PRICE-BASED RESOURCE ALLOCATION IN CDMA NETWORKS AND STABILITY ANALYSIS

Bo Yang*, Xinping Guan*,1, Chengnian Long*

*Institute of Electrical Engineering, Yanshan University,
Qinhuangdao, 066004, P.R. China
Email: {yangbo, xpguan, dylcn}@ysu.edu.cn

Abstract: We present an optimal model based on utility functions and pricing for downlink radio resource allocation in code division multiple access (CDMA) systems. The aim of this model is to achieve the throughput maximization via dynamic price adjustment. Due to the existence of action delays, we give globally asymptotical stability results using Lyapunov-Razumikhin arguments for rate and price evolvement closed-loop systems. Considering total power constraint for base station, we introduce a power allocation policy, which need only users' local information. Simulation results validate the effectiveness of the joint rate and power control scheme.

Keywords: rate control, power control, wireless networks, price, stability, time delays

1. INTRODUCTION

Emerging multimedia services and applications will increase the demand for bandwidth in wireless networks. Efficient radio resource allocation is one of key technologies for providing quality of service (QoS) to the bandwidth limited wireless networks. In wireless networks, QoS represents the satisfactory level that a user can obtain when using the resource provided by networks. Within the framework of microeconomics, satisfactory level can be represented by the concept of utility function. The basic idea of utility-based resource allocation is to adjust the resource price according to system load and channel conditions to control users' behaviour.

In code division multiple access (CDMA) systems, resources are referred to transmission power and data rate. In a cell the resource allocation between the base station (BS) and users can be divided into two categories: uplink and downlink. In downlink case we consider the transmission from base stations (BS) to mobiles and vice versa for uplink.

[1]Supported in part by NSFC (60174010, 60404022), Key Research Project of Ministry of Instruction (204014) and Yanshan University (YDJJ2003010).

Utility-based or price-based resource allocation has received significant development in recent years. In particular, the downlink power control without power constraint is formulated in (Xiao *et al.,* 2003) as a non-cooperative *N*-person game. In (Alpcan and Başar, 2004) the authors present uplink power controls based on non-cooperative game theory and prove the existence of a unique Nash equilibrium for the system without considering power constraint. They also establish the global convergence of the dynamics with delay and switch. In (Lee, *et al.,* 2002) downlink power control with constraint is treated as a partially cooperative *N* person game with dynamic price but without showing how to allocate data rate. In (Siris, 2002) the author proposed resource allocation frameworks for downlink and uplink, respectively. The difference for downlink resource allocation algorithms from uplink is that the downlink is total power constrained at BS instead of individual power limits at mobiles.

In this paper we focus on the downlink throughput optimization problem in CDMA systems. We propose an economic model for joint downlink rate and power control. We notice that the power control in physical layer is faster than rate control. After some manipulations there are only rate variables and bit-energy-to-noise-density ratio (E_b/N_o) in the optimal model. Based on this model a dynamic price charged by BS is adjusted adaptively according to load and environment to balance the difference of user demand and BS supply. While the joint power control operates at a fast time scale with the change of rate control at upper layer to maintain target E_b/N_o. In cellular systems, the target E_b/N_o is determined by speech quality considerations. In this paper, we seek a target E_b/N_o with respect to maximizing the utility function of throughput. Moreover we consider the action delays existing in measurement or signalling information between BS and mobiles. The authors in the aforementioned literatures omitted this lag except for Alpcan and Başar (Alpcan and Başar, 2004). However they considered only pure uplink power control. In this paper, the proposed downlink rate control policy can be regarded as a closed-loop nonlinear differential equation with heterogeneous action delays. With Lyapunov-Razumikhin theory we prove that the stability can be guaranteed by scaling down step size to overcome large delay. In addition, we give an upper bound of the price dependent on delay and channel information, which is useful in estimating the minimum transmitting rates. From simulations we see that the joint power control within the limit can maintain QoS. In addition, simulations also backup that the dynamic scheme can improve the throughput compared with a static price scheme.

The remainder of this paper is organized as follows. In section 2, we describe the optimal model of downlink resource allocation. We formulate the dynamic price-based rate adjustment in section 3, where we also present the stability results with delay. Then in section 4 the joint power control and description of the algorithm are given. Section 5 presents the simulation results. Finally we conclude in section 6.

2. SYSTEM MODEL

In this section, we consider power and data rate allocation for the downlink of a CDMA system with *N* users. Denoted by r_i the transmission rate, by p_i the transmission power, and by h_i the average channel gain from the BS in the target cell to mobile *i*. Denote W/r_i and I_i as the spreading gain and the other cell average interference plus thermal noise variance seen by user *i*. Let *W* be the system chip rate. We assume that the target cell has a maximum transmission power of p_T. For a given user $f_i(\gamma_i)$ is the efficiency function, which describes the percentage of successfully transmitted information bits to overall bits transmitted. Thus, user *i's* throughput is defined as $x_i = r_i f_i(\gamma_i)$. We can describe the received E_b/N_o as

$$\gamma_i = \frac{W}{r_i} \frac{h_i p_i}{h_i \sum_{j=1, j\neq i}^{N} p_i + I_i} \tag{1}$$

where $h_i p_i$ denotes the received signal power at mobile *i*. $h_i \sum_{j=1,j\neq i}^{N} p_j + I_i$ denotes the interference power from other connections and intracell noise.

Since the aim of this paper is to obtain power and data rate allocation to maximize the total expected throughput of the system, individual user's function describing the perceived QoS can be described as $U_i(x_i)$. In (Siris, 2002), Siris pointed out that the utility function U_i for elastic traffic is typically increasing concave. Mathematically, our resource allocation problem is formulated as maximization of the system overall utilities:

$$\max_{P,R} \sum_{i=1}^{N} U_i(x_i)$$
$$s.t. \begin{cases} \gamma_i = \gamma_i^* \\ \sum_{i=1}^{N} p_i \leq p_T \end{cases} \tag{2}$$

where $P = \{p_1, p_2, \cdots, p_N\}$, $R = \{r_1, r_2, \cdots, r_N\}$. γ_i^* is the target E_b/N_o. Eq. (2) is a nonlinear optimal problem with 2*N* decision variables, *P* and *R*. Since $U_i(x_i)$ is increasing function of x_i, maximizing utility of user *i* is equivalent to maximizing x_i. First we rewrite x_i as a function of γ_i invoking Eq. (1).

$$\max_{\gamma_i} x_i = \max_{\gamma_i} \frac{W}{\gamma_i} \frac{h_i p_i}{h_i \sum_{j=1,j\neq i}^{N} p_j + I_i} f_i(\gamma_i) \tag{3}$$

γ_i^* is solved from $f_i(\gamma_i^*) = \gamma_i f_i^{'}(\gamma_i^*)$ (Goodman and Mandayam, 2001). We now make the following assumptions.

A1. There are a large number of users in the system without a single dominant one. In other words each mobile uses a small portion of the wireless resource. Therefore, we have $W/r_i\gamma_i >> 1$. This assumption is consistent with those in (Siris, 2002).

A2. The underlying power control mechanism can achieve the target E_b/N_o, i.e. $\gamma_i = \gamma_i^*$.

As to A2, we will give the power control at last. In wide-band CDMA, the power control operates at a frequency of 1500Hz. On the other hand, the rate remains constant within a single frame, whose minimum duration is 10 milliseconds. Thus in order to maintain $\gamma_i = \gamma_i^*$ a change in the transmission rate would require adjusting the transmission power. Based on Eq. (1), the total power constraint can be rewritten as a constraint on r_i and γ_i, for $i = 1, \cdots, N$. The mathematical manipulation is as follows.

$$\frac{Wp_i}{r_i \gamma_i} = \left(\sum_{j=1,j\neq i}^{N} p_j\right) + \frac{I_i}{h_i} \tag{4}$$

Adding p_i to both sides of Eq. (4) and sum up Eq. (4), for all users and applying assumption *A1* we get

$$p_i \approx \frac{r_i \gamma_i^*}{W}\left(\sum_{j=1}^{N} p_j + \frac{I_i}{h_i}\right) \tag{5}$$

Invoking the total power constraint $\sum_{i=1}^{N} p_i \le p_T$ after some simple manipulations we obtain

$$\sum_{i=1}^{N} \alpha_i r_i \gamma_i^* \le p_T \tag{6}$$

where $\alpha_i = (I_i + p_T h_i)/(W h_i)$. Thus the power constraint can be rewritten as the constraint on the target E_b/N_o and data rate r_i. With the constraint (6) we get the following equivalent optimization problem.

$$\max_R \sum_{i=1}^{N} U_i\left(r_i f_i\left(\gamma_i^*\right)\right) \tag{7}$$

subject to condition (6). In the following sections, we will derive a dynamic pricing scheme to solve this optimization problem.

3. DOWNLINK RATE CONTROL AND STABILITY ANALYSIS WITH DELAY

3.1 Downlink rate control

From the microeconomics point of view, the network can be treated as the provider who provides the network resources, such as data rates. We regard the mobile user as demanders who consume resources. In order to balance the difference between suppliers and consumers, networks should dynamically adjust the price to achieve the maximization problem (6) (7). Using this interpretation, we define the Lagrangian function,

$$L(R,\lambda) = \sum_{i=1}^{N} U_i\left(r_i f_i\left(\gamma_i^*\right)\right) - \lambda\left(\sum_{i=1}^{N} \alpha_i \gamma_i^* r_i - p_T\right) \tag{8}$$

We convert (8) into its dual model

$$\min_{\lambda}\left|p_T - \sum_{i=1}^{N} \alpha_i \gamma_i^* r_i\right| \tag{9}$$

$$r_i^* = \arg\max_{r_i}\left\{U_i\left(r_i f_i\left(\gamma_i^*\right)\right) - \lambda \alpha_i \gamma_i^* r_i\right\} (i = 1,2,\cdots,N) \tag{10}$$

We can interpret λ as the price per power charged by BS, $\alpha_i \gamma_i^* r_i$ as the power needed to transmit data at rate r_i while maintaining target E_b/N_o . Therefore, $\lambda \alpha_i \gamma_i^* r_i$ is the power cost or user demand to maximize user *i's* throughput. p_T is the resource provided by BS.

Since $U_i(\cdot)$ are concave and the constraints are linear there is no duality gap and dual optimal price, which is Lagrange multiplier exists (Bertsekas, 1995). Since $U_i\left(r_i f_i\left(\gamma_i^*\right)\right) - \lambda \alpha_i \gamma_i^* r_i$ is a concave function of r_i. The solution of the sub problem (10) can be written as

$$r_i = U_i'^{-1}\left(\lambda \alpha_i \gamma_i^*\right) \tag{11}$$

According to (lemma1, Lee, 2002), if we find a λ^* such that $\sum_{i=1}^{N} \alpha_i \gamma_i^* r_i^* = p_T$ the social optimal solution of (6) (7) or an approximate solution of (2) can be obtained.

In the following, we present a gradient pricing algorithm to find λ^*.

$$\dot{\lambda}(t) = \delta\left(\sum_{i=1}^{N} \alpha_i \gamma_i^* r_i - p_T\right)_{\lambda}^{+} \tag{12}$$

$$\left(g(z)\right)_z^+ \hat{=} \begin{cases} g(z)\, if\ z > 0\ or\ z = 0\ and\ g(z) \ge 0 \\ 0\ if\ z = 0\ and\ g(z) < 0 \end{cases} \tag{13}$$

where $\delta > 0$ is the step size. Noticing Eq. (11) and Eq. (12), the closed loop dynamics are the following:

$$\dot{\lambda}(t) = \delta\left(\sum_{i=1}^{N} \alpha_i \gamma_i^* U_i'^{-1}\left(\lambda \alpha_i \gamma_i^*\right) - p_T\right)_{\lambda}^{+} \tag{14}$$

Remark 1. A mobile *i* far from BS usually has a small path gain, h_i. Thus, both far user and large intracell noise contribute large α_i . Large α means hostile transmission environment. Large α results in large price for (12) and small transmission rate from (11), since $U'^{-1}(\cdot)$ is a decreasing function. Therefore, this dynamic pricing scheme can add intelligence to BS.

Remark 2. Hostile environment would result in low transmission rate to save power. On the other hand, this scheme shows a kind of unfairness to far users. But this is beneficial to improve the downlink throughput, see (Xiao *et al.* 2003).

3.2 Stability analysis under heterogeneous action delays

Note that in this scheme BS requires mobiles to provide target E_b/N_o and received path gain as well as noise. Measurement, processing and signalling of these information result in action delays. Since propagation delays are negligible for cellular wireless networks (Alpcan and Başar, 2004), there are fixed action delays. Considering the action delays τ_i $i = 1,2,\cdots,N$ between BS and mobiles, the closed loop system will be converted into

$$\dot{\lambda}(t) = \delta\left(\sum_{i=1}^{N} \alpha_i \gamma_i^* U_i'^{-1}\left(\alpha_i \gamma_i^* \lambda(t-\tau_i)\right) - p_T\right)_{\lambda}^{+} \tag{15}$$

To investigate the stability of a single cell rate control, we state the following lemma first.

Lemma 1. There exists the unique non-negative solution λ^* to the closed-loop system (14).

PROOF. To proof one can use the similar technique in (Paganini, 2002). The proof first argues that $(\cdot)^+$ is redundant and then mimics the Picard theorem (Khalil, 1996) to show the existence of unique non-negative equilibrium. We skip details for space. □

The following theorem will be useful in estimating the minimum data rate.

Theorem 1. Consider the closed-loop system (15), where $U_i(\cdot)$ satisfies for $i = 1,2,\cdots,N$, $U_i''(\cdot) \geq -\eta_1$, where $\eta_1 > 0$ is a constant. For large t and $\tau_m = \max_i \tau_i$, the price is upper bounded by

$$\lambda(t) \leq \lambda^*\left(1 + \tau_m \delta \eta_1^{-1} \sum_{i=1}^N \left(\alpha_i \gamma_i^*\right)^2\right) \tag{16}$$

PROOF. From lemma 1, Eq. (15) and applying the Mean Value Theorem, we get

$$\begin{aligned}
\dot{\lambda} &= \delta\left(\sum_{i=1}^N \alpha_i \gamma_i^* \left(U'^{-1}_i\left(\alpha_i \gamma_i^* \lambda(t-\tau_i)\right) - U'^{-1}_i\left(\alpha_i \gamma_i^* \lambda^*\right)\right)\right)_\lambda^+ \\
&\leq \delta\left(-\sum_{i=1}^N \left(\alpha_i \gamma_i^*\right)^2 \eta_1^{-1}\left(\lambda(t-\tau_i) - \lambda^*\right)\right) \\
&\leq \delta \sum_{i=1}^N \left(\alpha_i \gamma_i^*\right)^2 \eta_1^{-1} \lambda^*
\end{aligned} \tag{17}$$

For $U_i''(\cdot) \geq -\eta_1$ we get $\left(U'^{-1}_i(\cdot)\right)' < -\eta_1^{-1}$, then apply Mean Value Theorem and the definition of (13) to obtain (17). Consider U'^{-1} is a decreasing function, if $\lambda(\xi) > \lambda^*$ for $\xi \in [t-\tau_m, t]$, $\dot{\lambda}(t) < 0$. We will use the similar time scale analysis as in (Wang and Paganini, 2002) for the following arguments. For a given instant $t_1 > \tau_m$ suppose (16) is invalid. Thus, following inequality (17), we get $\lambda(t_1 - \tau) > \lambda^*$ for $\tau \in [0, \tau_m]$. However $\dot{\lambda}(t) < 0$ when $t \geq t_1$, and $\lambda(t)$ will decrease until $t = t_2$ when $\lambda(t_2) = \lambda^*$. Moreover we will argue that $\lambda(t)$ will not cross the upper bound of (16) any more. On the contrary, assume that it reached this upper boundary at some instant t_3 and $\dot{\lambda}(t_3 + \varepsilon) > 0$ for some $\varepsilon > 0$. However, similar to the above we have $\lambda(t_3 + \varepsilon - \tau) > \lambda^*$, for $\tau \in [0, \tau_m]$. So we conclude that $\dot{\lambda}(t_3 + \varepsilon) \leq 0$, a contradiction. Therefore, we get the theorem 1. □

Considering that some traffic services require the minimum rate, we can apply the above results to obtain the maximum price then minimum data rate. In the following, we will present a global stability result with time delay for the closed-loop system (15). In order to make problem tractable, we omit the non-negative saturation of (13).

Theorem 2. Consider the delayed feedback nonlinear system (15) without saturation and suppose that $U_i(\cdot)$, $i = 1,\cdots,N$ are such that $-\eta_1 \leq U_i'' \leq -\eta_2$ with $\eta_1 > \eta_2 > 0$. If δ is small enough, the pricing based rate allocation can guarantee globally asymptotic stability with diverse action delays.

PROOF. We now define the Lyapunov candidate as:

$$V\left(\tilde{\lambda}(t)\right) = \frac{1}{2}\tilde{\lambda}^2(t)$$

where $\tilde{\lambda}(t) = \lambda(t) - \lambda^*$. Consider the set of $\tilde{\lambda}(t)$ such that, for some $\theta > 1$ and $\tau_m = \max_i(\tau_i)$, $\tilde{\lambda}^2(t-\varsigma) \leq \theta^2 \tilde{\lambda}^2(t)$, for $\varsigma \in [0, 2\tau_m]$. Then we will show the derivative of V under the solution of (15) without saturation is decreasing.

$$\begin{aligned}
\dot{V}\left(\tilde{\lambda}(t)\right) &= \tilde{\lambda}(t)\sum_{i=1}^N \delta\left[\alpha_i \gamma_i^* U'^{-1}_i\left(\alpha_i \gamma_i^* \lambda(t-\tau_i)\right) - p_T\right] \\
&= \tilde{\lambda}(t)\sum_{i=1}^N \delta \alpha_i \gamma_i^* \begin{bmatrix} \left(U'^{-1}_i\left(\alpha_i \gamma_i^* \lambda(t)\right) - U'^{-1}_i\left(\alpha_i \gamma_i^* \lambda^*\right)\right) \\ -\left(U'^{-1}_i\left(\alpha_i \gamma_i^* \lambda(t)\right) - U'^{-1'}_i\left(\alpha_i \gamma_i^* \lambda(t-\tau_i)\right)\right) \end{bmatrix} \\
&\leq \tilde{\lambda}(t)\sum_{i=1}^N \delta \alpha_i \gamma_i^* \left[-\alpha_i \gamma_i^* \eta_1^{-1} \tilde{\lambda}(t) - \int_{t-\tau_i}^t \frac{\partial U'^{-1}_i}{\partial \lambda}(\lambda(\rho))\dot{\lambda}(\rho)d\rho\right] \\
&\leq \tilde{\lambda}(t)\sum_{i=1}^N \delta \alpha_i \gamma_i^* \left[-\alpha_i \gamma_i^* \eta_1^{-1} \tilde{\lambda}(t) + \eta_2^{-2}\int_{t-\tau_i}^t \alpha_i \gamma_i^* \delta \sum_{i=1}^N \alpha_i \gamma_i^* \left|\tilde{\lambda}(\rho - \tau_i)\right| d\rho\right]
\end{aligned}$$

Considering the assumption of $\tilde{\lambda}^2(t-\varsigma) \leq \theta^2 \tilde{\lambda}^2(t)$ for $\varsigma \in [0, 2\tau_m]$, we proceed with the above inequality

$$\dot{V}\left(\tilde{\lambda}(t)\right) \leq \tilde{\lambda}^2(t)\sum_{i=1}^N \delta\left(\alpha_i \gamma_i^*\right)^2\left(-\eta_1^{-1} + \eta_2^{-2}\theta \tau_m \delta \sum_{i=1}^N \alpha_i \gamma_i^*\right)$$

According to Lyapunov-Razumikhin arguments, the global asymptotic stability follows from

$$\delta < \frac{\eta_1^{-1}}{\eta_2^{-2}\theta \tau_m \sum_{i=1}^N \alpha_i \gamma_i^*} \tag{18}$$

This completes the proof. □

Remark 3. In theorem 1 and theorem 2 we assume $-\eta_1 \leq U_i'' \leq -\eta_2$. In practice there are the minimum transmission rate requirement and maximum rate limitation for mobile hosts and BS, respectively. So this assumption is reasonable.

4. DOWNLINK POWER ALLOCATIONS AND ALGORITHM DESCRIPTION

4.1 Downlink power allocations

Complementary slackness condition states that at optimality, the product of the dual variable and the associated primal constraint must be zero. So, if $R^* = [r_1^* \cdots r_N^*]$ is the optimal solution of (6) (7) we must have $\sum_{i=1}^N \alpha_i r_i^* \gamma_i^* = p_T$, which is the approximation of $\sum_{i=1}^N p_i = p_T$. Thus the first principle of optimal power allocation to maximize (2) should satisfy $\sum_{i=1}^N p_i^* = p_T$ or at least $\sum_{i=1}^N p_i \leq p_T$. The other aim of power control is to guarantee $\gamma_i = \gamma_i^*$ for all users. Based on this aim we present the following result.

Proposition 1. If the transmission power assigned to user *i* is selected by:

$$p_i \frac{\frac{1}{W/r_i\gamma_i^* + 1} \cdot \sum_{j=1}^N \frac{I_j/h_j}{W/r_j\gamma_j^* + 1}}{1 - \sum_{j=1}^N \frac{1}{W/r_j\gamma_j^* + 1}} + \frac{I_i/h_i}{W/r_i\gamma_i^* + 1} \tag{19}$$

and

$$\sum_{i=1}^N \frac{p_T + I_i/h_i}{1 + W/r_i\gamma_i^*} = p_T \tag{20}$$

holds, then the power allocation scheme can guarantee the target E_b/N_o for all users as well as that BS transmits at its maximum power limit p_T.

PROOF. Summing up Eq. (19) for *N* users, we have

$$\sum_{i=1}^{N} p_i = \frac{\sum_{i=1}^{N} \frac{I_i/h_i}{1+W/r_i\gamma_i^*}}{1-\sum_{i=1}^{N}\frac{1}{1+W/r_i\gamma_i^*}} \quad (21)$$

Thus if Eq. (20) holds, following Eq. (21), we obtain $\sum_{i=1}^{N} p_i^* = p_T$. Substituting Eq. (21) into the following, and invoking (19) we get

$$\left(h_i \sum_{j=1}^{N} p_j + I_i\right) = p_i h_i \left(W/\gamma_i^* r_i + 1\right) \text{ for } i=1,\ldots,N \quad (22)$$

Subtract $h_i p_i$ from both sides of Eq. (22). Then it is straightforward that

$$\frac{W h_i p_i}{r_i\left(h_i \sum_{j=1, j\neq i}^{N} p_j + I_i\right)} = \gamma_i^*$$

This completes the proof. □

4.2 Algorithm descriptions

Suppose the following system parameters are given or calculated in advance: the BS total power constraint p_T, the utility functions $U_i(\cdot)$ and inverse characteristic $U_i'^{-1}(\cdot)$ and target E_b/N_o. At each resource allocation point, the power and rate joint allocation algorithm performs the following steps:

1) Users measure received inference $I_i^{(k)}$. Update path gain $h_i(k)$ and calculate $\alpha_i^{(k)}$ for *i*=1,2,... Users provide their channel gain and interference information to BS. Set $r_i^{(1)} = r_{i,\min}$, where $r_{i,\min}$ is user's minimum rate requirement, for $i = 1,2,\cdots,N$
2) $r_i^{(1)} = r_{i,\min}$, BS selects mobiles. Mobiles are ordered by $r_1^{(k)} < r_2^{(k)} < r_3^{(k)} < \ldots r_N^{(k)}$

   For $$\sum_{i=1}^{N} \frac{p_T + I_i/h_i}{1+W/r_i\gamma_i^*} \le p_T \quad (23)$$

   $N{+}{+}$

   End

   $\lambda^{(1)} = \max_i \left(U'\left(r_i^{(1)}\right)/\alpha_i^{(1)}\gamma_i^*\right)$
3) At BS $\lambda^{(k+1)} = \left[\lambda^{(k)} + \delta\left(\sum_{i=1}^{N} \alpha_i^{(k)}\gamma_i^* r_i^{(k)} - p_T\right)\right]_{\lambda}^{+}$;
4) $r_i^{(k+1)} = U_i'^{-1}\left(\lambda^{(k)}\alpha_i^{(k)}\gamma_i^*\right)$ for $i = 1,2,\cdots,N$
5) $$p_i^{(k)} = \frac{\frac{1}{W/r_i^{(k)}\gamma_i^*+1}\cdot\sum_{j=1}^{N}\frac{I_j^{(k)}/h_j^{(k)}}{W/r_j^{(k)}\gamma_j^*+1}}{1-\sum_{j=1}^{N}\frac{1}{W/r_j^{(k)}\gamma_j^*+1}} + \frac{I_i^{(k)}/h_i^{(k)}}{W/r_i^{(k)}\gamma_i^*+1}$$
6) $k \leftarrow k+1$, go to step 1).

Remark 4. In practice, to enhance throughput BS usually turns down mobile hosts located at the boundary of a cell. In our rate adjustment scheme, far users are allocated lower rates. The aim to arrange mobiles in that order at step 2) is to overcome the near-far unfairness. In step 2) we make an implicit assumption that $r_{i,\min}$ is enough to satisfy (23).

5. SIMULATION RESULTS

In this section, we present simulations that exhibit the stability of our scheme with delay and how data rate, transmission power and signal quality depend on network environments. We also compare our dynamic price-based resource allocation with congestion price for downlink scheme in (Siris, 2002). Suppose there are four mobile hosts in a cell. The parameters for these users are shown in Table 1.

Table 1 Parameters of environments for four users

Parameter	Path gain	Noise (W)
USER 1	2.5e-6	(5e-2, 1e-8)
USER 2	2.5e-6	(2e-2, 1e-6)
USER 3	2e-9	(5e-5, 1e-10)
USER 4	1e-9	(5e-5, 2e-10)

In Table 1, we assume the background noise $I_i \sim \left(\mu, \sigma^2\right)$ W. We consider four users have the same type of utility function and efficiency function, such as, $U(x) = 1 - e^{-0.4x}$, $f(\gamma) = \left(1 - 0.5e^{-\gamma}\right)^{60}$. $\gamma^* = 5$ is determined from $f(\gamma^*) = \gamma^* f'(\gamma^*)$. The total power limit of BS is 10W, 10% load. Chip rate $W = 3.84$ MHz. In our scheme, we let step size $\delta = 1e-4$. The action delays between four users and BS are 0.1sec. Simulation time is 50 sec.

Fig. 1 shows that the downlink transmission rate controlled by the dynamic pricing scheme. The rate is dependent on the mobile's distance from the BS. Smaller distance means larger channel gain and large transmission rate. This fact also can be determined by Eq. (11). Note that BS transmits to near user at large rate can enhance the throughput, i.e. utility maximization, see (Xiao *et al.,* 2003). Fig. 1 also shows that as to the users with same distances from BS, BS will allocate larger rate to one with small background interference. This policy will save energy for BS, since users with larger rates and larger background noise will require BS to allocate more power to maintain target E_b/N_o. Fig. 2 depicts the rate evolvement controlled by congestion price for downlink in (Siris, 2002). We can find the same distance and interference dependence as our algorithm. However BS with our dynamic pricing algorithm exhibits larger throughput. Since our dynamic price can be adjusted with environments and load.

From Fig. 3 we find the total power assigned to four users is approximatively equal to 10 percentage of total power constraint, which is consistent with the simulation setup. Since user 1 receives more background noise than user 2, the former must need more power to maintain its E_b/N_o. Though the farthest user 4 is allocated the smallest transmission rate, it needs the most power to achieve γ^*.

Fig. 4 validates that the power control (19) is effective in achieving target γ^*. From the simulations we make sure that the selected parameters can guarantee the globally stability with 0.1 sec. action delays.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we present an optimal model for joint rate and power allocation in CDMA systems to maximize the throughput. Particularly, we focus on downlink direction. After reformulating the original nonlinear optimal problem into a two-time scale or two layer control problem, we propose a dynamic pricing algorithm to adjust the transmission rate for mobiles. Due to the existence of non-negligible action delay we turn to Lyapunov-Razumikhin arguments and give the global stability results with delay. Considering total power constraint and achieving target signal noise ratio, we propose a power adjustment policy, which need individual users to provide their local information. Future work will focus on the trade off between near far fairness and throughput maximization as well as combination with TCP.

REFERENCES

Alpcan, T. and Başar, T. (2004). Hybrid systems model for power control in multicell wireless data networks. *Performance Evaluation*, **57(4):** 477-495.

Bertsekas, D. (1995). *Nonlinear programming,* Athena Scientific.

Goodman, D., and Mandayam, N. (2001). Network assisted power control for wireless data. *Mobile Networks and applications,* **6,** 409-415.

Khalil, H.K. (1996). *Nonlinear systems,* Prentice-Hall, Englewood Cliffs, NJ.

Lee, J.W., Mazumdar, R.R., Shroff N.B. (2002). Downlink power allocation for multi-class CDMA wireless networks. In: *Proc. IEEE INFOCOM'02*, pp. 1480-1489. New York City, USA.

Paganini, F. (2002). A global stability result in network flow control. *System & control letters,* **46,** pp. 165-176.

Siris, V.A. (2002). Resource allocation for elastic traffic in CDMA networks. In: *Proc. ACM Mobicom'02*, pp. 193-204, Atlanta, USA.

Wang, Z.K., Paganini, F. (2002). Global stability with time delay in network congestion control. In: *Proc. IEEE CDC'02*, Las Vegas, Nevada, USA

Xiao, M., Shroff, N.B., Chong, E. (2003). A utility-based power-control scheme in wireless cellular systems. *IEEE/ACM Trans. Networking,* **11,** pp. 210-211.

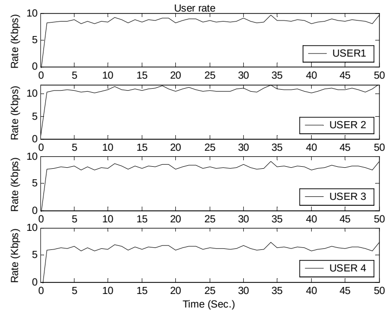

Fig. 1. User rate evolvement controlled by dynamic pricing.

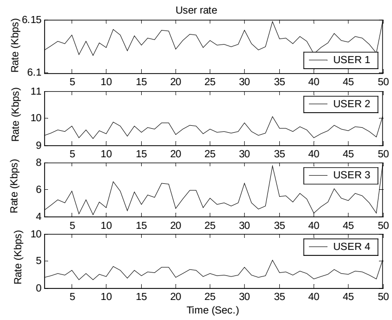

Fig. 2. User rate evolvement controlled by congestion price.

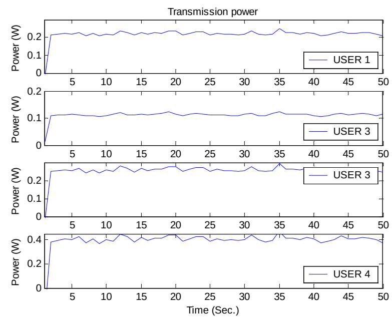

Fig. 3. Joint downlink power allocation.

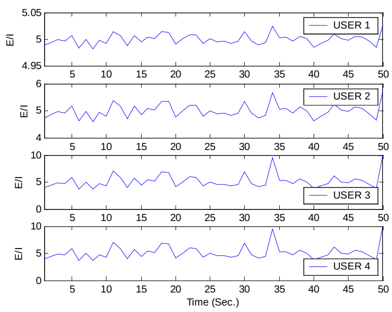

Fig. 4. Received E_b/N_o at mobiles.