STABILITY SWITCHES AND REVERSALS OF LINEAR SYSTEMS WITH COMMENSURATE DELAYS: A MATRIX PENCIL CHARACTERIZATION [1]

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Abstract: This paper addresses the problem of asymptotic stability of linear time-delay systems including commensurate delays. More precisely, we focus on the characterization of stability switches and reversals using a matrix pencil approach. The proposed approach makes use of the generalized eigenvalue distribution with respect to the unit circle of some appropriate finite-dimensional matrix pencils. Classical problems, as for example, hyperbolicity and delay-independent/delay-dependent stability characterizations are reconsidered, and simple computational conditions are derived. *Copyright ©2005 IFAC*

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1. INTRODUCTION

The main ideas for developing algebraic criteria, numerically tractable for the stability analysis of delay systems including single or multiple commensurate delays can be found in the works of (Kamen, 1980, 1982), (Cooke and Grossman, 1982) , and (Rekasius, 1980) back in the 80s.

In parallel to the approach of Kamen, (Rekasius, 1980) proposed a different way to handle the problem (the so-called *pseudo-delay* technique, see also MacDonald 1989), which consists in using a bilinear transformation (or substitution) for rewriting the delay system as a parameter-dependent polynomial, relation which is valid only on the imaginary axis. In this framework, the stability problem is reduced to the analysis of the roots distribution of some algebraic polynomial, using the connection between the original quasipolynomials and the corresponding parameter-dependent polynomials with respect to the imaginary axis (Thowsen 1981), combined with the standard Routh-Hurwitz stability criteria.

In a completely different framework, Cooke and Grossman, 1982) focused on the complete characterization of the asymptotic stability of a second-order linear delay differential equation including a single delay, and their ideas concerning switches characterization, root crossings and related properties are more general, as reflected by (Cooke and van den Driessche, 1986).

Each approach received a lot of consideration as suggested by the contributions (Hertz *et al.*, 1984) (Kamen's approach), (Olgac and Sipahi, 2002) (Rekasius approach), and (Beretta and Kuang, 2002) (Cooke, Grossman and van den Driessche

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approach). Next, an interesting new approach was proposed by (Walton and Marshall, 1987) where, excepting some overlappings with (Cooke and van den Driessche, 1986), the authors proposed a nice method to reduce the commensurate delays case to the study of a some particular quasipolynomial including only one delay, and, as in the previous cases, the correspondence between these forms is with respect to the imaginary axis. Finally, we arrive to the contributions of Chen, Gu and Nett (Chen *et al.*, 1995) (state-space representations), where the authors suggested the use of matrix pencils to characterize the *delay-independent* stability, as well as the first delay-interval guaranteeing stability, otherwise. Some extensions of the method have been proposed in (Niculescu, 1998). Further discussions, comments, and other references can be found in (Gu *et al.*, 2003), (Niculescu, 2001).

This paper focuses on the characterization of *stability switches* (roots of the characteristic equation crossing the imaginary axis from stability to instability) and *reversals* (roots crossing the imaginary axis from instability to stability) for a class of linear time-delay systems including *commensurate* delays (delays in rational dependence) in the light of the contributions of (Cooke and van den Driessche, 1986). The proposed approach makes use of the distribution with respect to the unit circle of the generalized eigenvalues of some appropriate finite-dimensional matrix pencil (Chen *et al.*, 1995), (Niculescu, 1998), and the derived conditions are *simple*, *easy to check*, and to the best of the authors' knowledge, there does not exist any similar result in the literature. Various extensions, and applications are also proposed, as for example, the *hyperbolicity* (Hale *et al.*, 1985), and *delay-interval* stability (Kharitonov and Niculescu, 2003).

The remaining paper is organized as follows: Section 2 includes some preliminary results concerning the roots of characteristic function of linear systems with commensurate delays. Section 3 is devoted to the main results, i.e. crossing roots characterization using the computation of generalized eigenvalues of appropriate matrix pencils. Various applications are presented in Section 4, and some concluding remarks end the paper.

Notations: The following notations will be used throughout the paper: $\mathbb{R}$ ($\mathbb{C}$) denotes the set of real (complex) numbers, $\mathcal{C}(0,1)$ denotes the unit circle in the complex plane; $\sigma(M)$ represents the set of eigenvalues of the complex matrix $M \in \mathbb{C}^{n\times n}$; $\mathbb{C}^+$ ($\mathbb{C}^-$) denotes the open right (left) half complex plan; $In(M) = (\pi(M), \nu(M), \delta(M))$ is the inertia of the complex matrix $M \in \mathbb{C}^{n\times n}$, where $\pi(M)$, $\nu(M)$ and $\delta(M)$ denote the number of eigenvalues with negative ($\mathbb{C}^-$), positive ($\mathbb{C}^+$) and zero real parts ($j\mathbb{R}$). Next, $\mathcal{C}_{n,\tau} = \mathcal{C}([-\tau, 0], \mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$ with the topology of uniform convergence. Finally, the following norms will be used: $\| \cdot \|$ refers to the Euclidean vector norm; $\| \phi \|_c = \sup_{-\tau\le t\le 0} \| \phi(t) \|$ stands for the norm of a function $\phi \in \mathcal{C}_{n,\tau}$. Moreover, we denote by $\mathcal{C}^v_{n,\tau}$ the set defined by $\mathcal{C}^v_{n,\tau} = \{\phi \in \mathcal{C}_{n,\tau} \;:\; \| \phi \|_c < v\}$, where v is a positive real number.

2. PRELIMINARY RESULTS

Consider the following class of linear systems with commensurate delays:

$$\dot{x}(t) = Ax(t) + \sum_{k=1}^{n_d} A_k x(t - k\tau), \qquad (1)$$

with the initial condition:

$$x_{t_0}(\theta) = \phi(t_0 + \theta), \qquad \forall \theta \in [-n_d\tau, 0], \qquad (2)$$

where $\phi \in \mathcal{C}_{n,n_d\tau}$, and $x \in \mathbb{R}^n$. The delay τ is assumed strictly positive, and n_d is a positive integer denoting the number of commensurate delays.

The characteristic function associated to (1)-(2) is defined by:

$$\mathcal{P}(s) := \mathbf{det}\left(sI_n - A - \sum_{k=1}^{n_d} A_k e^{-sk\tau} \right). \qquad (3)$$

It is well-known that the system (1)-(2) is *asymptotically stable* if all the roots of the quasipolynomial (3) are located in $\mathbb{C}^-$ (see, for instance, (Bellman and Cooke, 1963)).

In the sequel, we shall focus on the analysis of the behavior of the roots of the characteristic quasipolynomial with respect to the parameter τ, when the delay is increased from 0 to $+\infty$. More explicitly, we shall compute *all* the *delay values* for which at least one root of the characteristic function lies on the imaginary axis $j\mathbb{R}$. In this sense, introduce the following applications $u, l : \mathbb{R}_+ \times \mathbb{R}_+ \mapsto \overline{\mathbb{R}}$ (u, l for upper and lower, respectively):

$$u(\tau,\alpha) = \mathbf{max}\bigg\{ Re(\lambda) \le 0 : \; \mathbf{det}(\lambda I_n - A - \sum_{k=1}^{n_d} A_d e^{-j\lambda\alpha k\tau}) = 0 \bigg\}, \qquad (4)$$

$$l(\tau,\alpha) = \mathbf{min}\Big\{ Re(\lambda) \geq 0 : \mathbf{det}(\lambda I_n - A - \sum_{k=1}^{n_d} A_d e^{-j\lambda\alpha k\tau}) = 0 \Big\}, \quad (5)$$

with $u(\tau,\alpha) = -\infty$ and $l(\tau,\alpha) = +\infty$ if the corresponding sets are empty. It was proved in (Datko, 1978) (Hale *et al.*, 1985) that both functions u and l are *continuous* in the variables α and τ. Furthermore, using the same arguments as in (Cooke and Grossman, 1982) (based on the Rouché's theorem), one can prove that:

Proposition 1. As the delay continuously varies in $\mathbb{R}_+$, the number of zeros of the characteristic function $\mathcal{P}$ of (1)-(2) on the right half-plane of the complex plane can change only if one zero appears on or crosses the imaginary axis.

The functions u and l defined above together with the results of the Proposition 1 give the complete picture of the behavior of the roots λ of the quasipolynomial $\mathcal{P}$ as functions of τ. Indeed, the computation of the delay values corresponding to the crossing roots will define, after reordering, *all* the delay-intervals guaranteeing asymptotic stability or with a given number of strictly unstable roots, since the number of roots in $\mathbb{C}^+$ does not change between two "adjacent" delays (Proposition 1 above). In conclusion, we get the so-called *D-subdivision* of the stability/instability regions in the sense precised by (Neimark, 1949).

3. MAIN RESULTS

Consider the system (1)-(2), and introduce now the following matrix pencil (Chen *et al.*, 1995):

$$\Sigma = zM + N, \quad (6)$$

associated to the dynamical system (1)-(2), where $M,\ N \in \mathbb{R}^{(2n_d n^2)\times(2n_d n^2)}$ are given by:

$$M = \begin{bmatrix} I_{n^2} & 0 & \dots & 0 & 0 \\ 0 & I_{n^2} & \dots & 0 & 0 \\ & & \ddots & & \\ 0 & 0 & \dots & I_{n^2} & 0 \\ 0 & 0 & \dots & 0 & B_{n_d} \end{bmatrix}, \quad (7)$$

$$N = \begin{bmatrix} 0 & -I_{n^2} & 0 & \dots & 0 \\ 0 & 0 & -I_{n^2} & \dots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \dots & -I_{n^2} \\ B_{-n_d} & B_{-n_d+1} & B_{-n_d+2} & \dots & B_{n_d-1} \end{bmatrix} \quad (8)$$

with B_{-k} $(k = \overline{1,n_d})$, B_i $(i = \overline{1,n_d})$ given by:

$$B_{-k} = I_n \otimes A_k^T, \qquad B_i = A_i \otimes I_n, \qquad B_0 = A \oplus A^T,$$

where $\otimes$, $\oplus$ denotes the product and the sum of Kronecker, respectively.

Using the definitions, properties and notations above, we have the following result:

Proposition 2. (crossing characterization). Assume that the matrix pencil Σ is regular. Then the quasipolynomial $\mathcal{P}$ has a crossing root on the imaginary axis for some positive delay value τ_0 if and only if the following conditions are satisfied simultaneously:

(i) The matrix pencil Σ has generalized eigenvalues on the unit circle;
(ii) There exists some $z_0 \in \sigma(\Sigma) \cap \mathcal{C}(0,1)$, such that:

$$\sigma\left(A + \sum_{k=1}^{n_d} A_k z_0^k\right) \cap j\mathbb{R}^* \neq \emptyset. \quad (9)$$

Furthermore, for some z_0 satisfying the condition (ii) above, the set of delays corresponding to the induced crossing is given by:

$$\mathcal{T}(z_0) = \Big\{ \frac{Log(\overline{z_0})}{j\omega_0} + \frac{2\pi\ell}{\omega_0} > 0 : j\omega_0 \in \sigma\left(A + \sum_{k=1}^{n_d} A_k z_0^k\right) - \{0\}, \quad \ell \in \mathbb{Z} \Big\} \quad (10)$$

where $Log(\cdot)$ denotes the principal value of the logarithm.

Proof: $\Leftarrow$ The condition is derived straightforwardly by construction. Indeed, it follows from (9) that there exists some $j\omega_0 \neq 0$ on the imaginary axis such that:

$$\mathbf{det}\left(j\omega_0 I_n - A - \sum_{k=1}^{n_d} A_k z_0^k\right) = 0.$$

The remaining problem is to prove that there exists at least one positive delay value τ_0 satisfying $z_0 = e^{-j\omega_0\tau_0}$, which is true since the general solution τ of the complex equation above has the form:

$$\tau = \frac{Log(\overline{z_0})}{j\omega_0} + \frac{2\pi\ell}{\omega_0},$$

with $\ell \in \mathbb{Z}$, and we can always choose a large positive $\frac{2\pi\ell}{\omega_0}$, that makes the corresponding quantity τ positive. Next, it is clear that the set of all delays τ generating such a root crossing is not finite, and is given by $\mathcal{T}(z_0)$ as defined by the relation (10).

$\Rightarrow$ By contradiction. Assume that the quasipolynomial $\mathcal{P}$ has at least one (non-zero) root on the imaginary axis $j\omega_0$ for some delay $\tau_0 > 0$, but the matrix pencil Σ does not have any generalized eigenvalues satisfying the conditions *(i)-(ii)* above.

In such a case, it follows that the characteristic equation associated to the quasipolynomial (3) is satisfied by $s = j\omega_0 \neq 0$, that is:

$$\mathbf{det}\left(j\omega_0 I_n - A - \sum_{k=1}^{n_d} A_k e^{-j\omega_0 k\tau_0}\right) = 0, \quad (11)$$

and thus, if we define $z_0 = e^{-j\omega_0\tau_0}$, the condition (9) is satisfied. Thus, since we assumed that the matrix pencil Σ does not have any generalized eigenvalues satisfying the conditions *(i)-(ii)* above, it follows that $z_0 \notin \sigma(\Sigma)$. In the same time, if $j\omega_0$ satisfies (11), it follows that $-j\omega_0$ satisfies:

$$\mathbf{det}\left(-j\omega_0 I_n - A^T - \sum_{k=1}^{n_d} A_k^T \overline{z_0}^k\right) = 0. \quad (12)$$

In conclusion,

$$\begin{aligned} &D(z_0) \\ &= \mathbf{det}\left[\left(A + \sum_{k=1}^{n_d} A_k z_0^k\right) \oplus \left(A + \sum_{k=1}^{n_d} A_k z_0^k\right)^H\right] \\ &= 0, \end{aligned} \quad (13)$$

since the matrices defining the Kronecker sum in (13) have complex conjugate roots $\pm j\omega_0$ on the imaginary axis. Next, using the developments in (Chen *et al.*, 1995) (see also (Gu *et al.*, 2003), (Niculescu, 2001)), it follows that

$$D(z_0) = z_0^{n_d}\mathbf{det}\left(z_0 M + N\right),$$

with M, N given by (8). Finally, since $z_0 \neq 0$, the matrix pencil Σ has, at least, one generalized eigenvalue on the unit circle z_0, for which (9) is satisfied, which contradicts the assumption. The proof is complete.

Definition 3. A complex z_0 satisfying the conditions (ii) in Proposition 2 will be called a crossing generator, and denote σ_g the set of all such crossing generators. Then

$$\mathcal{T} = \bigcup_{z\in\sigma_g} \mathcal{T}(z) \quad (14)$$

will be called the delay crossing generator set.

Using the definition above, Proposition 2 simply says that *the existence of crossing roots is equivalent to the property that the delay crossing generator set is not empty.*

Remark 4. (simple/multiple crossings). The Proposition 2 describes all the cases for which crossings may appear, and it does not make any distinction between simple, and multiple root crossings.

Such a distinction will be addressed in the next paragraphs, where the *crossing direction* (from left to right i.e. towards *instability*, or from right to left, i.e. towards *stability*) will be characterized, under the assumption that the crossing roots are *simple*.

Remark 5. (generalized eigenvalues properties). It is important to notice that *not all* the generalized eigenvalues of Σ will generate *crossing roots*.

Indeed, it follows from the definition of Σ that the solutions z on the unit circle of the complex plane of the equation (Chen *et al.*, 1995), (Niculescu, 1998):

$$\mathbf{det}\left[\left(A + \sum_{k=1}^{n_d} A_k z^k\right) \oplus \left(A + \sum_{k=1}^{n_d} A_k z^k\right)^H\right] = 0$$

include not only the crossing generators, but also all the *symmetric roots* (with respect to the origin of the complex plane) of the characteristic function (3), when the delay is seen as a *free parameter*.

Remark 6. (Algorithm). The procedure for the computation of the crossing roots is straightforward:

- First, we compute the generalized eigenvalues of the matrix pencil Σ on the unit circle of the complex plane (under the assumption of regularity of Σ);
- Next, for each generalized eigenvalue $z \in \mathcal{C}(0,1) \cap \sigma(\Sigma)$, we compute the eigenvalues on the imaginary axis of the complex matrix $A + \sum_{k=1}^{n_d} A_k z^k$, and the corresponding delay crossing generator set $\mathcal{T}(z)$.

For the sake of simplicity, we shall focus only on the characterization of *simple* root crossings. The general case can be treated in a similar manner, and is omitted.

We have the following result:

Proposition 7. Assume that the crossing roots are simple, and let $z_0 \in \sigma_g$ be a crossing generator of some root $j\omega_0 \neq 0$ of the delay system (1)-(2).

Then, we have a root crossing the imaginary axis towards instability (stability) if and only if:

$$Re\left\{\frac{\omega_0}{ju^*v}\sum_{k=1}^{n_d} k z_0^k u^* A_k v\right\} > 0 (< 0), \quad (15)$$

where u^* and v are row and respectively column eigenvectors of the corresponding $j\omega_0$-eigenvalue of the complex matrix $A + \sum_{k=1}^{n_d} A_k z_0^k$.

The proof is based on the so-called Jacobi's formula for computing the differential of the determinant of some square matrix M:

$$d\mathbf{det}(M) = \mathbf{Tr}\left(\mathbf{Adj}(M) dM\right),$$

where dM, and $d\mathbf{det}(M)$ define the differentials of A, and of its determinant, respectively.

A special case study easy to check is represented by the situation when all the matrices A_k are of *rank* one of some special form $A_k = vu_k^T$, for all $k = 1, \ldots n_d$. Then Proposition 7 rewrites as follows:

Proposition 8. Assume that the matrices $A_k = vu_k^T$ are of rank one, for all $k = 1, \ldots, n_d$, and assume also that the crossing roots are simple, and let $z_0 \in \sigma_g$ be a crossing generator of some root $j\omega_0 \neq 0$ of the delay system (1)-(2).

Then, we have a root crossing the imaginary axis towards instability (stability) if and only if:

$$Re\left\{\frac{\omega_0}{j}\sum_{k=1}^{n_d} k z_0^k v^T u_k\right\} > 0 (< 0). \quad (16)$$

Remark 9. (Numerics). The computation of the root crossing direction is straightforward, and it makes use only of the computation of the *eigenvectors* of the complex matrix $A + \sum_{k=1}^{n_d} A_k z_0^k$.

Remark 10. The results in Proposition 7 represent a natural extension of the root crossing characterization proposed by (Cooke and van den Driessche, 1986), and (Cooke and Grossman, 1982) to the state-space representation of the delay system. To the best of the authors knowledge, there does not exist any similar results in the literature.

4. APPLICATIONS

In this section, we shall reconsider various problems encountered in the literature in the light of the results proposed above. First, we focus on characterizing the case when there are no crossings with respect to the imaginary axis for any delay value, that is the so-called *hyperbolicity* in the sense mentioned by Hale, Infante and Tsen in (Hale *et al.*, 1985). In other words, the number of strictly unstable roots is *constant* for all delay values, including also the case free of delay.

We have the following result (Niculescu, 1998):

Proposition 11. Assume that the system free of delays has no roots on the imaginary axis. Then the delay system (1)-(2) is hyperbolic if and only if the set $\mathcal{T}$ defined by (14) is empty.

Remark 12. As seen in (Hale *et al.*, 1985), the assumption on the system free of delay can be relaxed to the condition that the matrix $A + \sum_{k=1}^{n_d} A_k$ is *nonsingular*.

In terms of generalized eigenvalues characterization for the corresponding matrix pencil Σ, the condition on the delay crossing generator set $\mathcal{T}$ has a more complicated description (see, for instance, (Niculescu, 1998, 2001)):

Proposition 13. The delay crossing generator set $\mathcal{T}$ is empty if and only if the matrix pencil Σ has no generalized eigenvalues on the unit circle of the complex plane, or if it does, all its zeros z_0 should be either roots of the matrix polynomial $\mathcal{P}_2(z) = A + \sum_{k=1}^{n_d} A_k z_0^k$, either they should satisfy the inertia condition:

$$In\left(A + \sum_{k=1}^{n_d} A_k z_0^k\right) = In\left(A + \sum_{k=1}^{n_d} A_k\right). (17)$$

Remark 14. As seen in (Niculescu, 1998), we can use a second matrix pencil for computing the roots of the matrix polynomial $\mathcal{P}_2$. Thus, the result can be more simplified using the relations between the generalized eigenvalues of the corresponding matrix pencils.

5. CONCLUDING REMARKS

This paper addressed the characterization of roots distribution with respect to the imaginary axis of some class of delay systems with delays in rational dependence. More explicitly, we proposed a simple way to compute the switches (roots crossing the imaginary towards instability) and reversals (crossing roots towards stability) as functions of the delay parameter. The corresponding conditions are expressed in terms of generalized eigenvalue distribution of some appropriate matrix pencil defined by the associate characteristic function.

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