

# MODELING AND CONTROL OF A FLEXIBLE SENSOR STRUCTURE IN MICROASSEMBLY

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Abstract: This paper aims at developing a force-guided microassembly technology with in-situ flexible polyvinylidene fluoride (PVDF) beam force sensing and hybrid force/position control based on an infinite dimensional model of the flexible sensor structure. Besides the designed 1-D PVDF cantilever based micro-force sensor, as the soft sensor structure itself is installed at the free end of micromanipulator, during manipulation, the sensor beam is necessary to be considered as a distributed parameter flexible link of the manipulator either. Then a hybrid micro-force/position control scheme on the basis of this infinite dimensional sensor/link model is developed. Experimental results verified the effectiveness of the hybrid control scheme as well as the performance of the developed micro-force sensor. Ultimately the technology will provide a critical and major step towards the development of automated manufacturing processes for batch assembly of micro devices. *Copyright* © 2005 IFAC

Keywords: Force/position control, Force sensor, Flexible beam, Micro assembly

## 1. INTRODUCTION

Manufacturing processes which are capable of efficiently assembling micro devices have not been developed, partially because, at the micro-scale, structures are fragile and easily breakable. They typically break at the micro-Newton force range that cannot be reliably measured by the most existing force sensors, and cannot be effectively regulated in a safety margin (Fatikow *et al.*, 2000). As a result, this situation decreases overall yield and drives up the cost of micro devices.

To address the problems above mentioned, this paper is to develop a hybrid force/position control scheme for microassembly based on an infinite dimensional sensor/link model, i.e., modeling and designing a highly sensitive force sensor to measure the micro-force information, and then enables an on-line regulation of both micro-force and position during microassembly. The sensor is designed with a flexible PVDF cantilever beam structure. As the sensor installed at the free end of the micromanipulator is a rather soft beam, when assembly is performed, it is necessary to consider the beam as an infinite dimensional flexible link of micromanipulator, so as to avoid the inaccuracy and the spillover problem in the control, due to the ignored high frequency dynamics of the beam (Matsuno *et al.*, 1994)(Matsuno and Kasai, 1998)(Ge *et al.*, 1998)(Ching and

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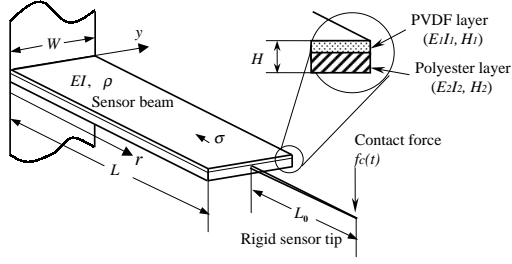


Fig. 1. Illustration of the 1-D PVDF composite sensing beam.

Wang, 1999)(Siciliano and Villani, 2000). Then based on the feedback of the self-sensing link, a hybrid micro-force/position control scheme is developed for microassembly. The closed-loop stability of the system is proven based on the partial differential equations of the system dynamics which govern the motion of the flexible sensor structure. The developed controller only requires the measurements of translational displacement, translational velocity and the micro-force acting at the tip, which leads to an easy engineering implementation. The experiments demonstrated the performances of the developed micro scale control schemes using a 1-D PVDF sensor structure. This could be an important step to enhance the microassembly technology.

## 2. MODELING OF PVDF MICRO-FORCE SENSOR

Fig.1 shows a developed 1-D PVDF sensing beam structure. Based on piezoelectric effect and the mechanics of materials for composite cantilever beam (IEEE, 1987), using an equivalent circuit model of a resistor  $R_P$  in parallel with a capacitor  $C_P$  for the PVDF film, the output voltage  $V(t)$  across the PVDF film due to charge  $Q(t)$  generated by external micro-force  $f_c(t)$ , can be described by (Shen *et al.*, 2003)

$$\frac{V(t)}{R_P} + \dot{V}(t)C_P = \frac{dQ}{dt}. \quad (1)$$

Continually, by combining this open-circuit sensor model and a differential charge amplifier processing circuit, as shown in Fig. 2. The global sensing transfer function of the PVDF sensory system is

$$\frac{V_{out}(s)}{f_c(s)} = \frac{K_c B}{R_P C_f} \frac{\lambda s}{(1 + \lambda s)(1 + \tau_1 s)} \quad (2)$$

where  $\lambda$  and  $\tau_1$  are the time constants of the PVDF film and the low pass filter, respectively.  $B$  is the constant depending on the PVDF film.  $K_c$  is the gain of the amplifier.  $C_f$  is the feedback capacitor of the amplifier. The function is a bandpass type filter. Based on this equation, we can obtain the micro-force  $f_c(t)$  by measuring the

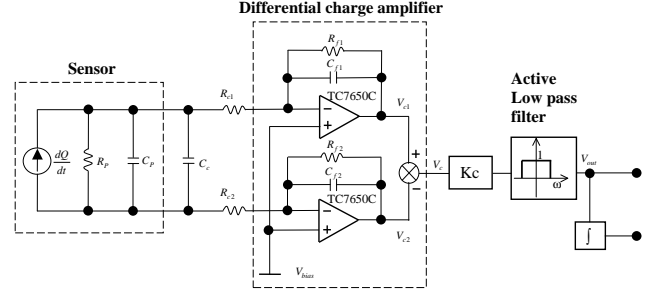


Fig. 2. Schematics of the developed electronic circuit for the PVDF sensor.

output voltage of the sensory system when the initial values  $f_c(t_0)$  and  $V_{out}(t_0)$  are known.

## 3. HYBRID CONTROL OF FLEXIBLE SENSOR STRUCTURE

### 3.1 Dynamics of Flexible Sensor Structure

The structure of the developed high sensitive micro-force sensor has large flexibility due to the use of the PVDF films. It makes the integrated sensor and micromanipulator system a flexible robot system. As shown in Fig. 3, during manipulation, we consider the sensor as a one-link flexible arm with a rigid tip. It is driven by a linear motor along the Z-axis and X-axis in the horizontal and vertical direction, respectively.  $EI$ , the uniform flexural rigidity of composite beam ( $EI = E_1 I_1 + E_2 I_2$ );  $\rho$ , the uniform mass per unit length of the composite beam;  $M_m$ , the mass of the translational motor base;  $\rho_t$ , the uniform mass per unit length of the tip;  $F_x(t), F_z(t)$ , the control input forces applied to the linear motor base along X and Z axes;  $x(t), z(t)$ , the positions of the motor base along X and Z axes;  $\omega(L, t)$ , the elastic deflection at the free end of the flexible link; and  $p(r, t) := z(t) - \omega(r, t)$ , the displacement variable along Z axis. Since we consider to regulate the micro-force along Z axis, an assumption of small deflection of the beam is made, then we can ignore the effects of centripetal acceleration and axial compression of the beam.

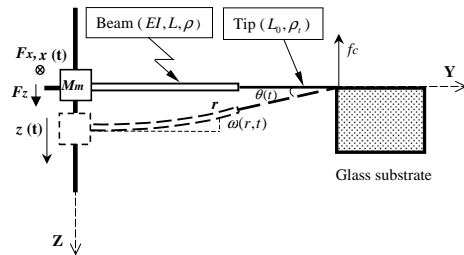


Fig. 3. A flexible sensor structure with a rigid tip driven by a linear motor (planar motion).

The position vector of contact point of the rigid tip is given

$$P_t = \begin{bmatrix} P_{ty} \\ P_{tz} \end{bmatrix} = \begin{bmatrix} (L + L_0)\cos\theta(t) \\ z(t) - \omega(L, t) - L_0\sin\theta(t) \end{bmatrix} \quad (3)$$

where  $\theta(t) = \omega'(L, t) \approx \sin\theta(t) \approx \frac{\omega(L, t)}{L}$ .

Then, a constraint surface  $\Phi(z(t), \omega(L, t), \omega'(L, t)) = 0$  for the system is found by

$$\begin{aligned} P_{tz} &= \Phi(z(t), \omega(L, t), \omega'(L, t)) \\ &\approx z(t) - \omega(L, t) - L_0\omega'(L, t) \\ &\approx z(t) - \omega(L, t)\left(\frac{L + L_0}{L}\right). \end{aligned} \quad (4)$$

Based on the system motion (i.e. planar motion), the total kinetic energy  $E_K$  is given by

$$\begin{aligned} E_K &= \frac{1}{2}M_m\dot{z}^2(t) + \frac{\rho}{2}\int_0^L \dot{p}^2(r, t)dr + \frac{1}{2}I_b\omega_b^2 \\ &\quad + \frac{1}{2}I_t\dot{\theta}^2(t) \end{aligned} \quad (5)$$

where  $I_b = \frac{\rho L^3}{12}$  is the moment of inertia of the flexible beam.  $\omega_b = \frac{\dot{\theta}(t)L_0}{L}$  is the angular velocity of the flexible beam.  $I_t = \frac{\rho_t L_0^3}{3}$  is the moment of inertia of the rigid tip. And the total potential energy  $E_P$  is

$$E_P = \frac{EI}{2}\int_0^L (\omega''(r, t))^2 dr. \quad (6)$$

In the system, the virtual work  $\delta W$  is given by

$$\delta W = \delta z(t)F_z(t). \quad (7)$$

Substituting the above equations into the Extended Hamilton's Principle (Meirovitch, 1975):

$$\int_{t_1}^{t_2} (\delta E_K - \delta E_P + \delta W + \xi \delta \Phi) dt = 0 \quad (8)$$

where let  $\xi$  be a Lagrange multiplier associated with the above constraint (4). The contact force between the constraint surface and the rigid tip can be represented in the term of the Lagrange multiplier  $\xi$ . Then we obtain the following dynamic equations of the constrained one-link flexible arm along Z axis.

$$M_m\ddot{z}(t) + \rho \int_0^L (\ddot{z}(t) - \ddot{\omega}(r, t)) dr = F_z(t) + \xi \frac{\partial \Phi}{\partial z(t)} \quad (9)$$

$$\ddot{\omega}(r, t) + \frac{EI}{\rho} \omega''''(r, t) = \ddot{z}(t) \quad (10)$$

and the corresponding boundary conditions as follows

$$\left(\frac{\rho L_0^2}{12L} + \frac{\rho_t L_0^3}{3L^2}\right)\ddot{\omega}(L, t) - EI\omega''''(L, t) = \xi \frac{\partial \Phi}{\partial \omega(L, t)} \quad (11)$$

$$EI\omega''(L, t) = \xi \frac{\partial \Phi}{\partial \omega'(L, t)} \quad (12)$$

$$\omega(0, t) = \omega'(0, t) = 0. \quad (13)$$

Moreover, the relationship between the contact force  $f_c$  and the Lagrange multiplier  $\xi$  can be given by

$$J_c^T f_c \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \xi \begin{bmatrix} \frac{\partial \Phi}{\partial z(t)} \\ \frac{\partial \Phi}{\partial \omega(L, t)} \\ \frac{\partial \Phi}{\partial \omega'(L, t)} \end{bmatrix} \quad (14)$$

where  $J_c$  is Jacobian matrix between the Cartesian coordinate and the generalized coordinate  $[z(t) \ \omega(L, t) \ \omega'(L, t)]^T$ . From equation (14), we obtain

$$f_c(t) = \xi(t). \quad (15)$$

To realize the micro contact force control, we consider the relation of  $f_c(t)$ ,  $\omega(r, t)$ , and  $z(t)$ . Let  $f_c^d$  be the desired contact force,  $\omega_d(r)$  the related static deformation of the flexible beam, and  $z_d$ , the related static position of the motor base at the equilibrium state. At the equilibrium state ( $f_c^d, \omega_d(r), z_d$ ), the relation is found as

$$\begin{aligned} \dot{z}(t) &= \ddot{z}(t) = 0, \\ \dot{\omega}(r, t) &= \ddot{\omega}(r, t) = 0 \end{aligned} \quad (16)$$

and based on the above equation and equation (10), we have

$$\omega_d''''(r, t) = 0. \quad (17)$$

By considering the kinematics of the sensor structure, the relation of the quasi-static deformation  $\omega_d(r)$  (assumed the bending of the beam mostly appears in the first shape mode in this microassembly.) and the desired contact force  $f_c^d$  is given as

$$\omega_d(r) = \frac{1}{6EI}(3Lr^2 - r^3 + 3L_0r^2)f_c^d. \quad (18)$$

From the constrained condition of system motion in eqn. (4), then the relationship between the  $z_d$  of the motor base and the desired contact force  $f_c^d$  is given

$$z_d = \omega_d(L) \frac{L + L_0}{L} = \frac{2L^3 + 5L^2L_0 + 3L_0^2L}{6EI} f_c^d. \quad (19)$$

This relationship can be used to set a  $z_d$  corresponding to a desired contact force  $f_c^d$ .

In addition, considering the linear motor moves along X axis, we assume that the motion along X axis (vertical motion) doesn't affect the bending of flexible link, so we have the dynamics of system along X axis as follows,

$$M_m \ddot{x}(t) + (m_b + m_t) \ddot{x}(t) = F_x(t) - G \quad (20)$$

where  $m_b$  is the mass of the beam,  $m_t$  the mass of the tip.  $G$  is gravity of the system.

### 3.2 Controller Design of Flexible Sensor Structure

Based on the dynamic equations (9), (10) and the boundary conditions (11)~(13), the force controller in the constrained direction Z has the following general form

$$F_z(t) = -k_p(z - z_d) - k_d \dot{z} - \xi \frac{\partial \Phi}{\partial z(t)} + (f_c - \text{sgn}(f_c^d \dot{z}) f_c^d) \quad (21)$$

where  $k_p$ , and  $k_d$  are positive gains.  $z_d$  can be set according to equation (19) if a desired contact force  $f_c^d$  is required.  $\xi = f_c(t)$  can be measured by using the PVDF sensor.  $z(t)$  and  $\dot{z}(t)$  can be obtained by the encoder.

From equation (20), the position controller in the unconstrained direction X has the following general form

$$F_x(t) = -k_{px}(x - x_d) - k_{dx}(\dot{x} - \dot{x}_d) + G \quad (22)$$

where  $k_{px}$ , and  $k_{dx}$  are positive gains. The controller in equation (22) is a traditional scheme for position control and trajectory tracking, we put less words to describe it. Here, we use the PVDF force sensor to detect the setting desired force, once the detected force approaches the desired value, the position control in the unconstrained direction X will be started.

*Theorem 1:* If the control force applied to the motor base is decided by the controller equation (21), then the distributed parameter, infinite dimensional system (9~13) is stable.

*Proof:* Consider the following Lyapunov function:

$$\mathbf{V}(t) = E_K + E_P + \frac{1}{2} k_p (z(t) - z_d)^2 + \|f_c^d(z(t) - z_d)\|. \quad (23)$$

In a differential form, we have

$$\dot{\mathbf{V}}(t) = \dot{E}_K + \dot{E}_P + k_p \dot{z}(t)(z(t) - z_d)$$

$$+ \text{sgn}(f_c^d \dot{z}) f_c^d \dot{z}. \quad (24)$$

Assume that the deflection  $\omega(r, t)$  satisfies

$$\frac{d}{dt} \int_0^L \dot{\omega}^2(r, t) dr = \int_0^L \left[ \frac{d}{dt} \dot{\omega}^2(r, t) \right] dr. \quad (25)$$

Then using eqn. (10), time derivative of  $E_K$  (5) is given by

$$\begin{aligned} \dot{E}_K &= M_m \dot{z} \ddot{z} + \rho \int_0^L \dot{p}(r, t) \ddot{p}(r, t) dr + I_b \omega_b \dot{\omega}_b + I_t \dot{\theta} \ddot{\theta} \\ &= M_m \dot{z} \ddot{z} + \rho \int_0^L (\dot{z} - \dot{\omega})(\ddot{z} - \ddot{\omega}) dr + \Delta \Delta \\ &= M_m \dot{z} \ddot{z} + \dot{z} \rho \int_0^L (\ddot{z} - \ddot{\omega}) dr - EI \int_0^L \dot{\omega} \omega'''' dr + \Delta \Delta \end{aligned} \quad (26)$$

where  $\Delta \Delta = I_b \omega_b \dot{\omega}_b + I_t \dot{\theta} \ddot{\theta}$ , from eqn. (11), it can be rewritten as

$$\Delta \Delta = \left( \frac{\rho L_0^2}{12L} + \frac{\rho_t L_0^3}{3L^2} \right) \ddot{\omega}_L \dot{\omega}_L = \xi \frac{\partial \Phi}{\partial \omega_L} \dot{\omega}_L + EI \dot{\omega}_L \omega_L''''$$

where  $\omega_L := \omega(L, t)$  for simplicity. Combine eqn. (26) and  $\Delta \Delta$ , notice that the terms  $\nabla = -EI \int_0^L \dot{\omega} \omega'''' dr + EI \dot{\omega}_L \omega_L''''$  can be further written as (Ge *et al.*, 1998):

$$\nabla = EI [\dot{\omega}'_L \omega''_L - \dot{\omega}'_0 \omega''_0 - \int_0^L \omega'' \dot{\omega}'' dr + \dot{\omega}_0 \omega_0''']$$

where  $\omega_0 := \omega(0, t)$  for simplicity. From the boundary conditions (12) and (13), consequently, we have

$$\nabla = EI \dot{\omega}'_L \omega''_L - EI \int_0^L \omega'' \dot{\omega}'' dr = \xi \frac{\partial \Phi}{\partial \omega_L} \dot{\omega}'_L - \dot{E}_P.$$

Re-substituting  $\dot{E}_K$  into eqn. (24), the rewritten  $\dot{\mathbf{V}}(t)$  is obtained. Continually substituting eqn. (9) into the rewritten  $\dot{\mathbf{V}}(t)$ , yields

$$\dot{\mathbf{V}}(t) = \dot{z}(t) F_z(t) + \xi \dot{\Phi} + k_p \dot{z}(t)(z(t) - z_d) + \text{sgn}(f_c^d \dot{z}) f_c^d \dot{z}. \quad (27)$$

Notice that  $\Phi = 0$ , then  $\dot{\Phi} = 0$ . Now substituting the controller (21) into the equation, leads to

$$\dot{\mathbf{V}}(t) = -k_d \dot{z}^2(t) \quad (28)$$

which means negative semi-definite. Thus the closed-loop system is stable.

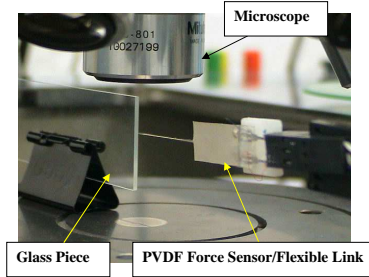


Fig. 4. Experimental set-up.

It is clear that the controller (21) is less dependent of system parameters, and thus possesses robust stability to the system parameter uncertainties. Based on the force feedback from the flexible PVDF beam itself, the controller is also easy to be implemented, no high-order signal measurements are needed. Due to the infinite dimensionality of the system, it is difficult to prove the asymptotic stability. However, due to the existence of internal structural damping in the flexible sensor in practice (not include in the proof of Theorem 1), and reconsider the controller (21), we find that the flexible sensor structure can only possibly stop at the final position  $z = z_d$  when  $\dot{z} = 0$ , which implies the micro-force regulation can be achieved corresponding to the constrained condition (19), then the practically asymptotic behavior of the flexible sensor structure can be shown.

## 4. EXPERIMENTS

### 4.1 Experimental Set-up

The experiments were conducted in the set-up shown in Figure 4. The experimental set-up mainly consists of a 3-D SIGNATONE Computer Aided Probe Station (linear motor controlled) and a Mitutoyo FS60 optical microscope system. The PVDF force sensor is installed at the free end of the micro probe (micromanipulator) as a flexible link for manipulation, illustrated in Figure 5. The sensor has the following dimensions and parameters:  $L_0 = 0.0225m$ ;  $L = 0.0192m$ ;  $w = 0.0102m$ ;  $H_1 = 28\mu m$  (PVDF film);  $R_p = 1.93 \times 10^{12}\Omega$ ;  $C_p = 0.90 \times 10^{-9}F$ ;  $C_f = 0.2 \times 10^{-10}F$ ;  $E_1 = 2 \times 10^9 N/m^2$ ;  $E_2 = 3.8 \times 10^9 N/m^2$ ;  $\rho = 1.911g/m$ .  $\rho_t = 0.089g/m$ .

The sampling rate is 1KHz in the experiments. The maximum encoder frequency of micromanipulator motor is  $8 \times 10^6$  counts/s with 14-bit DAC resolution. The loop time of the force sensing and control system is about 2ms. To reduce the vibrations from the environment, an active vibration isolated table was used during the experiments. All experiments were done at a stable room temperature.

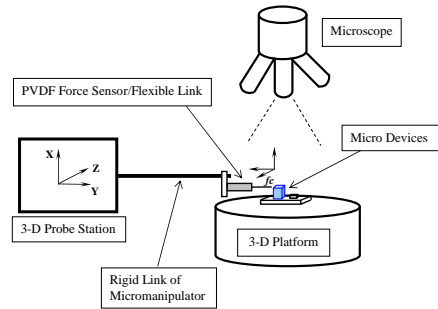


Fig. 5. Illustration of the flexible sensor/link work cell.

### 4.2 Force Sensing

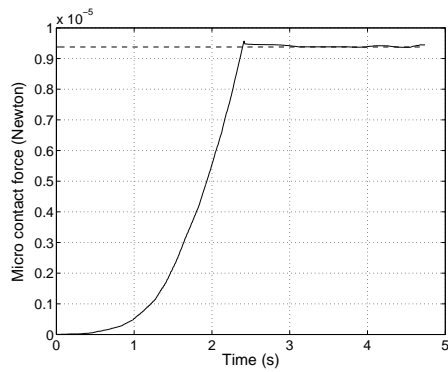
Calibration on the force sensing had been conducted based on the strain energy method of bending beam (Shen *et al.*, 2003). By preliminary calibration, the sensitivity of the PVDF sensor was estimated to be  $4.6602V/\mu N$ , the resolution of the sensor was in the range of  $\mu N$ . The output dynamic range of the sensor is 84.3 dB.

### 4.3 Hybrid Micro-Force/Position Control

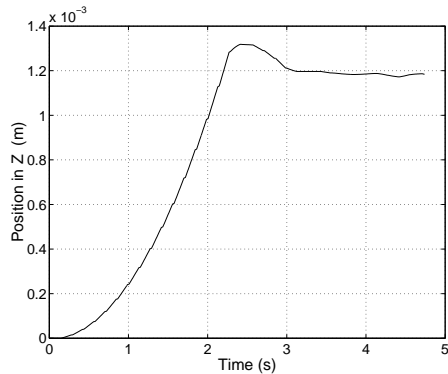
As shown in Fig. 3, we used a linear motor to move the 1-D PVDF sensor tip to contact a glass surface along Z axis horizontally, the micro contact force regulation on the basis of the infinite dimension model was then tested. In the first experiment, when the motor moves from an initial position, assuming the sensor tip continue to contact at a point on the glass surface, the PVDF beam starts to bend, like a flexible link of micromanipulator. By using the developed sensing and control method, the micro-force of tip is regulated to a desired value. Figure 6 shows the results of force control of flexible structure. Notice that in all force figures, the dash line represents the desired force  $f_c^d$ . Another experimental result shows after a period of tip force regulation along Z axis, starting from 2.8s, the sensor tip is controlled to move a straight line along X direction on the constrained glass surface from the contact point, at the same time, the contact force is still regulated in the contact direction (Z direction). Figure 7 shows the experimental result with the gains  $k_p = 20$ ,  $k_d = 15$ . The results verified the performance of the developed hybrid force/position control scheme, that is, with the position  $z(t)$  approaches to the desired value  $z_d$ , the force regulation law makes the force error go to zero asymptotically.

## 5. CONCLUSION

This paper presents the development of a micro force-guided microassembly technology with in-situ PVDF beam force sensing and hybrid

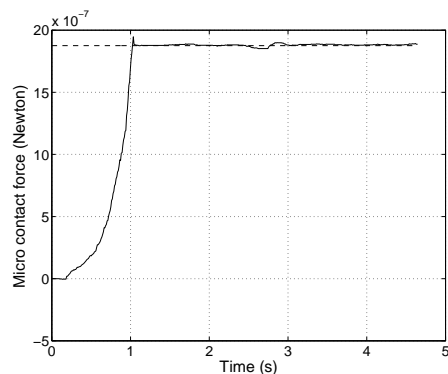


(a)

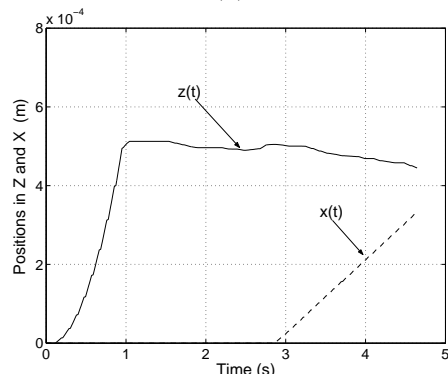


(b)

Fig. 6. Micro contact force control: (a) micro contact force along Z, (b) position  $z(t)$  in Z.



(a)



(b)

Fig. 7. Hybrid control: (a) micro contact force along Z, (b) positions  $z(t)$  and  $x(t)$  in Z and X.

force/position control based on an infinite dimensional model. The resolution of the developed sensor is in the range of  $\mu\text{N}$ . Considering this flexible sensor structure as a robot link, the force/position controller is developed. The stability of the closed-loop system can be proven based on the partial differential equations which govern the motion of the flexible sensor structure. Subsequently, the developed controller can avoid the low accuracy of microassembly performance associated with model truncation, and it is simple and easy to be implemented. Experimental results of the 1-D PVDF flexible sensor structure verify the performance of the developed control scheme. Ultimately the technology will provide a major step towards the development of automated assembly of micro devices.

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