RECEDING HORIZON NEURAL $\mathcal{H}_\infty$ CONTROL FOR A CLASS OF NONLINEAR UNKNOWN SYSTEMS

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Abstract: In this paper, we present new RHNHC (Receding Horizon Neural $\mathcal{H}_\infty$ Control) for nonlinear unknown systems. First, we propose LMI (Linear Matrix Inequality) condition on the terminal weighting matrix for stabilizing RHNHC. Under this condition, noninceasing monotonicity of the saddle point value of the finite horizon dynamic game is shown to be guaranteed. Then, we propose RHNHC for nonlinear unknown systems which guarantees the infinite horizon $\mathcal{H}_\infty$ norm bound and the internal stability of the closed-loop systems. Since RHNHC can deal with input and state constraints in optimization problem effectively, it does not cause an instability problem or give a poor performance in contrast to the existing neural $\mathcal{H}_\infty$ control schemes.

Keywords: Receding horizon control, Neural networks, $\mathcal{H}_\infty$ control, Nonlinear systems, Unknown systems

1. INTRODUCTION

Receding horizon control (RHC) has been widely investigated as a successful feedback strategy (Kwon and Pearson, 1977; Kwon and Pearson, 1978; Kwon *et al.*, 1983; Lee *et al.*, 1998; Kwon and Kim, 2000). For the closed-loop stability of the RHC, one approach is to impose infinite terminal weighting which is equivalent to setting a zero terminal weighting matrix for the inverse Riccati equation (Kwon and Pearson, 1977; Kwon and Pearson, 1978). This is referred to as the terminal equality condition. Since imposing infinite terminal weighting is demanding, use of finite terminal weighting matrices has been investigated (Kwon *et al.*, 1983; Lee *et al.*, 1998; Kwon and Kim, 2000).

Multilayer neural networks have some attractive properties, such as the universal approximation capability and the possibility for on- and off-line learning, which motivates their use for control applications (Gupta and Rao, 1994). In spite of these successful neural control application, there are not many stability analysis in neural control (Chen and Liu, 1994; Feng and Michel, 1999; Poznyak *et al.*, 1998; Suykens *et al.*, 2000). Recently, stability conditions for a multilayer neural system, which is regarded as a linear differential inclusion (LDI) system, have been derived in (Limanond and Si, 1998) and (Tanaka, 1996). However, they had not analyzed the stability of neural control systems by considering modeling errors resulting from approximation of a plant with neural networks. With regard to $\mathcal{H}_\infty$ control by neural networks, there has been little work published (Suykens *et al.*, 1996; Suykens *et al.*, 1999). In (Lin and Lin, 2001; Lin and Lin, 2002*a*; Lin and

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Lin, 2002b), the linear state-feedback controller based on the LDI representation was designed. Since the neural networks approximation is performed by using input and state data in local compact sets and linear controller of the form $u = Kx$ employed in neural $\mathcal{H}_\infty$ control may not satisfy input and state constraints, the existing neural $\mathcal{H}_\infty$ controller (Lin and Lin, 2001; Lin and Lin, 2002a; Lin and Lin, 2002b) may cause the instability problem or give a poor performance.

In this paper, we propose new RHNHC scheme based on the LDI representation of neural networks in order to overcome the violation problem for input and state constraints. Since the proposed RHNHC scheme can deal with input and state constraints in optimization problem very effectively, some problems such as the instability and the poor performance due to the violation of input and state constraints can be removed by RHNHC. In addition, the LMI condition on the finite terminal weighting matrix for guaranteeing the internal stability and the infinite horizon $\mathcal{H}_\infty$ performance of RHNHC is proposed.

In Section 2, we give a LDI description of neural networks and formulate the problem. In Section 3, the main results such as cost monotonicity condition, RHNHC are presented. Section 4 provides the numerical example to demonstrate the proposed RHNHC. In Section 5, the conclusion is given.

2. NEURAL NETWORKS DESCRIPTION AND PROBLEM FORMULATION

Consider the unknown nonlinear systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x(t)) + g(u(t)) + d(t) \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $f(\cdot)$ is continuous function but not assumed a *prior* known with zero initial condition, $g(\cdot)$ is continuous function but not assumed a *prior* known with zero initial condition. A and B are constant matrices of appropriate dimensions. $d(t) \in \mathcal{W}$ is the external disturbance or unmodeled dynamics which satisfies the following assumption

$$\mathcal{W} = \{d \in \mathcal{L}_2^n[0, \infty) : d^T Dd \leq 1\} \quad (2)$$

for some $D > 0$.

Let the L-layered neural networks $NN_f(x(t), W_1, W_2, ..., W_L)$ and $NN_g(u(t), V_1, V_2, ..., V_L)$ where $W_i(i = 1, ..., L) \in R^{n_i^x \times n_{i-1}^x}$ and $V_i(i = 1, ..., L) \in R^{n_i^u \times n_{i-1}^u}$ denotes the weight matrices from the $(i-1)-th$ layer to the $i-th$ respectively, be trained to approximate the unknown structure $f(x(t))$ and $g(u(t))$ respectively. The description of neural networks is given as

$$\begin{aligned} NN_f(x(t), W_1, W_2, ..., W_L) \\ = \Psi_L[W_L ... \Psi_2[W_2 \Psi_1[W_1 x(t)]]...] \quad (3) \\ NN_g(u(t), V_1, V_2, ..., V_L) \\ = \tilde{\Psi}_L[V_L ... \tilde{\Psi}_2[V_2 \tilde{\Psi}_1[V_1 u(t)]]...] \quad (4) \end{aligned}$$

where the activation function vector $\Psi[\cdot] : R^n \mapsto R^n$ and $\tilde{\Psi}[\cdot] : R^n \mapsto R^n$ are defined as $\Psi[z] = [\psi_1(z_1)...\psi_n(z_n)]^T$ and $\tilde{\Psi}[z] = [\tilde{\psi}_1(z_1)...\tilde{\psi}_n(z_n)]^T$, respectively which can be determined via a learning algorithm such as the backpropagation. This paper assumes the sigmoid type activation functions $\psi(\cdot) \in F_x$ in the hidden layers and $\psi(\cdot) \in F_u$ in the output layer where

$$F_x = \left\{ \psi(\cdot) : R \mapsto R | \psi(z) = \lambda \left(\frac{2}{1 + \exp(-\frac{z}{q})} - 1 \right) \right\}$$
$$F_u = \{\psi(\cdot) : R \mapsto R | \psi(z) = \lambda z; \lambda > 0\}$$

for $\lambda > 0$ and $q > 0$. For the approximation accuracy ϵ_1 and ϵ_2 over the compact sets $S_x \in R^n$ and $S_u \in R^m$, it can be shown that the optimal constant approximation weights $W^* = \{W_1^*, ..., W_L^*\}$ and $V^* = \{V_1^*, ..., V_L^*\}$ exist, defined by

$$W^* = \arg \min_{W \in \Omega_W} \max_{x(t) \in S_x} ||f(x(t)) - NN_x(x(t), W_1, ..., W_L)|| \quad (5)$$

$$V^* = \arg \min_{V \in \Omega_V} \max_{u(t) \in S_u} ||g(u(t)) - NN_u(u(t), V_1, ..., V_L)|| \quad (6)$$

where Ω_W and Ω_V are constant sets for W and V, respectively, such that

$$\begin{aligned} ||f(x(t)) - NN_x(x(t), W_1^*, ..., W_L^*)|| \\ \leq \epsilon_1 ||x(t)||, \qquad \forall x(t) \in S_x, \quad (7) \\ ||g(u(t)) - NN_u(u(t), V_1^*, ..., V_L^*)|| \\ \leq \epsilon_2 ||u(t)||, \qquad \forall u(t) \in S_u. \quad (8) \end{aligned}$$

Using the LDI representation of the neural networks can be found in (Limanond and Si, 1998; Tanaka, 1996; Lin and Lin, 2001; Lin and Lin, 2002a; Lin and Lin, 2002b), (3) can be represented by

$$NN_f(x(t), W^*) = \sum_{\omega \in \oplus_{i=1}^L \Upsilon_{n_i^x}} \mu(\omega) A_\omega(\omega, \Psi, W^*) x(t) \quad (9)$$

where $n_L^x = n$ and $A_\omega(\omega, \Psi, W^*) = diag[\xi_{Li}(\omega_{Li}, \psi_{Li})] W_L^* \cdots W_2^* diag[\xi_{1i}(\omega_{1i}, \psi_{1i})] W_1^*$. In a similar ways, NN_u are represented by the following LDI form:

$$NN_g(u(t), V^*) = \sum_{\eta \in \oplus_{i=1}^L \Upsilon_{n_i^u}} \mu(\eta) B_\eta(\eta, \tilde{\Psi}, V^*) u(t) \quad (10)$$

where $n_L^u = n$ and $B_\eta(\eta, \tilde{\Psi}, V^*) = diag[\xi_{Li}(\eta_{Li}, \tilde{\psi}_{Li})] V_L^* \cdots V_2^* diag[\xi_{1i}(\eta_{1i}, \tilde{\psi}_{1i})] V_1^*$. Note that the following property holds

$$\sum_{\omega \in \oplus_{i=1}^L \Upsilon_{n_i^x}} \mu(\omega) = \sum_{\eta \in \oplus_{i=1}^L \Upsilon_{n_i^u}} \mu(\eta) = 1 \quad (11)$$

with $\mu(\omega) = \mu(\omega_{ij}, \Psi_i, W_i^*) = h_{Ln_L^x}(i_{Ln_L^x}) \cdots h_{L1}(i_{L1}) \cdots h_{2n_2}(i_{2n_2^x}) \cdots h_{21}(i_{21}) h_{1n_1^x}(i_{1n_1^x}) \cdots h_{11}(i_{11}) \geq 0, \forall \omega_{ij}$ and $\mu(\eta) = \mu(\eta_{ij}, \tilde{\Psi}_i, V_i^*) = h_{Ln_L^u}(i_{Ln_L^u}) \cdots h_{L1}(i_{L1}) \cdots h_{2n_2}(i_{2n_2^u}) \cdots h_{21}(i_{21}) h_{1n_1^u}(i_{1n_1^u}) \cdots h_{11}(i_{11}) \geq 0, \forall \eta_{ij}$.

Then, the nonlinear systems (1) can be represented by the LDI form with error bounds:

$$\dot{x}(t) = Ax(t) + Bu(t) + \Psi_L[W_L^* ... \Psi_2[W_2^* \Psi_1[W_1^* x(t)]]...] + \tilde{\Psi}_L[V_L^* ... \tilde{\Psi}_2[V_2^* \tilde{\Psi}_1[V_1^* u(t)]]...] + v_f(t) + v_g(t) + d(t) \tag{12}$$

$$= \left(A + \sum_\omega \mu(\omega) A_\omega\right) x(t) + \left(B + \sum_\eta \mu(\eta) B_\eta\right) u(t) + v_f(t) + v_g(t) + d(t) \tag{13}$$

where $A_\omega = A_\omega(\omega, \Psi, W^*)$, $B_\eta = B_\eta(\eta, \tilde{\Psi}, V^*)$, $v_f(t) = f(x(t)) - \sum_\omega \mu(\omega) A_\omega(\omega, \Psi, W^*) x(t)$, and $v_g(t) = g(u(t)) - \sum_\eta \mu(\eta) B_\eta(\eta, \tilde{\Psi}, V^*) u(t)$ with $||v_f(t)|| \leq \epsilon_1 ||x(t)||$ and $||v_g(t)|| \leq \epsilon_2 ||u(t)||$. It should be noted that we need not know the the exact values of μ_ω and μ_η since they are not used in RHNHC design actually. In order to obtain the finite terminal weighting matrix for the stabilizing RHNHC, we only need know the values of A_ω and B_η in (13).

For the stabilizing RHNHC, the following finite horizon cost is associated with the system (12):

$$J(x(t_0), t_0, t_1) = \int_{t_0}^{t_1} [x^T(s) Q x(s) + u^T(s) R u(s) - \gamma^2 d(s)^T d(s)] ds + x^T(t_1) Q_f x(t_1) \tag{14}$$

where $t_0 \geq 0$ is an initial time, t_1 is a final time, $Q = C^T C \geq 0$, $R > 0$, and $Q_f > 0$, $\gamma > 0$ is the disturbance attenuation level. The state and control variables are restricted to fulfill the following constraints:

$$x(t) \in S_x, \qquad u(t) \in S_u, \qquad \forall t \geq 0. \tag{15}$$

where

$$S_x = \{x(t) |\ x^- \leq x(t) \leq x^+\ \}, \tag{16}$$

$$S_u = \{u(t) |\ u^- \leq u(t) \leq u^+\ \}, \tag{17}$$

$x^-, x^+ \in R^n$ and $u^-, u^+ \in R^m$ are constant vectors.

The finite horizon optimal differential game consists of the minimization with respect to $u(s), (t_1 \leq s \leq t_0)$, and the maximization with respect to $d(s), (t_1 \leq s \leq t_0)$ of the cost function (14). The minimization and the maximization of (14) must performed under the following three conditions:

1. the neural networks dynamics, $s \in [t_0, t_1]$

$$\dot{x}(s) = Ax(s) + Bu(s) + \Psi_L[W_L^* ... \Psi_2[W_2^* \Psi_1[W_1^* x(s)]]...] + \tilde{\Psi}_L[V_L^* ... \tilde{\Psi}_2[V_2^* \tilde{\Psi}_1[V_1^* u(s)]]...] + d(s), \tag{18}$$

2. the constraints (15), $s \in [t_0, t_1]$

$$x(s) \in S_x, \qquad u(s) \in S_u, \tag{19}$$

3. the terminal state constraint $x(t_1) \in S$ where the terminal constraint set is defined by

$$S = \{x \in R^n | x^T Q_f x \leq \alpha\} \tag{20}$$

for some $\alpha > 0$.

If a feedback saddle-point solution for the finite horizon optimal differential game exists, we denote the solution as $u^*(s)$, $(t_0 \leq s \leq t_1)$ and as $d^*(s)$, $(t_0 \leq s \leq t_1)$, respectively. In the following, the optimal value of the finite horizon optimal differential game will be denoted by $J^*(x(t_0), t_0, t_1)$. RHNHC is then obtained by solving the finite horizon optimal differential game of cost (14) with the initial time t_0 and the terminal time t_1 replaced by the current time t and $t+T$, respectively, where $T > 0$ is constant. The stability of the proposed RHNHC depends on the choice of the finite terminal weighting matrix Q_f. The purpose of this paper is to show that the finite horizon RHNHC with the cost in (14) guarantees the infinite horizon $\mathcal{H}_\infty$ performance for nonlinear unknown systems (1) under certain conditions on Q_f.

3. MAIN RESULTS

3.1 The monotonicity of the saddle point value function

We obtain the condition of the finite terminal weighting matrix Q_f under which the nonincreasing monotonicity of the saddle point value function is guaranteed.

Theorem 1. For given $\gamma > 0$, assume that there exist $X = X^T > 0$ and Y such that

$$\begin{bmatrix} (1,1) & X & X & Y^T & Y^T & I \\ X & -Q^{-1} & 0 & 0 & 0 & 0 \\ X & 0 & -\frac{1}{\epsilon_1^2} I & 0 & 0 & 0 \\ Y & 0 & 0 & -R^{-1} & 0 & 0 \\ Y & 0 & 0 & 0 & -\frac{1}{\epsilon_2^2} I & 0 \\ I & 0 & 0 & 0 & 0 & -\frac{\gamma^2}{2\gamma^2+1} I \end{bmatrix} \leq 0, \tag{21}$$

$$\forall \sigma \in \oplus_{i=1}^L \Upsilon_{n_i^x}, \qquad \forall \eta \in \oplus_{i=1}^L \Upsilon_{n_i^u}$$

where

$$(1,1) = (A + A_\sigma)X + (B + B_\eta)Y + [(A + A_\sigma)X + (B + B_\eta)Y]^T.$$

Then the optimal cost $J^*(x(\tau), \tau, \sigma)$ satisfies the following relation:

$$\frac{\partial J^*(x(\tau), \tau, \sigma)}{\partial \sigma} \leq 0, \qquad \tau \leq \sigma. \tag{22}$$

Proof: Due to the page limitation, we omit the proof. Refer to (Ahn *et al.*, 2004) for the detailed proof. ∎

Remark 2. Recently, numerous useful algorithms to solve linear matrix inequality (LMI) problems have been developed. That is why we can obtain Q_f very easily by solving the feasibiliy problem of LMI with the existing convex optimization softwares such as MATLAB LMI Toolbox.

In the following theorem, it will be shown that if the monotonicity of the optimal cost holds once, it holds for all subsequent times.

Theorem 3. If $\frac{\partial J^*(x(\tau'),\tau',\sigma)}{\partial\sigma} \le 0$ for some τ', then $\frac{\partial J^*(x(\tau''),\tau'',\sigma)}{\partial\sigma} \le 0$ where $\tau' \le \tau'' \le \sigma$.

Proof: Refer to (Ahn *et al.*, 2004). ∎

3.2 RHNHC(Receding Horizon Neural $\mathcal{H}_\infty$ Control)

In this section, RHNHC scheme is proposed for unknown nonlinear systems under the external disturbance. Let the saddle point value function be $V(x(t_0),T) \triangleq J^*(x(t_0),x^*(t_0),d^*(t_0),T) \triangleq J^*(x(t_0),t_0,t_1)$ where $T = t_1 - t_0$ is the horizon size. Under this notation, $\frac{\partial J^*(x(\tau),\tau,\sigma)}{\partial\sigma} \le 0$ of Theorem 1 is equivalent to $\frac{\partial V(x(\tau),T)}{\partial T} \le 0$. RHNHC is obtained by replacing t_0 by t, t_1 by $t+T$, and $x(t_0)$ by $x(t)$.

The next two results are used later to show that RHNHC scheme can guarantee the infinite horizon $\mathcal{H}_\infty$ performance.

Lemma 1. The saddle point value of the finite horizon optimal differential game satisfies

$$V(x(t),T) \ge 0 \tag{23}$$

for all nonnegative constant T.

Proof: Given $d^*(s) = 0$, $s \in [t, t+T]$, for every $x^*(s)$, we have $J(x(t),x^*(t),0,T) \ge 0$ and then

$$\begin{aligned} V(x(t),T) &= J(x(t),x(t)^*,d(t)^*,N) \\ &\ge J(x(t),x(t)^*,0,T) \ge 0 \end{aligned} \tag{24}$$

This completes the proof. ∎

Lemma 2. Under the monotonicity condition of the saddle point value (21), the saddle point value satisfies

$$V(0,T) = 0 \tag{25}$$

for all nonnegative constant T.

Proof: If $x(0) = 0$, because of Theorem 1 and Lemma 1,

$$\begin{aligned} 0 &\le V(0,T) \le V(0,T-\delta) \le V(0,T-2\delta) \\ &\le \cdots \le V(0,T-N\delta) = V(0,0) = 0 \end{aligned} \tag{26}$$

where $\delta = \frac{T}{N} > 0$ and N is a positive integer. This completes the proof. ∎

Next, we introduce the definition of the robust ellipsoid invariance set for selection of the auxiliary control gain K at the terminal time $t+T$.

Definition 1. A set in R^n is said to be robust invariant if all the trajectories starting from within it will remain in it regardless of $w \in \mathcal{W}$. An ellipsoid set (20) can be called as a robust ellipsoid invariant set if $\frac{d}{dt}(x^TQ_fx) \le 0$ for all $d(t) \in \mathcal{W}$ and all $x \in \partial S$, the boundary of S.

Based on this definition, we present the following lemma for the robust invariance of the neural networks (12) subject to the constraints (16)-(17) and the disturbance (2) with a state feedback controller.

Lemma 3. Suppose that $X > 0$ and Y satisfying (21) also satisfy the following LMIs for some symmetric matrices Z and P with $X = Q_f^{-1}$ and $Y = KX$:

$$\begin{bmatrix} \frac{1}{\alpha}Z & Y \\ Y^T & X \end{bmatrix} \ge 0, \; \textit{with} \;\; Z_{jj} \le \overline{u}_j^2, \quad j = 1,...,m \tag{27}$$

$$\begin{bmatrix} \frac{1}{\alpha}P & I \\ I & X \end{bmatrix} \ge 0, \; \textit{with} \;\; P_{jj} \le \overline{x}_j^2, \quad j = 1,...,n \tag{28}$$

$$\begin{bmatrix} (1,1) & X & Y^T & I \\ X & -Q^{-1} & 0 & 0 \\ Y & 0 & -R^{-1} & 0 \\ I & 0 & 0 & -\gamma^2 I \end{bmatrix} \le 0 \tag{29}$$

where

$$(1,1) = -X - \frac{1}{\alpha}D^{-1}.$$

Also, $\overline{u}_j$ and $\overline{x}_j$ are defined by $\overline{u}_j = \min(-u_j^-, u_j^+)$ and $\overline{x}_j = \min(-x_j^-, x_j^+)$. u_j^-, u_j^+, x_j^-, and x_j^+ are the j-th elements of u^-, u^+, x^-, and x^+, respectively, and Z_{jj} and P_{jj} are (j,j) elements of the matrices Z and P, respectively. Then, the state-feedback controller $u(t) = Kx(t)$ guarantees the robust invariance of the systems under the external disturbance $d(t) \in \mathcal{W}$ for all initial states $x(0) \in S$ while satisfying the constraints (16)-(17). The resultant state trajectory $x(t)$ always remains in the region S regardless of $d(t) \in \mathcal{W}$.

Proof: Refer to (Ahn *et al.*, 2004). ∎

The following result shows that RHNHC scheme can guarantee the infinite horizon $\mathcal{H}_\infty$ performance.

Theorem 4. Assume that the finite terminal weighting matrix Q_f satisfies LMI conditions (21) ,(27), (28), and (29), the unknown nonlinear system (1) controlled by RHNHC scheme guarantees the infinite horizon $\mathcal{H}_\infty$ performance.

Proof: Due to the page limitation, we omit the proof. Refer to (Ahn *et al.*, 2004) for the detailed proof. ∎

3.3 Internal Stability of RHNHC

In this subsection, without the external disturbance ($d(t) = 0$), we investigate the internal stability property of the proposed RHNHC which can be stated as the following theorem.

Theorem 5. Without the external disturbance, if $\left.\frac{\partial J^*(x(t),t,\sigma)}{\partial \sigma}\right|_{\sigma=t+T} \leq 0$, the unknown nonlinear systems (1) with RHNHC is asymptotically stable.

Proof:

$$J^*(x(t), t, t+T) = \int_t^{t+\mu} [x^{*T}(s)Qx^*(s) + u^{*T}(s)Ru^*(s)]ds + J^*(x(t+\mu), t+\mu, t+T)$$

According to Theorem 2, $\left.\frac{\partial J^*(x(t),t,\sigma)}{\partial \sigma}\right|_{\sigma=t+T} \leq 0$ implies $\left.\frac{\partial J^*(x(t+\mu),t+\mu,\sigma)}{\partial \sigma}\right|_{\sigma=t+T} \leq 0$ for any $0 < \mu < T$. Hence,

$$J^*(x(t), t, t+T) \geq \int_t^{t+\mu} [x^{*T}(s)Qx^*(s) + u^{*T}(s)Ru^*(s)]ds + J^*(x(t+\mu), t+\mu, t+T+\mu)$$

which means that $J^*(x(t), t, t+T)$ is strictly decreasing. Therefore, $J^*(x(t), t, t+T) \to c > 0$ as $t \to \infty$. Furthermore, form (30), it is clear that $\int_t^{t+\mu} [x^{*T}(t)Qx^*(t) + u^{*T}(t)Ru^*(t)]dt \to 0$ as $t \to \infty$. Finally, $x(t) \to 0$ and $u(t) \to 0$ as $t \to \infty$. This completes the proof. ∎

This theorem states that the nonincreasing monotonicity of the optimal cost is a sufficient condition for the stability of RHNHC without disturbance. Using the result of Theorem 5, we obtain the following corollary on the internal stability of RHNHC with the finite terminal weighting matrix.

Corollary 6. Without the external disturbance, assume that the finite terminal weighting matrix Q_f satisfies the conditions of Theorem 1. Then, the unknown nonlinear system (1) with RHNHC is asymptotically stable.

Proof: Since the existence of Q_f satisfying the conditions of Theorem 4 guarantees $\left.\frac{\partial J^*(x(t),t,\sigma)}{\partial \sigma}\right|_{\sigma=t+T} \leq 0$, the stability result follows form Theorem 5. ∎

4. NUMERICAL EXAMPLE

Consider the following nonlinear systems

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0.25 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} u + \begin{bmatrix} \exp(-(x_1+x_2))\cos(x_1+x_2) - 1 \\ 1.5\cos^2(u)\sin^2(u) \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

where d_1 and d_2 are the external disturbance with random entries, chosen from a normal distribution with mean zero and variance one. A 2-layer neural network with 3 hidden nodes and 1 output node was used to approximate $\exp(-(x_1+x_2))\cos(x_1+x_2) - 1$. And another 2-layer neural network with 2 hidden nodes and 1 output node was chosen to approximate $1.5\cos^2(u)\sin^2(u)$. The set of index vector $\Upsilon_{n_i^x}$ $\Upsilon_{n_i^u}$ can be stated as follows

$$\Upsilon_{n_1^x} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\},$$

$$\Upsilon_{n_2^x} = \{0, 1\},$$

$$\Upsilon_{n_1^u} = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\},$$

$$\Upsilon_{n_2^u} = \{0, 1\}.$$

The weights of the neural networks were trained in

$$S_x = \left\{ x(t) \middle| \begin{bmatrix} -1 \\ -1 \end{bmatrix} \leq x(t) \leq \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \tag{30}$$

$$S_u = \{u(t)| -1 \leq u(t) \leq 1\} \tag{31}$$

with uniformly distributed random numbers. Refer to (Ahn *et al.*, 2004) for the detailed information on some variables in simulation. Applying Theorem 4, we obtain

$$Q_f = \begin{bmatrix} 0.0096 & 0.0133 \\ 0.0133 & 0.1957 \end{bmatrix}, \qquad K = \begin{bmatrix} -0.2851 & -2.8417 \end{bmatrix}$$

with $\alpha = 0.0218$. Using the results of (Lin and Lin, 2001; Lin and Lin, 2002*a*; Lin and Lin, 2002*b*), we obtain the following neural $\mathcal{H}_\infty$ controller (NHC):

$$u(t) = \begin{bmatrix} -1.4614 & -14.2538 \end{bmatrix} x(t).$$

Now, we compare the results of RHNHC and NHC when the initial state $x(0)$ is in S_x and not in S_x, respectively. For the case of $x(0) = (-0.4, 0.2) \in S_x$, Figure 1 shows that the proposed RHNHC guarantees the faster convergence around the origin than the existing NHC. For the case of $x(0) = (-1.2, 1.3) \notin S_x$, Figure 2 shows that the proposed RHNHC achieves the good convergence property around the origin. But, the existing NHC scheme fails to achieve the desired control objective. Next, we compare the $\mathcal{H}_\infty$ cost of RHNHC and NHC for the cases of $x(0) = (-0.4, 0.2) \in S_x$ and $x(0) = (-1.2, 1.3) \notin S_x$, respectively. From Table 1, in case of $x(0) = (-0.4, 0.2) \in S_x$, the $\mathcal{H}_\infty$ performance of the proposed scheme is better than that of the NHC scheme. In addition, it is shown that the proposed RHNHC satisfies the desirable $\mathcal{H}_\infty$ norm bound ($\gamma^2 = 0.0484$), even with the initial state which is not in S_x. But, the existing NHC does not guarantee the $\mathcal{H}_\infty$ norm bound ($\gamma^2 = 0.0484$) for the case 2. Therefore, though the initial state is not in S_x, it can be seen that the suggested RHNHC scheme has the potential to obtain the desirable $\mathcal{H}_\infty$ performance.

	RHNHC	NHC
Case 1: $x(0) = (-0.4, 0.2) \in S_x$	0.0027	0.0081
Case 2: $x(0) = (-1.2, 1.3) \notin S_x$	0.0483	0.9803

Table 1. $\mathcal{H}_\infty$ costs of RHNHC and NHC

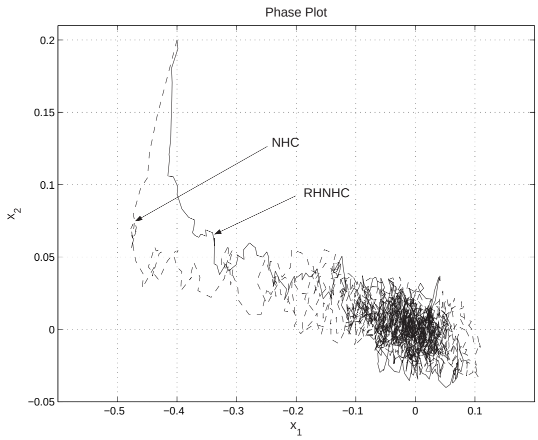

Fig. 1. Phase plot for $x(0) = (-0.4, 0.3)$

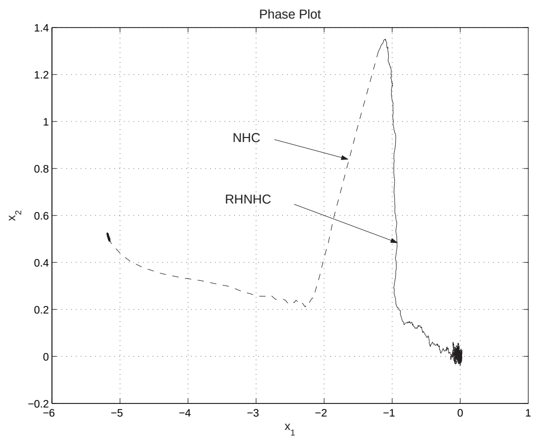

Fig. 2. Phase plot for $x(0) = (-1.2, 1.3)$

5. CONCLUSION

In this paper, we propose the new RHNHC scheme for a class of unknown nonlinear systems. The LMI condition on the terminal weighting matrix for nonincreasing monotonicity of the saddle point value is proposed. Under this condition, RHNHC guarantees the infinite horizon $\mathcal{H}_\infty$ performance and the internal stability for nonlinear unknown systems. Through the simulation result, RHNHC guarantees the better performance than the existing neural $\mathcal{H}_\infty$ control schemes and removes some problems of them. This result can be regarded as the first result on receding horizon control problem for nonlinear unknown systems.

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