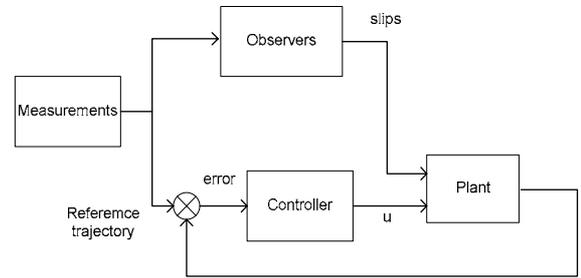


Fig.3 control scheme

The kinematics model of the vehicle is expressed as discrete-time representation to feedback trajectory. The control system incorporates an extended Kalman filter which will estimate the vehicle's slip parameters including slip on the left track, slip on the right track and slip angle as plant inputs, thus ensuring the vehicle to follow the prescribed trajectory. The Kalman filter estimation component is done off-line for the simplicity.

The Kalman filter receives the control input from the measurements data files and generates the estimation of slip parameters. The estimated slip parameters combined with measured angular velocities of drive sprockets are fed to the plant to obtain the estimated trajectory and then used as feed backs. The controller is designed to minimize the error in the estimated trajectory and measured trajectory with the aid of the observers.

## 5. ESTIMATION RESULTS

The estimation experiments are conducted using unoptimized MATLAB code and executed using a 3.0 GHz Pentium IV processor. Experimental data such as tractive forces, angular velocities of the sprockets, directional speed of the vehicle and heading angle are provided by Le (Le, 1999). Track slips are obtained by solving nonlinear equation in (Wong, 2001) using numeric methods given the soil properties  $(c, \phi, K)$  and tractive forces. As mentioned above, slip angle is a function of directional speed of the vehicle. It is stated that the slip angle is a function of angular velocity of the vehicle (Le, 1999).

$$\alpha = \arctan \frac{\dot{\phi}^2}{4g\mu_l} \quad (10)$$

Where,  $\mu_l$  coefficient of longitudinal resistance. So slip angle is calculated using Equation (10). Vehicle trajectory is measured by PLS laser scanner produced by Sick Optick Electronic (Le, 1999). These data are defined as measurements. For all the observers, input variables are trajectory  $(X, Y)$  and heading angle  $(\phi)$ . Output variables are  $i_o, i_i, \alpha$ . The observers receive control inputs  $\omega_o, \omega_i$  and the trajectory  $X, Y, \phi$  from the measurement datasets. The observers are then driven to estimate slip parameters.

Table 1 shows the root-mean-square error in slip parameters by comparing results from observers with measurements. Time represents the computer processing unit time in second. In Table 2, noise is added to the model for the verification of robustness of the observers.

**Table 1**  
Measurements Without Noise

Slips		$i_o$	$i_i$	$\alpha$
SMO	RMS	0.017	0.026	0.037
	TIME	0.16		
EKF	RMS	0.0429	0.0353	0.148
	TIME	0.178		

**Table 2**  
Measurements With Noise

Slips		$i_o$	$i_i$	$\alpha$
SMO	RMS	0.024	0.033	0.045
	TIME	0.234		
EKF	RMS	0.0477	0.0399	0.148
	TIME	0.297		

The accuracy and superior performance of the sliding mode observer over EKF is shown in experimental results.

It can be easily seen that the error in trajectory is acceptable because slip parameters are bounded in the Extended Kalman Filter algorithm as shown in Fig.4. The error in trajectory becomes bigger only when the vehicle is steering. By comparison with EKF, Sliding Mode Observer tracks the trajectory well as shown in Fig.5. The estimated heading angle is very close to the measurement of heading angle. Finally, the accuracy and robustness of slip estimator is validated against measured data.

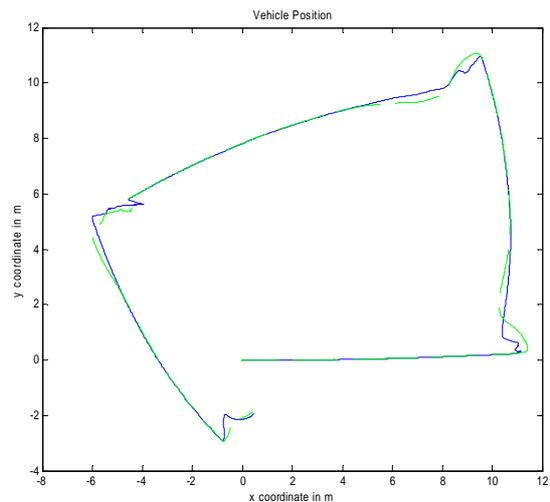


Fig.4 Comparison between measured (—) and predicted (---) vehicle trajectory using Extended Kalman Filter

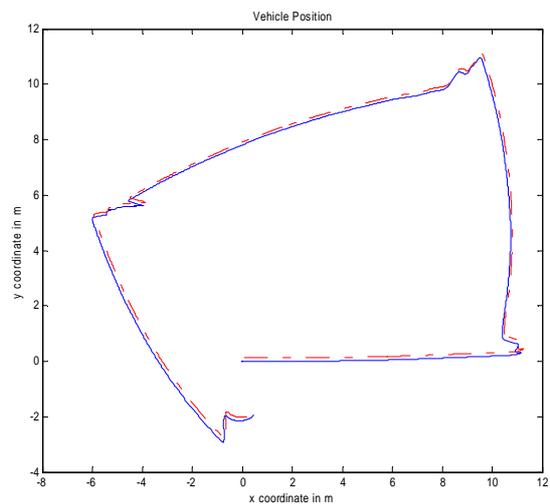


Fig.5 Comparison between measured (—) and predicted vehicle (---) trajectory using Sliding Mode Observer

The average relative error in x component of trajectory is 0.0001. The average relative error in x component of trajectory is 0.001. Fig.6. shows the comparison between the measured trajectory and controlled

