LONGITUDINAL AXIS FLIGHT CONTROL LAW DESIGN BY ADAPTIVE BACKSTEPPING

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Abstract: This paper develops an adaptive backstepping flight control law in order to bestow good flying qualities in longitudinal axis for all flight conditions. Using the backstepping procedure, a new type of controller was synthesized in order to accomplish desired responses under a wide range of flight envelope. Model-reference tuning of the backstepping controller is employed to achieve the desired responses for all flight conditions. Simulation results demonstrate that the proposed method is capable of giving desired closed-loop dynamic performance and robustness against uncertainties within the subsonic and supersonic flight conditions.

Keywords: Adaptation, flight control, integral action, Lyapunov stability, robustness.

1. INTRODUCTION

One of the most popular control schemes for flight control has been based on a conventional Proportional-Integral (PI) controller design method with desired specifications over entire flight envelope. Aside from PI control, the gain scheduling method has also been extensively used for controlling the nonlinear aircraft dynamics (Reichert, 1992; Nelson, 1998). Recently, the Lyapunov-based design method has been developed for aircraft flight control systems (Harkegard and Glad, 2000; Lee and Kim, 2001; Sharma and Ward, 2002). These studies utilized the backstepping method to construct stable nonlinear controllers in order to improve the performance of flight path control systems. Harkegard and Glad (2000) developed a backstepping controller that is globally stabilizing around the stall angle of attack. Sharma and Ward (2002) presented a neural adaptive backstepping controller that provides good command tracking and robustness against aerodynamic uncertainties. However, these studies only focused on a single flight condition or at low speed flight envelope. In addition to gain-scheduling control, few researchers have developed full envelope control laws (Reynolds, *et al.*, 1994). The flight control system probably doesn't carry out in the presence of an entire flight envelope of supersonic aircraft.

The objective of the paper is to use the backstepping design procedure to establish a backstepping longitudinal flight control law with good performance and robustness across a specified flight envelope. The model-reference design is used to derive the parameters adaptation rules of the backstepping controller via Lyapunov theory so that the closed-loop system has met specifications within the operating conditions. Approximation of the parameters of the backstepping controller as a function of aerodynamic coefficients and parameters restrictions is designed to achieve desired response in subsonic and supersonic flight regimes.

This paper extends the previous result in Ju, *et al.* (2004) to propose an integral and adaptive flight controller via backstepping. A novel adaptive backstepping flight control law is derived using the backstepping method with parameter adaptation to compensate for the uncertainties. This type of the control structure is developed by backstepping design methodology, and the control gain matrices are scheduled for all flight conditions. The longitudinal short-period requirements by the low-order equivalent systems were analyzed, and evaluate the performance and robustness of the proposed control through simulations in order to show the effectiveness of the adaptive control method.

The remaining parts of the paper are outlined as follows. Section II briefly recalls the well-established linearized aircraft dynamics. A backstepping design procedure is used in Section III to design a longitudinal control law across a specified flight envelope. The model-reference parameters tuning method is also adopted to bestow good handling qualities for all flight conditions. Section IV conducts computer simulation results. Section V concludes the paper.

2. AIRCRAFT DYNAMICS

A backstepping design for flight control deals with the longitudinal motion of the aircraft and the control of the rigid body dynamics. A detailed description of aircraft's small perturbation equations of motion can be found in Roskam (1979). The assumptions are consistent with initial straight and level flight with constant thrust. In this study, the basic flight control law uses the elevator as a primary control effector, and the change of a throttle command to maintain airspeed is assumed to be zero

2.1 Aircraft Dynamics Model

In this paper, a reference (fighter) aircraft model in longitudinal and vertical axes is used. The motion equations for a longitudinal aircraft with short period mode and phugoid mode are described by

$$\begin{aligned} \dot{u} &= X_u u + X_\alpha \alpha + X_\theta \theta + X_q q + X_{\delta e}\delta e \\ \dot{\alpha} &= Z_u u + Z_\alpha \alpha + Z_\theta \theta + Z_q q + Z_{\delta e}\delta e \\ \dot{q} &= M_u u + M_\alpha \alpha + M_\theta \theta + M_q q + M_{\delta e}\delta e \\ n_z &= \tfrac{V}{g}(q - \dot{\alpha}) \end{aligned} \quad (1)$$

where
u : longitudinal velocity (ft/s)
q : pitch rate (rad/s)
θ : pitch angle (rad)
α : angle of attack (rad)
δe : elevator surface deflection (rad)
n_z : normal acceleration at c.g. ('g)
V : aircraft velocity (ft/s)
X_* 、Z_* 、M_*: stability and control derivatives

2.2 Flight Envelope of the Aircraft

The boundaries of the flight envelope are indicated by the stall limit, load limit, temperature limit, and performance limit. In order to design flight control laws to cover a wide range of envelope, it is necessary to select a number of operation points to satisfy design specifications. The motion equations for a longitudinal aircraft include the short period and phugoid modes. A set of full envelope linear aircraft models has been used to represent the nonlinear aircraft system that should vary with airspeed, altitude, and dynamic pressure.

3. BACKSTEPPING CONTROL DESIGN

This section is devoted to deriving control laws for longitudinal aircraft using backstepping (Sharma and Calise, 2002; Dahlgren, 2002). This proposed control laws will provide the supersonic aircraft with significant desired responses. The proposed adaptive backstepping control algorithm consists of two parts: the integral backstepping control to eliminate the command tracking errors, and the parameter adaptation rules for parametric uncertainties. Finally, the model-reference approach is used to find appropriate parameters of the controllers in the presence of a wide range of flight envelope.

3.1 Integrator Backstepping

In this section, the backstepping design is used to derive control laws for the longitudinal motion of aircraft in the presence of a wide range of flight envelope. The motion equations for a longitudinal aircraft are described in (1). In this state-space model, the control input (δe) is a function of all the state variables (u, α, q, θ). However, equation (1) is not suited for designing flight control laws via backstepping. Hence, if the surface deflection (δe) on the lift (α) is neglected, the state-space model is on the correct form for backstepping design (Lee and Kim, 2001; Sharma and Ward, 2002). The lift force on the surface deflection is intentionally neglected for deriving the longitudinal flight control laws. Thus the dynamics of aircraft equations without airspeed control given by (1) can be rewritten as

$$\begin{aligned} \dot{\alpha} &= Z_\alpha \alpha + Z_\theta \theta + Z_q q \\ \dot{\theta} &= q \\ \dot{q} &= M_\alpha \alpha + M_\theta \theta + M_q q + M_{\delta e}\delta e \end{aligned} \quad (2)$$

The integral action to cope with steady errors as in the previous work will be used (Ju, *et al.*, 2004). The control objective is to track the angle of attack command without steady errors for longitudinal axis. In what follows shows how to design the flight control laws by backstepping.

Step 1: For the α-tracking objective, two new state variables can be defined as $z_1 = \alpha - \alpha_{ref}, z_2 = q - q_{des}$. The time derivative of z_1 is

$$\dot{z}_1 = \dot{\alpha} - \dot{\alpha}_{ref} \quad = Z_\alpha z_1 + Z_\theta \theta + Z_q q + Z_\alpha \alpha_{ref} - \dot{\alpha}_{ref} \quad (3)$$

Considering q as a control input for z_1-dynamics, one regards the desired value of q as the virtual control law of (3). In order for stabilization in (3), q can be chosen as

$$q_{des} = Z_q^{-1}(-Z_\alpha \alpha_{ref} + \dot{\alpha}_{ref} - Z_\theta \theta - K_1 z_1 - \lambda \int_0^t z_1(t)dt) \quad (4)$$

where λ is a constant value. In (4), the integral of the tracking error is introduced to the desired value of q. By adding the integral action into the stabilizing function, the tracking error will converge to zero. Substituting (4) into (3) yields

$$\dot{z}_1 = -(K_1 - Z_\alpha)z_1 - \lambda\chi_1 \tag{5}$$

where

$$\chi_1 = \int_0^t z_1(t)dt \tag{6}$$

Under the condition $K_1 > Z_\alpha$, the z_1-dynamics becomes asymptotically stable. Substituting (4) into z_2-dynamics yields

$$\begin{aligned} z_2 &= q - q_{des} \\ &= q - Z_q^{-1}(-Z_\alpha \alpha_{ref} + \dot{\alpha}_{ref} - Z_\theta \theta - K_1 z_1 - \lambda\chi_1) \end{aligned} \tag{7}$$

Then $\dot{z}_1$ can be rewritten as

$$\dot{z}_1 = -(K_1 - Z_\alpha)z_1 + Z_q z_2 - \lambda\chi_1 \tag{8}$$

and differentiating (7) give

$$\begin{aligned} \dot{z}_2 = {} & [M_\alpha - M_q Z_q^{-1} K_1 - Z_\theta Z_q^{-2} K_1 + Z_q^{-1} K_1 (Z_\alpha - K_1) \\ & + Z_q^{-1}\lambda] z_1 + (M_q + Z_q^{-1} Z_\theta + K_1) z_2 \\ & + (M_\alpha - M_q Z_q^{-1} Z_\alpha - Z_\theta Z_q^{-2} Z_\alpha)\alpha_{ref} \\ & + (M_\theta - M_q Z_q^{-1} Z_\theta - Z_\theta Z_q^{-2} Z_\theta)\theta \\ & + (M_q Z_q^{-1} + Z_\theta Z_q^{-2} + Z_q^{-1} Z_\alpha)\dot{\alpha}_{ref} + (-Z_q^{-1})\ddot{\alpha}_{ref} \\ & + (-M_q Z_q^{-1} - Z_\theta Z_q^{-2} - Z_q^{-1} K_1)\lambda\chi_1 + M_{\delta e}\delta e \end{aligned} \tag{9}$$

then (8) and (9) can be written as (assuming $\dot{\alpha}_{ref} = \ddot{\alpha}_{ref} = 0$)

$$\begin{aligned} \dot{z}_1 &= -(K_1 - Z_\alpha)z_1 + Z_q z_2 - \lambda\chi_1 \\ \dot{z}_2 &= \xi z_1 + \eta z_2 + \phi - \lambda\mu\chi_1 + M_{\delta e}\delta e \end{aligned} \tag{10}$$

where

$$\begin{aligned} \xi = {} & M_\alpha - M_q Z_q^{-1} K_1 - Z_\theta Z_q^{-2} K_1 \\ & + Z_q^{-1} K_1 (Z_\alpha - K_1) + Z_q^{-1}\lambda \end{aligned} \tag{11}$$

$$\eta = M_q + Z_q^{-1} Z_\theta + K_1 \tag{12}$$

$$\begin{aligned} \phi = {} & [M_\alpha - M_q Z_q^{-1} Z_\alpha - Z_\theta Z_q^{-2} Z_\alpha]\alpha_{ref} \\ & + [M_\theta - M_q Z_q^{-1} Z_\theta - Z_\theta Z_q^{-2} Z_\theta]\theta \end{aligned} \tag{13}$$

$$\mu = M_q Z_q^{-1} + Z_\theta Z_q^{-2} + Z_q^{-1} K_1 \tag{14}$$

Step 2: Define a Lyapunov function candidate as

$$V(z_1, z_2) = \frac{1}{2}\lambda\chi_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{15}$$

Using (14), the time derivative of V can be

$$\begin{aligned} \dot{V} = {} & -(K_1 - Z_\alpha)z_1^2 + (Z_q + \xi)z_1 z_2 \\ & + z_2(\eta z_2 + \phi - \lambda\mu\chi_1 + M_{\delta e}\delta e) \end{aligned} \tag{16}$$

In view of (16), the first term is negative definite as long as $K_1 > Z_\alpha$. In order to eliminate $z_1 z_2$ cross-term, the condition $\xi = -Z_q$ is chosen. To make the third term become negative definite, the control surface deflection can be selected as

$$\delta e = M_{\delta e}{}^{-1}(-K_2 z_2 - \phi + \lambda\mu\chi_1) \tag{17}$$

The control derivative of $M_{\delta e}$ remains non-zero for most aircraft control applications. Substituting (17) into (16) yields

$$\dot{V} = -(K_1 - Z_\alpha)z_1{}^2 + z_2{}^2(\eta - K_2) + (Z_q + \xi)z_1 z_2 \tag{18}$$

For $\dot{V}$ to be negative definite, the following constraints must obey

$$K_1 > Z_\alpha\,, \quad K_2 > \eta, \quad Z_q + \xi = 0 \tag{19}$$

Inserting $Z_q + \xi = 0$ into (11), λ can be expressed as

$$\lambda = K_1{}^2 + (M_q + Z_\theta Z_q^{-1} - Z_\alpha)K_1 - (M_\alpha Z_q + Z_q^2) \tag{20}$$

Step 3: This step aims to determine the control law. Using (7) and (13), the control law is of the form

$$\begin{aligned} \delta e &= M_{\delta e}{}^{-1} \begin{pmatrix} K_1 K_2 Z_q^{-1}(\alpha_{ref} - \alpha) - K_2 q \\ -(M_\theta - M_q Z_q^{-1} Z_\theta - Z_\theta Z_q^{-2} Z_\theta \\ \quad + K_2 Z_q^{-1} Z_\theta)\theta \\ +(-M_\alpha + M_q Z_q^{-1} Z_\alpha + Z_\theta Z_q^{-2} Z_\alpha \\ \quad - K_2 Z_q^{-1} Z_\alpha)\alpha_{ref} \\ +(\mu - K_2 Z_q^{-1})\lambda\chi_1 \end{pmatrix} \\ &= k_f \alpha_{ref} - k_\alpha \alpha - k_q q - k_\theta \theta + k_i \chi_1 \end{aligned} \tag{21}$$

where

$$\begin{aligned} k_f &= M_{\delta e}{}^{-1}(-M_\alpha + M_q Z_q^{-1} Z_\alpha + Z_\theta Z_q^{-2} Z_\alpha \\ &\qquad - K_2 Z_q^{-1} Z_\alpha + K_1 K_2 Z_q^{-1}) \\ k_\alpha &= M_{\delta e}{}^{-1}(K_1 K_2 Z_q^{-1}) \\ k_q &= M_{\delta e}{}^{-1} K_2 \\ k_\theta &= M_{\delta e}{}^{-1}(M_\theta - M_q Z_q^{-1} Z_\theta - Z_q^{-2} Z_\theta{}^2 + K_2 Z_q^{-1} Z_\theta) \\ k_i &= M_{\delta e}{}^{-1}(K_2 Z_q^{-1} - \mu)\lambda \end{aligned} \tag{22}$$

The resulting controller is parameterized by the five parameters K_f, K_α, K_q, K_θ and K_t. A trial-and-error method can be a way to find parameters in agreement with the constraints (19). It is time-consuming for full envelope control design. However, the automatic tuning of the parameters by model-reference approach is described in detail in the late subsections.

3.2 Adaptive Control Design

The previous design was based on the backstepping algorithm with integral action to improve steady-state control accuracy. In this subsection, the adaptive parameters of flight control law are designed to compensate for parametric uncertainties. In doing so, one rewrites (21) with adjustable parameters k_f, k_α, k_q, k_θ and k_i as

$$u = \hat{k}_f \alpha_{ref} - \hat{k}_\alpha \alpha - \hat{k}_q q - \hat{k}_\theta \theta + \hat{k}_i \chi_1 \tag{23}$$

where $\hat{k}_f, \hat{k}_\alpha, \hat{k}_q, \hat{k}_\theta$ and $\hat{k}_i$ are the estimates of $k_f, k_\alpha, k_q, k_\theta$ and k_i, respectively. To drive the parameter adaptation rules, a Lyapunov function candidate will be chosen by

$$\begin{aligned} V_1 &= \frac{1}{2}\lambda\chi_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \\ &+ \frac{1}{2r_f}\tilde{k}_f^2 + \frac{1}{2r_\alpha}\tilde{k}_\alpha^2 + \frac{1}{2r_q}\tilde{k}_q^2 + \frac{1}{2r_\theta}\tilde{k}_\theta^2 + \frac{1}{2r_i}\tilde{k}_i^2 \end{aligned} \tag{24}$$

where the parameter estimation errors are defined as

$$\begin{aligned} &\tilde{k}_f = k_f - \hat{k}_f, \quad \tilde{k}_\alpha = k_\alpha - \hat{k}_\alpha, \quad \tilde{k}_q = k_q - \hat{k}_q \\ &\tilde{k}_\theta = k_\theta - \hat{k}_\theta, \quad \tilde{k}_i = k_i - \hat{k}_i \end{aligned} \tag{25}$$

and their time derivatives are given by

$$\dot{\tilde{k}}_f = -\dot{\hat{k}}_f, \dot{\tilde{k}}_\alpha = -\dot{\hat{k}}_\alpha, \dot{\tilde{k}}_q = -\dot{\hat{k}}_q, \dot{\tilde{k}}_\theta = -\dot{\hat{k}}_\theta, \dot{\tilde{k}}_i = -\dot{\hat{k}}_i \tag{26}$$

Using (19) and (26), the time derivative of V_1 can be computed as

$$\begin{aligned} \dot{V}_1 &= \lambda\chi_1 z_1 + z_1\dot{z}_1 + z_2\dot{z}_2 + \frac{1}{r_f}\tilde{k}_f\dot{\tilde{k}}_f + \frac{1}{r_\alpha}\tilde{k}_\alpha\dot{\tilde{k}}_\alpha \\ &+ \frac{1}{r_q}\tilde{k}_q\dot{\tilde{k}}_q + \frac{1}{r_\theta}\tilde{k}_\theta\dot{\tilde{k}}_\theta + \frac{1}{r_i}\tilde{k}_i\dot{\tilde{k}} \end{aligned} \tag{27}$$

Substituting (22) into (13) yields

$$\begin{aligned} \phi &= M_{\delta e}(-k_q Z_q^{-1} Z_\alpha + k_\alpha - k_f)\alpha_{ref} \\ &+ M_{\delta e}(k_\theta - k_q Z_q^{-1} Z_\theta)\theta \end{aligned} \tag{28}$$

In (10), $\dot{z}_2$ can be written as

$$\dot{z}_2 = \xi z_1 - (K_2 - \eta)z_2 + K_2 z_2 + \phi - \lambda\mu\chi_1 + M_{\delta e}u \tag{29}$$

The term $(K_2 z_2 + \phi - \lambda\mu\chi_1 + M_{\delta e}u)$ in (29) can be calculated as

$$\begin{aligned} &K_2 z_2 + \phi - \lambda\mu\chi_1 + M_{\delta e}u \\ &= M_{\delta e}(-\tilde{k}_f\alpha_{ref} + \tilde{k}_\alpha\alpha + \tilde{k}_q q + \tilde{k}_\theta\theta - \tilde{k}_i\chi_1) \end{aligned} \tag{30}$$

then $\dot{z}_2$ can be rewritten as

$$\begin{aligned} \dot{z}_2 &= \xi z_1 - (k_2 - \eta)z_2 \\ &+ M_{\delta e}(-\tilde{k}_f\alpha_{ref} + \tilde{k}_\alpha\alpha + \tilde{k}_q q + \tilde{k}_\theta\theta - \tilde{k}_i\chi_1) \end{aligned} \tag{31}$$

Substituting (10) and (31) into (27) yields

$$\begin{aligned} \dot{V}_1 &= -(K_1 - Z_\alpha)z_1^2 - (K_2 - \eta)z_2^2 \\ &+ \tilde{k}_f(-z_2 M_{\delta e}\alpha_{ref} - \frac{1}{r_f}\dot{\hat{k}}_f) \\ &+ \tilde{k}_\alpha(z_2 M_{\delta e}\alpha - \frac{1}{r_\alpha}\dot{\hat{k}}_\alpha) + \tilde{k}_q(z_2 M_{\delta e} q - \frac{1}{r_q}\dot{\hat{k}}_q) \\ &+ \tilde{k}_\theta(z_2 M_{\delta e}\theta - \frac{1}{r_\theta}\dot{\hat{k}}_\theta) + \tilde{k}_i(-z_2 M_{\delta e}\chi_1 - \frac{1}{r_i}\dot{\hat{k}}_i) \end{aligned} \tag{32}$$

If the parameter adaptation rules are designed as

$$\begin{aligned} &\dot{\hat{k}}_f = -r_f z_2 M_{\delta e}\alpha_{ref}, \quad \dot{\hat{k}}_\alpha = r_\alpha z_2 M_{\delta e}\alpha, \\ &\dot{\hat{k}}_q = r_q z_2 M_{\delta e} q, \quad \dot{\hat{k}}_\theta = r_\theta z_2 M_{\delta e}\theta, \quad \dot{\hat{k}}_i = -r_i z_2 M_{\delta e}\chi_1 \end{aligned} \tag{33}$$

then time derivative of V_1 can be written as

$$\dot{V}_1 = -(K_1 - Z_\alpha)z_1^2 - (K_2 - \eta)z_2^2 \tag{34}$$

When conditions (19) are satisfied, $\dot{V}_1 \le 0$ is achieved. By using Barbalat's lemma (Krstic, *et al.*, 1995), it can be shown that $z_1 \to 0,\ z_2 \to 0$ as $t \to \infty$.

3.3. Model Reference Approach

In this subsection, the model-reference approach is used to find the parameters K_1 and K_2 of the backstepping controller so that conditions (19) are satisfied.

Based on the guidelines in flying qualities requirements of MIL-F-8785C, the ideal model for short period mode of longitudinal aircraft can be expressed as

$$\begin{bmatrix} \dot{\alpha}_m \\ \dot{q}_m \end{bmatrix} = \begin{bmatrix} a_{11m} & a_{12} \\ a_{21m} & a_{22m} \end{bmatrix} \begin{bmatrix} \alpha_m \\ q_m \end{bmatrix} + \begin{bmatrix} b_{1m} \\ b_{2m} \end{bmatrix} \delta_{em} \tag{35}$$

In the ideal model, the zero of the aircraft transfer function is decided by Z_α. It is assumed that the state feedback does not change the zero of the transfer function, and $a_{11m} = Z_\alpha$ was chosen. The characteristic equation of the ideal model can be expressed as

$$s^2 + (-a_{11m} - a_{22m})s + (a_{11m}a_{22m} - a_{21m}) = 0 \tag{36}$$

Based on the damping ratio (ξ_m) and the natural frequency (ω_{dm}) of the ideal model, a_{21m} and a_{22m} can be expressed by

$$a_{21m} = Z_\alpha(-Z_\alpha - 2\xi_m\omega_{dm}) - \omega_{dm}{}^2,\ a_{22m} = -Z_\alpha - 2\xi_m\omega_{dm} \tag{37}$$

Since the surface deflection of elevator on the lift is ignored, $b_{1m} = 0$ will be chosen. The dynamic equations can be written as

$$\begin{aligned} \dot{\alpha}_m &= Z_\alpha\alpha_m + q_m \\ \dot{q}_m &= (-Z_\alpha{}^2 - 2Z_\alpha\xi_m\omega_{dm} - \omega_{dm}{}^2)\alpha_m \\ &\quad + (-Z_\alpha - 2\xi_m\omega_{dm})q_m + b_{2m}\delta_{em} \end{aligned} \tag{38}$$

Using (38), b_{2m} is of the form

$$
\begin{aligned}
b_{2m} &= \delta_{em}^{-1}[(Z_\alpha^{\ 2} + 2Z_\alpha \xi_m \omega_{dm} + \omega_{dm}^{\ 2})\alpha_m \\
&\quad + (Z_\alpha + 2\xi_m \omega_{dm}) q_m] \\
&= \frac{\alpha}{\delta_{em}} \omega_{dm}^{\ 2}
\end{aligned}
\tag{39}
$$

$b_{2m}=\omega_{dm}^2$ will be chosen. The ideal model for the short period mode can be rewritten as

$$
\begin{bmatrix} \dot{\alpha}_m \\ \dot{q}_m \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 \\ Z_\alpha(-Z_\alpha - 2\xi_m\omega_{dm}) - \omega_{dm}^{\ 2} & -Z_\alpha - 2\xi_m\omega_{dm} \end{bmatrix} \begin{bmatrix} \alpha_m \\ q_m \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{dm}^{\ 2} \end{bmatrix} \delta_{em}
\tag{40}
$$

To match the response of the ideal short period mode, the control law given by (21) can be written as (assuming $z_1 \to 0$ as $t \to \infty$ and ignoring θ term)

$$
\begin{aligned}
u &= A_{lcf} x + B_{lcf} u_c \\
&= [-k_\alpha \quad -k_q] \begin{bmatrix} \alpha \\ q \end{bmatrix} + [k_f] \alpha_{ref}
\end{aligned}
\tag{41}
$$

where u_c is a command signal. The closed-loop flight control becomes

$$
\dot{x} = (A + BA_{lcf})x + (BB_{lcf})u_c \tag{42}
$$

The matrices A and B are states coefficients of the bare airplane. The (K_1, K_2) gains in (21) may be chosen by (19) and a prior knowledge of aero-coefficients. The convenient way to choose the values of the gains is based on the model-reference approach. The desired response model given by (35) can be rewritten as

$$
\dot{x}_m = A_m x_m + B_m u_c \tag{43}
$$

Define the error $e = x - x_m$, the time derivative of e is

$$
\dot{e} = A_m e + (A + BA_{lcf} - A_m)x + (BB_{lcf} - B_m)u \tag{44}
$$

For perfect model following, it implies that $A + BA_{lcf} = A_m$ and $BB_{lcf} = B_m$. This yields

$$
\begin{aligned}
&M_\alpha - M_{\delta e} k_\alpha = Z_\alpha(-Z_\alpha - 2\xi_m \omega_{dm}) - \omega_{dm}^{\ 2} \\
&M_q - M_{\delta e} k_q = -Z_\alpha - 2\xi_m \omega_{dm}, \quad M_{\delta e} k_f = \omega_{dm}^{\ 2}
\end{aligned}
\tag{45}
$$

Substituting (22) into (45) yields

$$
\begin{aligned}
K_1 &= \frac{Z_q (M_\alpha + Z_\alpha^{\ 2} + 2\xi_m \omega_{dm} Z_\alpha + \omega_{dm}^{\ 2})}{M_q + Z_\alpha + 2\xi_m \omega_{dm}} \\
K_2 &= M_q + Z_\alpha + 2\xi_m \omega_{dm}
\end{aligned}
\tag{46}
$$

Based on the damping ratio and aero-coefficients have been assigned for all flight conditions, the K_1 and K_2 will be found by (19). The design parameters K_1 and K_2 are chosen by constrains, which represent the freedom available to the designer to satisfy the Lyapunov stability theory. However, the design of the parameters K_1 and K_2 are tightened and bounded by flying qualities specifications.

4. ANALYSIS OF RESULTS

This section presents the results and analysis of the results from adaptive backstepping control laws described in the previous section. Flying qualities analyses based on MIL-F-8785C's short period approximation will be first presented. Then, the aircraft dynamic performance will be analyzed in the different flight conditions. Finally, the robustness to the off-normal value of the aerodynamic derivatives will be analyzed.

4.1 Flying Qualities Analysis

Several flying qualities measures are used to analyze the handling of manual flight control. MIL-F-8785C requires that the natural frequencies of the short period modes fall within the Level 1 bound. Fig. 1 depicts the region for short period's natural frequencies and Control Anticipation Parameter (CAP). It shown the short-period responses for tested flight conditions were within the level 1 boundary.

4.2 Dynamic Performance Analysis

In the simulations, the aerodynamic coefficients are assumed to be known exactly. Fig. 2 only depicts the simulated results of the controller with parameter adaptation rules at five different flight conditions. The backstepping controller is designed to provide a faster response without overshot of normal acceleration within the subsonic and supersonic flight conditions. The steady-state errors are improved using the controller with integral action in subsonic flight conditions except supersonic conditions.

4.3 Robustness Analysis

For robustness analysis, the tracking responses with ±50% parameter variations of stability and control matrices are shown in Fig. 3. The trajectory command is one-degree α doublet with a period of 50 seconds. The flight condition for robustness analysis is at Mach 0.8 and 0 feet altitude with $\gamma_{j(j=f,\alpha,q,\theta,i)} = 0.05$. Although tracking performance is degraded at the first square command, the controller with adaptation rules could quickly follow command signals at the second square wave.

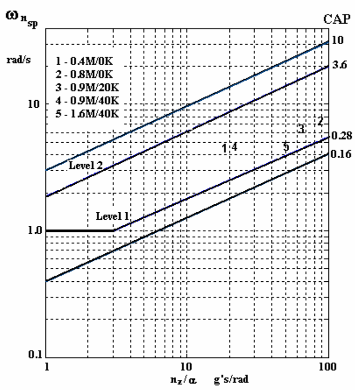

Fig. 1. MIL-F-8785C short period requirements

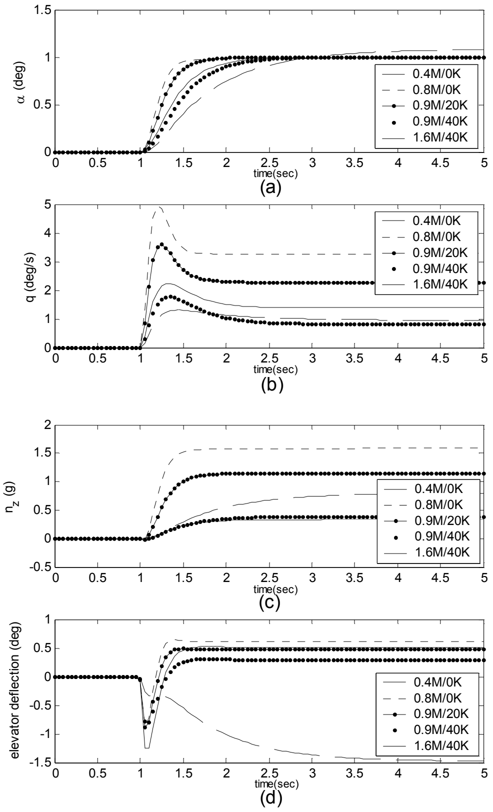

Fig. 2. Step responses of backstepping controller. (a) Angle-of-attack, (b) pitch rate, (c) normal acceleration and (d) surface deflection.

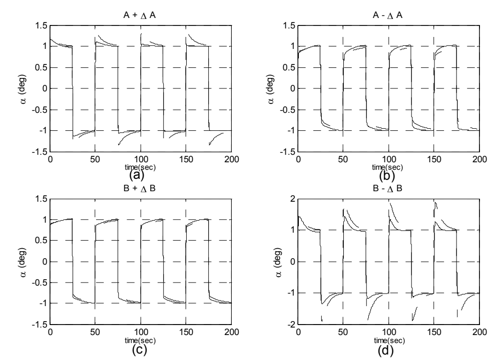

Fig. 3. The command tracking performance of the backstepping controller for the aerodynamic uncertainties. (a) A+ΔA, (b) A-ΔA, (c) B+ΔB and (d) B-ΔB (solid: adaptation rule, dash: non-adaptation rule)

5. CONCLUSIONS

In this paper, full envelope flight control laws for a supersonic aircraft have been established using an adaptive backstepping control with integral action. The longitudinal Lyapunov-based flight controller is designed by combining the backstepping and adaptive control. The control loop gains found by the model-reference approach are scheduled for all flight conditions. The desired responses of longitudinal axis have been achieved in a wide range of flight envelope. Both desired performance and robustness of the closed-loop aircraft have been proven via computer simulations. With adaptive backstepping designs with robustness against unmodeled dynamics, the designers are not intended to have a prior knowledge of the aerodynamic coefficients. The proposed adaptive control law is much simpler, and is easier to construct and realize. In the current work, only the longitudinal axis is considered; this backstepping control law can be easily extended to the lateral/directional axis. An important topic for future study might be to implement the proposed control laws in nonlinear simulations.

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