STEER-BY-WIRE SUSPENSION AND STEERING DESIGN FOR CONTROLLABILITY AND OBSERVABILITY

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Abstract: With steer-by-wire, suspension and steering systems are no longer governed by purely mechanical criteria but must instead be designed according to mechatronic considerations. This paper demonstrates an approach to steer-by-wire suspension design in which geometric design variables are chosen to render the tire reaction torque about the steer axis highly predictable. This predictability, in turn, clarifies the connection between the vehicle dynamics and steering dynamics, enabling full state observer and controller design.

Keywords: Steer-by-wire, Mechatronics, Observability, Controllability, Vehicle Dynamics

1. INTRODUCTION

Steer-by-wire technology promises to deliver significant benefits, ranging from relaxed packaging constraints to advanced safety systems such as lanekeeping assistance. While by-wire systems pose new engineering challenges, they also open up new design opportunities for suspension and steering systems. Currently, suspension and steering are designed to provide good vehicle handling characteristics and driver feel. With steer-by-wire technology, handling and feel become software features customizable on a per-driver basis. As suspension and steering make this transition to fully mechatronic systems, traditional design criteria must be re-examined. This paper presents a look at design from the perspective of inherent controllability and observability.

The key idea of the controller and observer structure assumed in this paper is that the steering motor dynamics and the vehicle dynamics are linked through the tire forces and their reactions about the steer axis. In Yih and Gerdes (2004), this relationship is used to develop an effective estimator for sideslip angle. Combined with a yaw rate measurement, the full vehicle state is then available for control, providing a range of possibilities for control algorithms (Yih and Gerdes, 2004; Vilaplana *et al.*, 2004). Gadda *et al.* (2004) showed that this link between vehicle and steering dynamics, if known, could also be exploited for developing steer-by-wire diagnostics. Thus the predictability of the connection between the steering system and the tire forces becomes central to observability, controllability, and diagnostics.

The geometry of the steering and suspension systems strongly influences the nature of the steer axis reaction torque. Thus the need for a predictable torque can be translated into specific constraints on system kinematics. The core of the paper focuses on describing these constraints and discussing how they are influenced by geometric design parameters. These ideas are further developed through an analysis and redesign of an existing steer-by-wire system for improved controllability and observability.

Fig. 1. Stanford's "P1" Steer-by-wire vehicle

2. STEER-BY-WIRE CONTROL PHILOSOPHY

In previous research at Stanford, the authors have modified a Corvette to steer-by-wire configuration (Yih *et al.*, 2003) and, more recently, developed P1, a by-wire vehicle with independent front-wheel steering (Figure 1). For both vehicles, the control scheme must account for both a lack of state information about the vehicle motion and disturbances on the steering system due to the front tire forces. By incorporating a physical model of the steering, these two problems can be linked and simultaneously solved. While vehicle dynamics are inherently nonlinear, the following derivation simplifies the problem to a linear framework to establish basic intuition behind the approach.

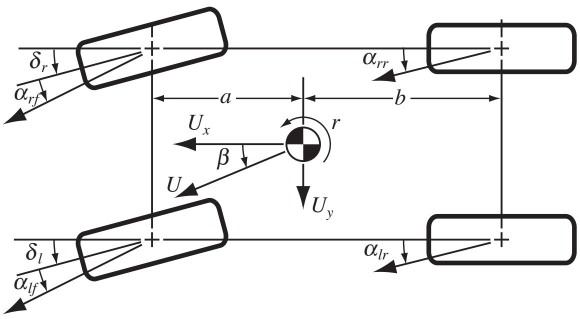

Fig. 2. Vehicle schematic and nomenclature

The planar dynamics of the vehicle can be modeled using the bicycle model, where the width of the vehicle is considered negligible. In this case, a slight extension to the bicycle model is used which considers left and right steer angles (δ_l and δ_r) separately (Figure 2). Small angle approximations are used and lateral tire force is assumed to be proportional to the tire slip angle, so that a linear model of the planar vehicle dynamics is developed, given by the following:

$$\dot{x_v} = A_v x_v + B_v \delta \tag{1}$$

where

$$x_v = \left[\, \beta \;\; r \,\right]^T$$
$$\delta = \left[\, \delta_l \;\; \delta_r \,\right]^T$$

$$A_v = \begin{bmatrix} -\dfrac{C_0}{mU_x} & \dfrac{C_1}{mU_x^2} - 1 \\ \dfrac{C_1}{I_z} & -\dfrac{C_2}{I_z U_x} \end{bmatrix}$$
$$B_v = \begin{bmatrix} \dfrac{C_{\alpha f}}{2mU_x} & \dfrac{C_{\alpha f}}{2mU_x} \\ \dfrac{C_{\alpha f} a}{2I_z} & \dfrac{C_{\alpha f} a}{2I_z} \end{bmatrix}$$
$$C_0 = C_{\alpha f} + C_{\alpha r}$$
$$C_1 = C_{\alpha r} b - C_{\alpha f} a$$
$$C_2 = C_{\alpha f} a^2 + C_{\alpha r} b^2$$

where δ_l and δ_r are the left and right steer angles, β is the sideslip angle, r is the yaw rate, I_z is the polar moment of inertia of the vehicle, $C_{\alpha f}$ and $C_{\alpha r}$ are the front and rear cornering stiffnesses, a and b are the distances from the center of gravity to the front and rear axles, m is the mass of the vehicle, and U_x is the forward velocity.

The steer-by-wire system at each wheel is a DC motor and gearbox connected via a parallelogram four-bar linkage to the wheel, enabling the following linear model of the steering:

$$\dot{x_m} = A_m x_m + B_m \left[\, i \;\; \tau_a \,\right]^T \tag{2}$$

where

$$x_m = \left[\, \delta \;\; \dot{\delta} \,\right]^T$$
$$A_m = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{b_m}{J_m} \end{bmatrix}$$
$$B_m = \begin{bmatrix} 0 & 0 \\ \dfrac{r_g \eta k_m}{J_m} & -\dfrac{1}{J_m} \end{bmatrix}$$

where δ is the steering angle, J_m is the effective moment of inertia of the steering system, b_m is the effective damping of the steering system, r_g is the gearbox ratio, η is the combined efficiency of the motor and gearbox, and k_m is the motor constant relating torque to current. The inputs to this model are the current to the motor, i, and the aligning torque, τ_a.

Tire forces generate a reaction torque about the steer axis. During most driving, the major contribution to this reaction torque is the aligning torque τ_a, which is the reaction torque due to lateral tire forces. It is related to the vehicle state by the following equation:

$$\tau_a = -C_{\alpha f}(t_p + t_m)(\beta + \frac{a}{U_x} r - \delta) \tag{3}$$

where t_p and t_m are the pneumatic and mechanical trails of the tire (Figure 3). Pneumatic trail is the distance from the center of the tire contact patch to the centroid of the force distribution on the tire contact patch. This value is typically about 20-25mm for passenger cars. Although constant for most handling regions, as the tire approaches its friction limit, the pneumatic trail decreases toward zero. In order to arrive at a linear model, t_m and t_p are assumed to be constant (as are $C_{\alpha f}$ and $C_{\alpha r}$), and the steer axis reaction torque is assumed to be equal to the aligning torque.

Combining (1), (2), and (3) yields a linear state-space model of the vehicle that is controllable and observable using measurements of only the steering angles and yaw rate. The accuracy of this model is limited by the assumptions that the aligning torque is a linear function of the vehicle states and that it is the only contributor to steer axis reaction torque. The next section examines how the steering system geometry can be designed to minimize the contributions to steer axis reaction torque not captured in this linear model.

3. CURRENT STEERING GEOMETRY AND DESIGN CONSIDERATIONS

The steering and suspension systems of the P1 by-wire vehicle are shown in Figure 4. The suspension is a double wishbone design with a four-bar steering linkage.

The mechanism by which a motor steers an individual wheel is comprised of two components: the steering knuckle (or upright), which defines the relative positions of the wheel and steer axis, and the steering linkage, which defines the relative positions of the steer axis and the motor axis. The design of each of these components can be analyzed separately.

3.1 Steering Knuckle Parameter Definitions

There are five main geometric parameters to consider in steering knuckle designs: the wheel radius (R_l), caster angle (θ_c), kingpin inclination angle (θ_k), scrub radius (d), and mechanical trail (t_m). These parameters are illustrated in Figure 3 with the car at its nominal suspension position and zero steer angle. Caster and kingpin inclination angles are defined relative to the car, while mechanical trail and scrub radius are defined relative to the wheel. Caster and kingpin inclination angles do not change with steer angle, but mechanical trail and scrub radius may. Most suspensions are designed such that the effect of vertical suspension travel on these measurements is small.

The values of these five parameters for the current design are within the typical ranges of most passenger cars. These values are given in Table 1.

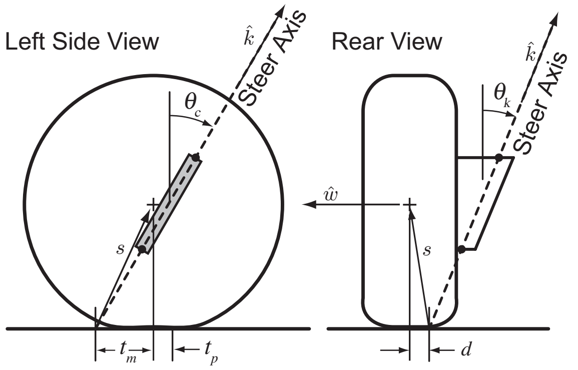

Fig. 3. Left and rear views of the left-hand wheel

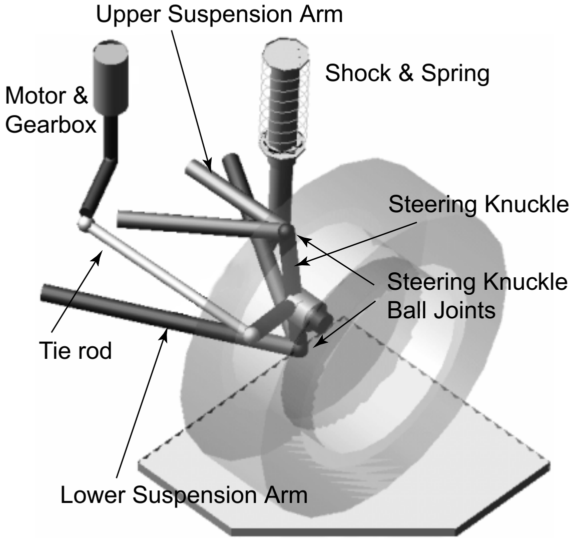

Fig. 4. View of current suspension and steering systems

Table 1. Steering Knuckle Parameters

Parameters at Zero Steer	Typical Values	Current P1 Design
Wheel Radius (R_l)	300 to 350 mm	320 mm
Caster Angle (θ_c)	4 to 7°	5.5°
Kingpin Inclination Angle (θ_k)	7 to 15°	13.2°
Scrub Radius (d)	-75 to 75 mm	50.5 mm
Mechanical Trail (t_m)	15 to 50 mm	28 mm

3.2 Effects of Knuckle Parameter Selection

Most important to steer-by-wire design is how these parameters effect the steer axis reaction torque. This is important from a mechanism design standpoint since it is what must be canceled by the steering actuator to maintain wheel position. It is also important from a state estimation standpoint since it is how tire forces (and, by a model, vehicle states) are observed by the steering torque.

There are three main contributions to the steer axis reaction torque:

- The cross-product of lateral tire forces with total trail
- The effect of suspension jacking

- The cross-product of longitudinal tire forces with scrub radius

The relative importance of these three parts depends on vehicle speed.

Effects on Steer Axis Reaction Torque from Total Trail At higher speeds (>5 m/s) when high lateral forces can be generated with relatively small steer angles, the cross-product of lateral tire forces with total trail dominates the steer axis reaction torque. This is the only contribution considered in the simple model given in (3). In a traditional steering system, this torque provides the return-to-center feel at higher speeds. In a steer-by-wire system, larger total trails not only will increase the gain used to sense lateral tire forces but will also increase the actuator effort needed to maintain a non-zero steer angle.

The following equations provide an analytic expression for mechanical trail (t_m) for the left wheel as a function of steer angle (δ):

$$k = \frac{1}{\sqrt{\tan^2\theta_c + \tan^2\theta_k + 1}} \begin{bmatrix} -\tan\theta_c \\ -\tan\theta_k \\ 1 \end{bmatrix} \quad (4)$$

$$P(\delta) = (1-\cos\delta)kk^T + \\ + \begin{bmatrix} \cos\delta & k_3\sin\delta & -k_2\sin\delta \\ -k_3\sin\delta & \cos\delta & k_1\sin\delta \\ k_2\sin\delta & -k_1\sin\delta & \cos\delta \end{bmatrix} \quad (5)$$

$$\boldsymbol{s} = [\hat{\boldsymbol{x}} \;\; \hat{\boldsymbol{y}} \;\; \hat{\boldsymbol{z}}]\, P(\delta)\, [-R_l\tan\theta_c \;\; d \;\; R_l]^T \quad (6)$$

$$\hat{\boldsymbol{w}} = [\hat{\boldsymbol{x}} \;\; \hat{\boldsymbol{y}} \;\; \hat{\boldsymbol{z}}]\, P(\delta)\, [0 \;\; 1 \;\; 0]^T \quad (7)$$

$$\hat{\boldsymbol{k}} = [\hat{\boldsymbol{x}} \;\; \hat{\boldsymbol{y}} \;\; \hat{\boldsymbol{z}}]\, k \quad (8)$$

$$\boldsymbol{l} = \boldsymbol{s} - R_l \frac{\hat{\boldsymbol{z}} - \hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{w}}\hat{\boldsymbol{w}}}{\|\hat{\boldsymbol{z}} - \hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{w}}\hat{\boldsymbol{w}}\|} \quad (9)$$

$$t_m = (\boldsymbol{l} - \frac{\hat{\boldsymbol{z}}\cdot\boldsymbol{l}}{\hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{k}}}\hat{\boldsymbol{k}}) \cdot \frac{\hat{\boldsymbol{w}} - \hat{\boldsymbol{w}}\cdot\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}}{\|\hat{\boldsymbol{w}} - \hat{\boldsymbol{w}}\cdot\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\|} \quad (10)$$

In (5), $P(\delta)$ is a rotation matrix which rotates about the steering axis, given by $\hat{\boldsymbol{k}}$ in (8). The orientation of the wheel is represented by $\hat{\boldsymbol{w}}$ in (7). The location of the center of wheel and the location of the contact patch, both relative to the nominal intersection of the steering axis with the ground, are given by $\boldsymbol{s}$ and $\boldsymbol{l}$ in (6) and (9). Vectors $\hat{\boldsymbol{k}}$, $\hat{\boldsymbol{w}}$, and $\boldsymbol{s}$ are illustrated in Figure 3.

A plot of mechanical trail for the current steer-by-wire design is given in Figure 5. Note that mechanical trail changes very significantly with steer angle, even passing through zero and going negative. When mechanical trail passes through zero, the vehicle states are unobservable, and it becomes impossible to estimate vehicle sideslip.

Effects on Steer Axis Reaction Torque from Suspension Jacking At lower speeds (<5 m/s) when the contribution from total trail and lateral forces is small, the effect of suspension jacking can dominate the steer axis reaction torque. As the wheels turn in and out, they lower and raise the car, respectively. Suspension jacking is the torque felt about the steer axis as a result of this motion. Jacking torque is simply the reaction torque about the steer axis from tire normal forces.

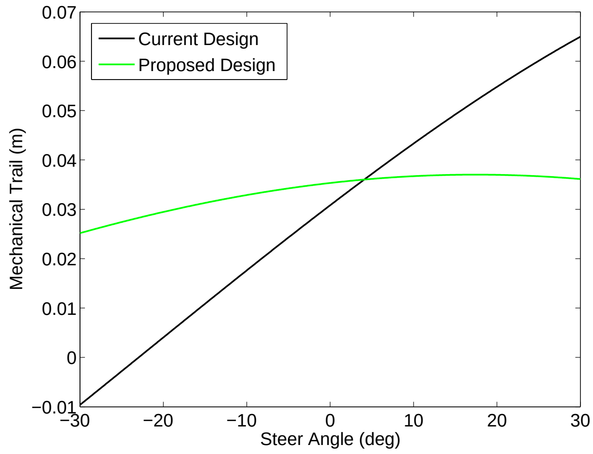

Fig. 5. Mechanical trail change on the left wheel as a function of steer angle

Jacking torque (τ_j) as a function of steer angle (δ) and normal force (F_z) is given by (4, 5, 6, 7, 8, 9) and the equation below:

$$\tau_j = \hat{\boldsymbol{k}} \cdot (\boldsymbol{l} \times F_z\hat{\boldsymbol{z}}) \quad (11)$$

Figure 6 illustrates the jacking torque effect on P1. The jacking torque tends to be large only when turning out at high steer angles. When the wheels are linked together in a traditional steering system, the jacking torques of each wheel nearly cancel at low steer angles. At high steer angles, the jacking torque of the wheel that is turning out dominates, resulting in a return-to-center feel. This is less important at high speeds since, for a given amount of steer angle, the torque due to lateral tire forces becomes much larger than jacking torque.

Effects on Steer Axis Reaction Torque from Scrub Radius The steer axis reaction torque contributions from scrub radius are generally undesirable and often minimized by design. The primary motivation for having a non-zero scrub radius is that at near-zero vehicle speed, it allows the tire to roll slightly while turning.

Turning with too little scrub radius at near-zero speeds would require very high actuator effort and result in excessive tire wear. However, a large scrub radius is also not desirable as it allows longitudinal tire forces (i.e. braking) to have a notable effect on steer axis reaction torque. When both

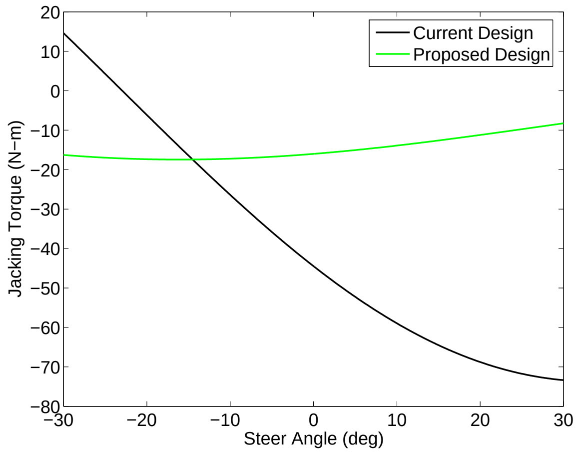

Fig. 6. Jacking torque on the left wheel as a function of steer angle

wheels are linked together in a traditional steering system, the contributions from the left and right sides generally cancel out. When the wheels are decoupled as in the P1 steer-by-wire system, the effects of longitudinal tire forces are undesirable for vehicle state estimation and control.

3.3 Steering Linkage Design

The P1 steer-by-wire vehicle uses a parallelogram four-bar steering linkage (Figure 4). This avoids all prismatic joints, maintaining high efficiency and backdrivability. It also maintains a 1:1 relationship between motor and steer angles so that effective inertias are constant.

The steering linkage should be designed to minimize roll steer. Because steering linkages tie together parts that do and do not move with vertical suspension travel, it is possible that the wheels may steer when the vehicle rolls. In traditional steering systems, this effect may be desirable to give a proper feel to the driver. In steer-by-wire systems, this creates an effect that is difficult to measure, causing a deterioration in state estimation ability. The roll steer characteristics are determined by the position of the inboard tie rod end, which can be selected with the same process used in traditional steering system design.

4. EXPERIMENTAL RESULTS

One way to validate the steer axis reaction torque model is to use experimental data from P1 to compare estimates of steer axis reaction torque to its measured value. The maneuvers were performed at a slow speed of 4 m/s to allow high steer angles which better illuminate steering nonlinearities. Longitudinal force contributions to the reaction torque are small enough to be neglected since the maneuvers were performed without braking.

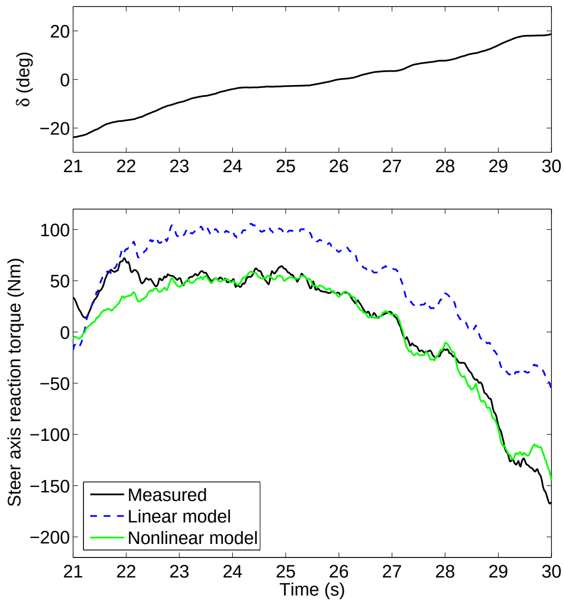

Fig. 7. Steer axis reaction torque estimation on left wheel

The measured value of steer axis reaction torque is computed using a measurement of steer motor current with a gearbox and motor model. This model includes friction, efficiency, damping, and inertia contributions. The estimates of steer axis reaction torque are computed using measurements of vehicle states and steer angles with two different models. The first is the linear aligning moment model given in (3); the second is the nonlinear model developed in the previous section that includes jacking torque and a varying mechanical trail.

The results for the left wheel are given in Figure 7. The largely constant offset between the two estimates is due primarily to the contribution of jacking torque. Near the end of the test, when the left wheel is turned far inward, this offset grows notably. This is due to the increasing mechanical trail. By including both of these, the nonlinear model tracks the measured value much better. Clearly, the effects of these two contributions cannot be neglected with P1.

5. AN IMPROVED STEERING GEOMETRY DESIGN

Based on this discussion, the suspension and steering systems on P1 can be improved by:

- Reducing mechanical trail changes with steer angle
- Reducing suspension jacking torque
- Reducing effects of longitudinal forces on steer axis reaction torque

A significant reduction in kingpin inclination angle will reduce mechanical trail changes and the effects of suspension jacking. A reduction in scrub radius will reduce the impact of longitudinal forces on steer axis reaction torque. These are altered by changing the position of the two ball joints on the steering knuckle (illustrated in Figure 4).

Changes to ball joint positions cannot be made arbitrarily. One reason is that the distance between the ball joints should not decrease much. This would increase the force-loading on suspension members, increasing unwanted compliance effects that reduce the accuracy of steer angle measurement. Another is packaging. The positions of wheels and brakes place constraints on the available positions for ball joints. There are three ways to position the ball joints:

- Both ball joints inside the rim of the wheel. Although this allows for low kingpin angles, it requires that they must be close together, giving undesirable force-loading characteristics.
- Lower ball joint inside the rim and upper ball joint inboard of the rim and tire. This allows the upper ball joint to be positioned higher and further away from the lower ball joint, but makes it difficult to attain both a small scrub radius and kingpin inclination angle.
- Lower ball joint inside the rim and upper ball joint above the rim and tire. This provides adequate separation distance between the ball joints and allows both a small scrub radius and kingpin inclination angle at the expense of consuming packaging space above the tire. This is known as a tall knuckle design.

For the P1 steer-by-wire vehicle, the last option above is the best choice. Due to packaging constraints, a zero kingpin inclination angle is undesirable. The proposed knuckle design is summarized by the parameters in table 2. The new parameters give mechanical trail and suspension jacking characteristics given in Figures 5 and 6.

Table 2. Steering Knuckle Parameters

Parameters at Zero Steer	Current P1 Design	Proposed Design
Wheel Radius (R_l)	320 mm	320 mm
Caster Angle (θ_c)	5.5°	6.3°
Kingpin Inclination Angle (θ_k)	13.2°	1.9°
Scrub Radius (d)	50.5 mm	28 mm
Mechanical Trail (t_m)	28 mm	33 mm

The basic form of the proposed design itself is not new; tall knuckle designs exist in production cars today. However, the motivations behind and the advantages of selecting this design are quite different. They are based not on traditional steering feel but rather their implications on controllability and observability.

6. CONCLUSIONS

The design considerations for individual wheel steer-by-wire are quite different than those for traditional steering systems. More suitable steering geometries can be developed by examining how the geometric parameters of the steering system influence observability and controllability.

The design criteria established in the paper compliment existing suspension and steering system design strategies by providing a framework with which to analyze the performance of steer-by-wire suspension designs. They provide the necessary link between traditional suspension and steering design and new steer-by-wire technology.

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