ACHIEVING X-SIGMA DELIVERIES IN SUPPLY CHAINS

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Abstract

Time-based competition and market globalization make it imperative for supply chains to have short and reliable order deliveries. This is difficult to achieve in view that activities of individual manufacturers are subject to various uncertainties such as unknown order arrivals and stochastic operations. Furthermore, delays of one manufacturer may propagate to it downstream manufacturers through precedence relationships. To stay competitive, it is critical to control variability and order lead-times across a chain, and to achieve delivering final products within specified target time windows with high probability. This is the key idea of achieving x-sigma delivery performance. In this paper, make-to-order supply chains with sequential workflows are considered. An effective solution methodology is developed to minimize overall order tardiness, earliness costs and delivery variability through effective scheduling and coordination. To accommodate new arrivals while fulfilling commitments of existing orders, a rescheduling approach is presented to generate high-quality schedules in a timely fashion. Numerical testing results demonstrate that the new approach is effective to schedule manufacturers across a chain to achieve the required three-sigma deliveries.

Keyword: Manufacturing scheduling, Supply chain, Six-sigma, Step penalty, Lagrangian relaxation

1. INTRODUCTION

Time-based competition and market globalization make it imperative for supply chains to achieve reliable on-time deliveries. This, however, is difficult in view that manufacturing activities are subject to various uncertainties such as unknown future order arrivals, stochastic operations, or unexpected machine breakdowns. In addition, manufacturers in a supply chain rely on their suppliers to provide component parts, and delays of one manufacturer may propagate to its downstream manufacturers through precedence relationships. The problem is particularly serious for make-to-order supply chains, where the flow of work is triggered by random arrivals of customer orders with little inventory to buffer against uncertainties. For example, it was reported that companies may have to reschedule up to 80% of existing orders to accommodate new ones, and this leads to poor delivery performance (Brown, 1988).

To stay competitive, there is a critical need to control order lead-times and variability of individual manufacturers across a chain to achieve delivering final products to customers within specified target time windows with high probability. This is the key idea of achieving x-sigma delivery, a hallmark for service quality and reliability (Hahn and Doganaksoy, 2000; Narahari, et. al., 2000). X-sigma delivery can be addressed by various means such as improving the reliability of machines through preventive measures, increasing inventory or expanding manufacturing capacities. They can also be approached through effective manufacturing scheduling and coordinating across a chain without major capital or labor investments, however, this has not been adequately studied with few corresponding methods available.

In this paper, after a brief review of the literature in Section 2, a supply chain with sequential workflow is formulated in Section 3, where manufacturers are modeled as job shops with stochastic operations. It is assumed that the manufacturing processes have been streamlined and machines are reliable through preventive measures. The goal of reliable on-time delivery of customer orders is translated to a goal of minimizing order tardiness, earliness costs, and variance of order lead-time times. In addition to machine capacity constraints and operation precedence constraints within and across manufacturers, order lead-time variances across the chain are required to be less than or equal to a certain x-sigma threshold determined based on the customer specified delivery time windows. In view that the problem is separable, it can be decomposed into order-level subproblems after relaxing all coupling constraints within and across manufacturers. An effective variance control technique is developed as an integrated piece of the scheduling process by using stochastic dynamic programming. Coordination operations within and across individual manufacturers including allocating the total variances across the chain is achieved through an iterative price updating process within a surrogate optimization framework.

The above is a description of the problem and the corresponding solution methodology. After scheduling, new orders become existing orders, with contract due dates systematically determined based on the estimated means and standard deviations of order completion times obtained from scheduling. In addition, a step penalty is added to prevent missing the contracted due date. In this way, the method strikes a balance between fulfilling existing commitments versus taking on new orders. To accommodate new order arrivals and occurrences of random events in a timely fashion, schedules are generated upon major order arrivals or periodically.

Numerical testing presented in Section 5 examines cases with different levels of uncertainties and new arrivals. The results demonstrate that the method is effective to reduce the variances of lead-times to achieve three sigma delivery performances, and for balancing the fulfillment of existing commitments versus taking in new arrivals.

2. LITERATURE REVIEW

Pioneered by Motorola, six sigma quality has become a hallmark of excellence for product or process quality (Hahn and Doganaksoy, 2000). Products with six sigma quality imply that there are no more than 3.4 defects per million parts in the presence of typical source of variation. This concept has been extended to delivery performance of manufacturing processes, where six sigma qualities imply that there are no more than 3 to 4 orders outside their target delivery time windows per million deliveries. This is difficult to achieve in view that manufacturing systems are usually subject to various sources of variability such as unknown order arrivals, stochastic operations or fluctuation in the manufacturing condition. Reducing variability is critical to improve delivery performances, and has been investigated by several papers, including Viswanadham (1999) and Tayur, et. al (1999). Research to buffer variability has been focused on the inventory management, smoothing production or expanding capacity at the strategic level, with limited exploration on inherent operation complexity and uncertainty. Additionally, achieving six-sigma deliveries also present challenges for supply chains, where variability of one manufacturer may propagate to another through precedence relationship. How to pool variability across a chain to reduce the propagation effects is therefore a major concern for supply chain management (Naraharim et. al., 2000; Garg et. al., 2002). A variance pool allocation technique has been developed to find optimal allocation of variability among component processes in a chain to achieve a required "delivery sharpness," a new metric defined to describe how concentrated that orders are delivered within specified time windows (Garg, et. al., 2002). The issues such as how to schedule and coordinate operations of individual manufacturers to achieve the overall six-sigma delivery performances, however, have not been adequately studied, and will be addressed in this paper.

3. PROBLEM FORMULATION

3.1 Problem Description and Modeling Convention

In this paper, a make-to-order model with sequential workflows is considered with schematic presented in Figure 1. There are F manufacturers in the chain. Orders have to go through operations in a series of manufacturers f-1, f, f+1 before completion. Orders arrival times are assumed to be deterministic, and operation-processing times are stochastic. A particular manufacturer f requires material or component parts from its upstream manufacturer f-1, and provides parts for its downstream manufacturer f+1. There are two types of orders within individual manufacturers: existing orders associated with fixed due dates in contract with downstream organizations; and new orders with requested delivery dates specified by downstream organizations. Final products (i.e., orders within the last manufacturer) are required to achieve x-sigma deliveries to customers within specified target time windows.

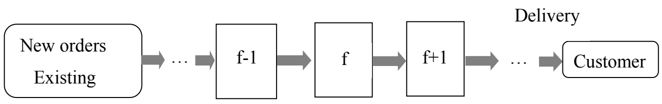

Figure 1. Schematic of a make-to-order supply chain model

From the above description, it can be seen that operations are subject to coupling constraints within and across individual manufacturers. And effective scheduling and coordinating across the chain are needed to achieve a shared goal of x-sigma delivery. In the following, constraints within individual manufacturers, cross-organization relationships and finally, the objective function will be presented.

3.2 Constraints within Individual Organizations

For simplicity, a particular manufacturer f is modeled as a job shop based on the model of Wang, et. al.(1997), and contains multiple machine types with the capacity of type h machine at time k given and denoted as M_{kh}^{f}. The i^{th} (i = 1, 2, …, I) order in manufacturer f is denoted as (f, i) and is associated with an upstream order (f-1, i) and a downstream order (f+1, i). Order (f, i) goes through a series of J_{fi} operations, with the j_{th} operation denoted as (f, i, j). Among the set of orders O_f, the subset of new orders is denoted as O_f^T, and the subset of existing orders is denoted as O_f^C. An existing order (f, i) has a due date d_{fi} in contract with the downstream manufacturer f+1 or the customer, and a promised delivery date $d_{f-1,i}^{p}$ offered by its upstream manufacturer f-1. Similarly, a new order (f, i) is associated with a requested delivery date d_{fi}^{r} specified by f+1, and a tentative delivery date $d_{f-1,i}^{t}$ offer by f-1. In addition, manufacturer f determines the requested delivery date $d_{f-1,i}^{r}$ for order (f, i). The constraints within individual manufacturers are briefly presented below.

Operation Processing Time Constraints. Each operation needs to be scheduled on a machine of the required type for a random amount of time.

$$c_{fij} = b_{fij} + p_{fij} - 1, \forall (f, i, j), \qquad (1)$$

where b_{fij} is the beginning time of (f, i, j), and c_{fij} the completion time; processing time p_{fij} is a nonnegative random variable with a given distribution. It is assumed that the processing times for different operations are independent.

Operation Precedence Constraints. Operation (f, i, j+1) cannot be started until its preceding operation (f, i, j) has been completed plus possibly a nonnegative slack time s_{fij}.

$$c_{fij} + s_{fij} + 1 \le b_{fi(j+1)},\ \forall\ (f, i, j \ne J_{fi}). \qquad (2)$$

There are other two sets of precedence constraints. First, delivery of (f, i) (as characterized by the tentative or promised delivery date) should after its completion time plus a required slack time (representing, e.g. transportation time). Second, orders can only be started after the material or component part arrival time (as characterized by the requested delivery date $d^{r}_{f-1,i}$) plus a required slack time.

Expected Machine Capacity Constraints. The number of active operations scheduled on a particular machine type h should be less than or equal to the capacity of that machine type at any time. In view of the complexity of stochastic scheduling as caused by the multitude of random event realizations, machine capacity constraints are approximated by the following expected versions (Luh, Chen, and Thakur, 1999):

$$E\left[\sum_{ij} \delta_{fijkh}\right] \le M^{f}_{kh}.\ \forall\ k \text{ and } h, \qquad (3)$$

where δ_{fijkh} is an operation indicator and defined to be one if the operation (f, i, j) is active at time k on machine type h and $\delta_{fijkh} \equiv 0$ otherwise. The above constraints couple decision variables belonging to different orders together, and are coupling constraints within individual manufacturers. They are to be satisfied in the expected sense in the core of the optimization algorithm, and to be strictly satisfied in the schedule implementation phase.

3.3 Cross-organization Relationship.

Cross-organization precedence constraints. Inter-organizational precedence relationship imposes constraints across manufacturers. For an existing order, the contracted due date may not be met during scheduling in view of disruptions caused by uncertainties such as unknown future order arrivals. For coordination purposes, a new promised delivery date d^{p}_{fi} is established and required to be less than or equal a new requested delivery date specified by f+1:

$$d^{p}_{fi} \le d^{r}_{fi},\ \forall (f, i) \in O^{C}_{f}. \qquad (4)$$

Similarly, for new orders, the tentative delivery date d^{t}_{fi} is required to be less than or equal to the requested delivery date specified by f+1.

These two constraints couple decision variables of adjacent organizations together, and are coupling cross-organization constraints.

X-sigma variance constraints. Assume that the means of order delivery times (as characterized by order completion times at the end of the chain) are centered at the middle of the corresponding customer-specified time windows without shift, x-sigma delivery requires that the standard deviation of completion times should be less than or equal to the lengths of target time windows divided by 2x. In view that order arrival times are deterministic, the completion variance is equivalent to the variance of order lead-time across the chain, and x-sigma variance constraints can be formulated as following:

$$\sigma^2_{c_i} = \sigma^2_{c_i - a_i} \le \left(\frac{U_i - L_i}{2x_i}\right)^2, \qquad (5)$$

where U_i and L_i are the given upper and lower limits of delivery time window specified by the customer, with $(U_i + L_i)/2$ equivalent to order contracted due dates at the end of the chain; $\sigma^2_{c_i}$ is the completion variance for the i[th] order, and $\sigma^2_{c_i - a_i}$ is the corresponding lead-time variance.

Assume that activities of individual manufacturers are independent and the transportation times between adjacent manufacturers are deterministic, the variance of lead-times across the chain can be abstracted as a sum of variances of order lead-times with individual manufacturers. Equation (5) therefore can be transformed into follows:

$$\sigma^2_{c_i - a_i} = \Sigma^{F}_{f=1} \sigma^2_{c_{fi} - a_{fi}} \le \left(\frac{U_i - L_i}{2x_i}\right)^2, \qquad (6)$$

where the term $\sigma^2_{c_{fi} - a_{fi}}$ represents variance for lead-times of order (f, i). The above constraints are additive, and this facilitate allocating and reducing variances through scheduling and coordinating individual manufacturers.

3.4 Overall Objective Function

As mentioned in Section 1, manufacturers have a shared goal of achieving on-time deliveries. This translates to minimize a weighted sum of penalties for expected order tardiness and earliness penalties. To achieve x-sigma delivery of finished orders to customers, it is critical to reduce variability for individual manufacturers across the chain. An additional term for penalizing lead-time variance is therefore introduced to the cost function of existing orders as follows:

$$J_{fi} \equiv E\left[w_{fi} T^2_{fi} + \beta_{fi} E_{fi} + \gamma_{fi} \text{Step}(T_{fi})\right] + w^{\sigma}_{fi} \sigma^2_{c_{fi} - a_{fi}},\ \forall (f, i) \in O^{C}_{f}, \qquad (7)$$

where parameters w_{fi}, β_{fi} and w^{σ}_{fi} are nonnegative penalty coefficients. The tardiness is defined as $T_{fi} = \max\ \{0, c_{fi} - d_{fi}\}$ and earliness $E_{fi} = \max\ \{0, \overline{b}_{fi} - b_{fi}\}$, where $\overline{b}_{fi}$ is the "desired beginning time." The term $\gamma_{fi}\text{Step}(T_{fi})$ represents a step penalty for missing the contracted due date as shown in Figure 2, with $\text{Step}(T_{fi})$ equals one if $T_{fi} > 0$ and 0 otherwise, and γ_{fi} the corresponding weight.

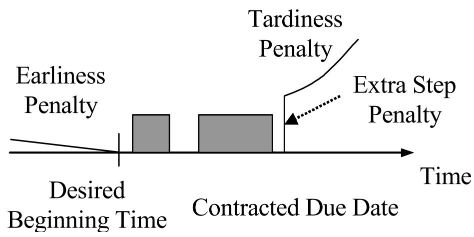

Figure 2. Penalty function for an existing order

In view that order arrival times are assumed to be deterministic, the lead-time variance $\sigma^2_{c_{fi} - a_{fi}}$ is equivalent to order completion variance:

$$\sigma^2_{c_{fi} - a_{fi}} = \sigma^2_{c_{fi}} \qquad (8)$$

This relationship will be used in derivations of Section 4.

The cost function for new orders are similarly defined, except that tardiness is calculated based on the requested delivery date as $T_{fi} = \max\{0, c_{fi} - d^r_{fi}\}$, and there is not extra step penalty term.

The overall problem is to minimize the sum of order cost functions:

$$\min J, J \equiv \sum_{fi} J_{fi}, \tag{9}$$

subject to constraints within the organizations including (1) -(3); and cross-organization constraints including (4) and (6). The decision variables are operation beginning times within all manufacturers, and the promised, tentative delivery dates as well as requested delivery dates for all the orders. The problem formulation is order-wise additive, and this motivates a Lagrangian relaxation based decomposition approach.

4. SOLUTION METHODOLOGY

To be consistent with the organizational structure, ideally a two-step relaxation is carried out, where coupling cross-organization constraints (4) and (6) are relaxed first by Lagrangian multipliers, and then machine capacity constraints (3) within individual manufacturers are relaxed. For simplicity of derivation, coupling constraints within and across organizations are relaxed at the same time and the overall problem (9) is decomposed into a set of order-level subproblems. Based on prices and the information received from upstream or downstream manufacturers (i.e., promised or tentative delivery dates and the requested delivery dates), each subproblem is solved by using stochastic dynamic programming (SDP, Wang, et. al., 1997). By intuitively decomposing lead-time variance into variability associated with individual operations, lead-time variances are calculated and reduced in a stage-wise fashion as an integrated piece of the SDP process. After the subproblems are solved, prices are updated to coordinate activities within and across the chain including allocating variances among individual manufacturers. This completes one iteration. The iteration repeats until algorithm converges.

After scheduling, a new order becomes an existing order, with contracted due date determined by expected completion time and the corresponding variance obtained from scheduling. To balance between fulfilling existing commitments versus maintaining agility to take on new orders, step penalties for avoiding missing the contracted due dates are imposed on existing orders. Rescheduling is triggered periodically or upon the arrivals of new orders. The details of this dynamic process are presented next.

4.1 Problem Decomposition

After relaxing cross-organization constraints (4) and (6) and machine capacity constraints (3) by using sets of multipliers $\{\eta\}$, $\{\lambda\}$ and $\{\pi\}$, a relaxed problem is obtained as:

$$\min L, L \equiv \sum_{fi} J_{fi} + \sum_f \left[\sum_{(f,i)\in O_f^T} (\eta_{fi}(d^t_{fi} - d^r_{fi})) \right] +$$

$$\sum_f \left[\sum_{(f,i)\in O_f^C} (\eta_{fi}(d^p_{fi} - d^r_{fi})) \right] + \sum_i \left[\lambda_i \left(\sum_{f=1}^{F} \sigma^2_{c_{fi}-a_{fi}} - \left(\frac{U_i - L_i}{2x_i} \right)^2 \right) \right]$$

$$+ \sum_f \sum_{kh} \pi_{kh} \left[\sum_{ij} E(\delta_{fijkh}) - M^f_{kh} \right], \tag{10}$$

subject to constraints within manufacturers (e.g., (1)-(2)).

4.2 Scheduling Individual Orders

The relaxed problem can be decomposed into a set of order-level subproblems within individual manufacturers. For simplicity of illustration, only formulation and solution of existing order subproblems will be elaborated in this paper. Similar ideas can easily apply to subproblems of new orders and are omitted. The subproblem for an existing order (f, i) is formulated as follows:

$$\min L_{fi}, L_{fi} \equiv E\left[w_{fi}T^2_{fi} + \beta_{fi}E_{fi} + \sum_j \sum_{k=b_{fij}}^{c_{fij}} \pi_{kh} + \gamma_{fi}\mathrm{Step}(T_{fi}) \right]$$

$$+ (\lambda_i + w^\sigma_{fi})\sigma^2_{c_{fi}} + \eta_{fi}d^p_{fi} - \eta_{f-1,i}d^r_{f-1,i}, \quad (f,i) \in O^C_f, \tag{11}$$

subject to (1)-(2). The decision variables are operation beginning times, the promised delivery date d^p_{fi} to be offered to the downstream manufacturer f+1, and the requested delivery date $d^r_{f-1,i}$ to be imposed upon the upstream manufacturer f-1. It should be noted that the lead-time variance has been replaced by the completion variance based on (8).

To solve the subproblem by using SDP (Wang, et. al., 1997), the challenge is to effectively compute and reduce order completion variances. To address this, a novel variance control technique is developed as an integrated piece of order scheduling. As the first step, for a given beginning time $b_{fi} = b_{fi1}$, the completion time for order (f, i) can be formulated as a sum of operation processing times and wait times as follows:

$$c_{fi} = b_{fi1} + \Sigma_{j=1}^{J}(p_{fij} + s_{fij}). \tag{12}$$

where the term $p_{fij} + s_{fij}$ is equivalent to the time interval between beginning times of operation j and its subsequent operation j+1. In view that these time intervals are independent for different operations; the completion variance can be calculated as follows:

$$\sigma^2_{c_{fi}} = \Sigma_{j=1}^{J}\sigma^2_{p_{fij}+s_{fij}}. \tag{13}$$

This facilitates computing and reducing the completion variance in an operation-wise fashion during the SDP process as illustrated below.

In SDP, a stage corresponds to an operation, and a state corresponds to a possible beginning time. The algorithm starts from the last stage J: given a particular beginning time b_{fiJ}, for each possible processing time p_{fiJ}, there is a corresponding completion time c_{fiJ}, and the completion variance is equivalent to variance of processing times based on (13). The terminal cost is therefore calculated as:

$$V_{fiJ}(b_{fiJ}) = E\left[w_{fi}T^2_{fi} + \sum_{k=b_{fiJ}}^{c_{fiJ}} \pi_{kh} + \gamma_{fi}\mathrm{Step}(T_{fi}) \right]$$

$$+ (\lambda_i + w^\sigma_{fi})\sigma^2_{p_{fiJ}} + \eta_{fi}d^p_{fi}, \tag{14}$$

Now move backward to stage J-1 from stage J. Given state $b_{fi,J-1}$, for each possible processing time of stage J-1, there is a corresponding decision of beginning time for stage J.

Based on (13), the completion variance for given $b_{fi,J-1}$ is calculated as:

$$\sigma^2_{c_{fiJ}} = \sigma^2_{p_{fi,J-1}+s_{fi,J-1}} + \sigma^2_{p_{fiJ}}. \quad (15)$$

The expected cumulative cost as the algorithm moving backward is then obtained recursively as:

$$V_{fij}(b_{fij}) = E\left[\beta_{fi}E_{fi}\Delta_{fij} + \sum_{k=b_{fij}}^{c_{fij}} \pi_{kh} + \min_{\{b_{fi,j+1}\}} V_{fi,j+1}(b_{fi,j+1})\right]$$

$$+ (\lambda_i + w^{\sigma}_{fi})\sigma^2_{p_{fij}+s_{fij}} - \Delta_{fij}\eta_{f-1,i}d^r_{f-1,i}, \quad (16)$$

where Δ_{fij} is an integer variable equal to one if j equals one and zero otherwise. The optimal L^*_{fi} is obtained as the minimal expected cumulative cost at the first stage subject to arrival time constraints:

$$L^*_{fi} = \min_{\{b_{fi1}\}} E\left[V_{fi1}(b_{fi1})\right]. \quad (17)$$

The solution from SDP is a policy describing what to do under which circumstances and therefore can be applied based on the occurrence of random events. Following the policy, optimal operation beginning times, as well as estimates of the means and variances of order completion times can be obtained by tracing the stage forward. Subproblems for new orders can be formulated and solved in the same way as presented above, except that order promised delivery dates are replaced by tentative delivery dates d^t_{fi} and contracted due dates are replaced by requested delivery dates d^r_{fi}.

4.3 Coordination Procedure to Convergence and Rescheduling

Given the optimal solutions of order subproblems, the high level dual problem is to select an optimal set of multipliers to maximize the dual function, i.e.,

$$\max_{\pi,\eta,\lambda} \tilde{q}(\eta,\lambda,\pi), \text{ with } \tilde{q}(\eta,\lambda,\pi) \equiv \tilde{L}^* \quad (18)$$

subject to non-negativity of all multipliers. To solve (18), the "Surrogate Subgradient Method" (SSGM, Zhao, et. al., 1999) was used to update the multipliers and allows efficient resolution of large problems. After the multipliers are updated to coordinate activities of manufacturers including allocate variances across the chain, the subproblems are resolved and the process continues until the prices are close to convergence. A new order then becomes an existing order after scheduling, with its tentative delivery date becomes the promised delivery date. To be consistent with x-sigma delivery, the contacted due dates for existing orders are determined in consultation with customers based on the estimated mean and standard deviation of order completion times obtained from scheduling, i.e.,

$$d_{fi} = E[c_{fi}] + x\sigma_{c_{fi}}. \quad (19)$$

Rescheduling is triggered upon the arrivals of new orders or periodically. Most decision variables are re-optimized except the contracted due dates of existing orders. To balance between fulfilling existing commitments versus maintaining agility to take on new orders, step penalties are imposed on existing orders to avoid missing their due dates during rescheduling.

4.4 Schedule Implementation and Evaluation

The solutions of individual subproblems, when put together, are generally not feasible in view that the coupling constraints (e.g., (3)) are approximated by expected versions and relaxed by Lagrangian multipliers. A greedy heuristics (Wang, et. al., 1997) is used at the on-line implementation phase to eliminate possible constraint violations. To evaluate algorithm performance, schedules are evaluated by using a simulation model embedded with the above-mentioned heuristics.

5. NUMERICAL RESULTS

The method presented above has been implemented in Matlab and tested on a PC with a Pentium IV 2.0 GHz processor and 512M SDRAM. Numerical testing has been performed on a simplified two-factory model with orders requiring a series of operations to process. Each factory contains three machine types, and each type for a specific operation. The operation processing times are uncertain and described by sets of symmetric three-value distributions with specified means and variances.

Two examples are tested. The first example presents cases with varying settings of operation uncertainty levels to demonstrate the value of the new approach to improve delivery performance by reducing variances of order lead-times. The performance of the new method is compared with that of the traditional SLR methods without x-sigma variance control technique (i.e., equivalent to $w^{\sigma}_{fi} = 0$ and without adding cross organization variance constraints), and the "weighted shortest processing time and critical ratio" (WSPT/CR) rule. The WSPT/CR rule gives priorities to operations with high tardiness weight and low processing times to reduce work-in-process inventory, while emphasizing the criticality to meet the due dates. It has been proved to be effective against many performance measures (Shafaei and Brunn, 1999). The second example presents cases containing different percentage of new arrivals to examine the effectiveness of step penalties to fulfill order delivery commitments.

For all the cases, the penalty weights for tardiness, earliness are set to be 1 and 0.1, respectively. For the new method, the penalty weights for lead-time variances are set to be 1. The orders are required to achieve approximately 3-sigma delivery performances (> 90% of orders delivered within target time windows). Algorithms are terminated after a fixed amount of computational time. Based on the scheduling policy obtained, 100 simulation runs are conducted for each case with random variable realized based on their distributions. To compare the performances of different algorithms, the same number of simulation runs is performed using the same set of random seeds.

Example 1. In this example, forty orders with 240 operations are to be scheduled on 6 machine types of two factories over a time horizon of 88 days. The number of machines per type is set to be 8 for both factories. Among the orders, there are 75% existing orders and 25% new orders. The step penalties for contract orders are set to 10. Two cases are tested. The variances of processing times are set to be 0.8 for Case 1, and 1.6 for Case 2 to represent low and high uncertainty levels, respectively. The results are summarized in Table 1 and Table 2. In the tables, the terms

"Mean Order Completion-time" "Standard deviation of Completion Times" and "Average Tardiness Cost" are average results based on 100 simulation runs. To reflect the performance of delivering finished orders to customers, the "Average Tardiness Cost" and the "Percentage of Late Delivery" are all computed based on the contracted due dates for finished orders at the end of the supply chain. The "CPU time" is the computation time for running the LR based algorithms for each case.

Table 1. Performance comparisons when uncertainty level is low

Case 1 (Low Uncertainty Level)	New LR/SSG	Traditional LR/SSG	WSPT /CR
Mean Order Completion (Day)	49.47	49.92	49.57
Deviation of Completion (Day)	1.71	2.82	3.47
Average Tardiness Cost	4.23	119.94	175.2
Percentage of Delay (%)	1.58	13.5	15.30
CPU time (Sec.)	200	200	/

Table 2. Performance comparisons when uncertainty level is high

Case 2 (High Uncertainty Level)	New LR/SSG	Traditional LR/SSG	WSPT /CR
Mean Order Completion (Day)	49.06	49.95	49.53
Deviation of Completion (Day)	3.23	4.02	4.76
Average Tardiness Cost	52.03	137.14	226.88
Percentage of Delay (%)	4.5	10.70	19.42
CPU time (Sec.)	200	200	/

From the tables, it can be seen that for both cases, the new approach outperforms the traditional SLR method and WSPT/CR rule by effectively reducing order completion variances and achieving lower delay rates as well as tardiness costs. For both cases, the delay rates are below 5% by using the new method; this implies that the new approach can generate high-quality schedules to achieve approximate 3-sigma order deliveries.

Example 2. In this example, 600 operation associated with 100 orders are to be processed on 150 machines of 6 types in two factories in a sequential manner. The variances of processing times are set to be 0.8. Two cases are tested, with Cases 3 and 4 containing 15% and 35% new orders, respectively. Each case is scheduled by using the new method with or without step penalties imposed on contracted due dates of existing orders (i.e., step penalties are set to be 10 and 0, respectively). The results of 100 simulation runs are presented in Table 3.

Table 3. Performance of the new method when scheduling orders with varying percentages of new arrivals

New orders	Compared items	Step Penalty = 10	Step Penalty = 0
15% Case 3	Mean Completion (Day)	48.52	48.33
	Deviation of Completion (Day)	1.83	1.79
	Average Tardiness Cost	4.33	7.96
	Percentage of Delay (%)	1.30	2.42
35% Case 4	Mean Completion (Day)	48.94	48.55
	Deviation of Completion (Day)	2.00	2.35
	Average Tardiness Cost	75.60	128.14
	Percentage of Delay (%)	4.55	8.83

From the table, it can be seen that as the percentage of new order arrivals increases, the average tardiness cost and order delay rate increase in view that new arrivals may cause original commitment to be compromised. By setting an appropriate step penalty, the new method strikes a balance between fulfilling the existing commitments versus taking in new orders by generating high-quality schedules with reduced tardiness costs.

6. CONLUSIONS

In this paper, a novel variance control technique is developed to achieve x-sigma supply chain delivery performance by accurately estimating and effectively reducing variances of lead-times through scheduling individual manufacturers and coordinating across a chain. Testing results supported by simulation demonstrates that 3-sigma delivery could be achieved without drastic increasing of the total cost. The effectiveness of the new approach to accommodate new orders while fulfilling commitments to existing orders is demonstrated, and this is of significance for practical applications.

7. ACNOWLEDGEMENT

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