ANALYSIS OF DYNAMIC SENSOR COVERAGE PROBLEM USING KALMAN FILTERS FOR ESTIMATION

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Abstract: We introduce a theoretical framework for the dynamic sensor coverage problem for the case with multiple discrete time linear stochastic systems placed at spacially separate locations. The objective is to keep an appreciable estimate of the states of the systems at all times by deploying a few limited range mobile sensors. The sensors implement a Kalman filter to estimate the states of all the systems. In this paper we present results for a single sensor executing two different random motion strategies. Under the first strategy the sensor motion is an independent and identically distributed random process and a discrete time discrete state ergodic Markov chain under the second strategy. For both these strategies we give conditions under which a single sensor fails or succeeds to solve the dynamic coverage problem. We also demonstrate that the conditions for the first strategy are a special case of the main result for the second strategy.

Keywords: Dynamic Sensor Coverage, Markov Chain, Kalman Filter

1. INTRODUCTION

Sensor coverage is the problem of deploying multiple sensors in an unknown environment for the purpose of automatic surveillance, cooperative exploration or target detection. Recent years have witnessed increased interest among the communication, control and robotics researchers in the area of mobile sensor networks. Each individual node in such a network has sensing, computation, communication and locomotion capabilities. When the environment is rapidly changing finding an efficient deployment strategy becomes a key issue for any application.

Coverage can be static (fixed sensors) or dynamic (mobile sensors). Static sensor coverage is desirable if the area to be covered is less than the union of the ranges of the sensor nodes. Static sensor coverage problem has been considered in (Cortes *et al.*, 2004) and in the references there in. The dynamic sensor coverage becomes necessary when a limited number of sensors is available and the area of interest can not be covered by a static configuration of sensors. There have been attempts to empirically solve the dynamic coverage problem using simulations and actual robots (Batalin and

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Sukhatme, 2002) but a sound theoretical base is still missing in the literature.

In this paper we consider N discrete time linear systems located at different points in space. One may think of dividing the uncertain area under consideration using a grid and then these N systems can be thought to represent the dynamics of local environment change at the grid points. We analyze the case when a single sensor is deployed. The sensor maintains discrete time Kalman filter estimates of the states of all the N systems. In order to model the limited range of the sensor, we constrain the sensor to receive measurements only for the system where it is physically located at that time instant. All the tools developed in this paper can be applied to the case where multiple grid points fall in the sensory range and hence the sensor receives measurements from more than one system. This extension requires only minor modifications, and is left as a future research direction.

For a system where the sensor is located, the sensor implements both the time update and measurement update laws of the Kalman filter. For all the other systems for which the sensor did not receive any measurements, only the time update law is implemented. The motion of the sensor is an i.i.d. random process under the first strategy and a discrete time discrete state (DTDS) Markov chain in the later strategy. For successful coverage the sensor needs to hop from one system to another such that the error covariance matrices of the estimates of states of all the N systems are bounded at all times. Intuition tells us that the sensor should spend more time at a location where the environment is changing rapidly than at one where the dynamics are relatively slow. The results we present in this paper satisfy this intuition. A similar set of results, developed independently, have been presented in (Gupta *et al.*, 2004)

In Section 2. we describe the problem mathematically. In Section 3 and 4 we present success and failure results for a single sensor moving according to two different motion strategies. In Section 5, we conclude and identify future research directions.

2. PROBLEM DESCRIPTION

Consider N independently evolving LTI systems, whose dynamics are given by

$$\begin{cases} x_{i,k+1} = A_i x_{i,k} + w_{i,k} \\ y_{i,k} = C_i x_{i,k} + v_{i,k} \end{cases} \tag{1}$$

where $x_{i,k},\ x_{i,k+1},\ w_{i,k} \in \mathbb{R}^{n_i}$ and $y_{i,k},\ v_{i,k} \in \mathbb{R}^{m_i}$, w_i and v_i are Gaussian random vectors with zero mean and covariance matrices Q_i and R_i respectively and i takes values in the set $\{1, 2, 3, \cdots, N\}$. Let $\mathbb{S}_n(\mathbb{S}_n^+)$ denote the set of symmetric positive semidefinite(definite) matrices of dimension n.

As already mentioned, the space to be covered can be discretized using a grid and the above N systems can be thought to represent the dynamics of certain local variables at the grid points. These variables can be temperature, barometric pressure in case of weather monitoring, threat emergence rate in case of surveillance, uncertain location of adversaries and friends in a situational awareness task and congestion measure at various routers in the case of a network.

In reality the independent evolution of the systems assumption may not always hold, as the dynamics of systems proximate in space may be highly dependent or even coupled. The results for the coupled environment case are under development, but the basic intuition and insight into the coverage problem remain the same.

There are N possible locations at which the sensor can be at a given time. If the sensor is in state i at time k it only has access to the measurement of the ith system at that time. The state transitions occur at a fixed time interval which is assumed to be the same as the sampling period of all the N systems without any loss of generality.

The sensor runs N Kalman filter recursions, one for each of the N systems. For system i the time update equations of the Kalman filter are implemented at all time instants, whereas the measurement update equations are implemented only at those time instants when the sensor happens to be at location i.

Let S_k be the stochastic process describing the motion of the sensor. S_k takes values in the set $\{1, 2, 3, \cdots, N\}$. Let $I_{i,k}$ be the indicator function describing whether or not the sensor is at location i at time k. Therefore $I_{i,k} = 1$ if and only if $S_k = i$. We model the covariance matrix of the measurement noise for the ith system in the following manner.

$$Var(v_{i,k}) = \begin{cases} R_i, & I_{i,k} = 1 \\ {\sigma_i}^2 I, & I_{i,k} = 0 \end{cases}.$$

When the sensor is not at the location i no observation is made for system i and this corresponds to the limiting case of $\sigma \to \infty$. Following a similar approach as in (Sinopoli *et al.*, 2004) we get the following Kalman filter equations:

$$\hat{x}^-_{i,k+1} = A_i \hat{x}_{i,k} \tag{2}$$

$$P^-_{i,k+1} = A_i P_{i,k} A_i^T + Q_i \tag{3}$$

$$\hat{x}_{i,k+1} = \hat{x}^-_{i,k+1} + I_{i,k+1} P^-_{i,k+1} C_i^T$$

$$\times \left(C_i P^-_{i,k+1} C_i^T + R_i\right)^{-1} (y_{i,k+1} - C_i \hat{x}^-_{i,k+1}) \quad (4)$$

$$P_{i,k+1} = P^-_{i,k+1} - I_{i,k+1} P^-_{i,k+1} C_i^T \times \left(C_i P^-_{i,k+1} C_i^T + R_i\right)^{-1} C_i P^-_{i,k+1}. \quad (5)$$

Eq. (2) and Eq. (3) are the time update relations for the estimate and the error covariance. It can be clearly seen from Eq. (4) and Eq. (5) that the measurement update is performed only when the sensor is at location i.

Using the above equations the recursive relation for the *a priori* error covariance matrix can be written as

$$P^-_{i,k+1} = A_i P^-_{i,k} A_i^T + Q_i - I_{i,k+1} A_i P^-_{i,k} C_i^T \times \left(C_i P^-_{i,k} C_i^T + R_i\right)^{-1} C_i P^-_{i,k} A_i^T. \quad (6)$$

For the rest of the paper we will drop the $-$ superscript from $P^-_{i,k}$. An important observation is that Eq. (6) is stochastic in nature due to presence of the random variable $I_{i,k+1}$. We now have N of these stochastic recursive equations, one for each of the N systems. So to maintain an appreciable estimate of the states of all N systems we would want that $\lim_{k\to\infty} \mathbb{E}[P_{i,k}]$ remains bounded for all i.

Since both $I_{i,k+1}$ and $P_{i,k}$ are random variables, we know that

$$\mathbb{E}[P_{i,k+1}] = \mathbb{E}\big[\mathbb{E}[P_{i,k+1}|P_{i,k}]\big] \quad (7)$$

where the inner expectation operator is over $I_{i,k+1}$ and the outer expectation is over $P_{i,k}$. Therefore

$$\mathbb{E}[P_{i,k+1}] = \mathbb{E}[A_i P_{i,k} A_i^T + Q_i - \rho_{i,k+1} A_i P_{i,k} C_i^T \times \left(C_i P_{i,k} C_i^T + R_i\right)^{-1} C_i P_{i,k} A_i^T] \quad (8)$$

where $\eta_{i,k+1} = \Pr[I_{i,k+1} = 1 | P_{i,k}]$.

Definition 1. We say that the dynamic sensor coverage problem has been successfully solved if the N limits

$$\lim_{k\to\infty} \mathbb{E}[P_{i,k}]\ ,\ i \in \{1, 2, \cdots, N\}$$

are finite for any set of initial conditions $P_{i,0} \geq 0$.

If there exists an $i \in \{1, 2, \cdots, N\}$ such that $\lim_{k\to\infty} \mathbb{E}[P_{i,k}]$ is unbounded for some $P_{i,0} \geq 0$, then the sensors have failed to solve the dynamic coverage problem.

Based on the above definition, we now present success and failure results for two different sensor motion strategies for a single sensor.

3. S_K INDEPENDENT AND IDENTICALLY DISTRIBUTED

Under this strategy at each time instant the sensor chooses to visit location i with probability π_i,

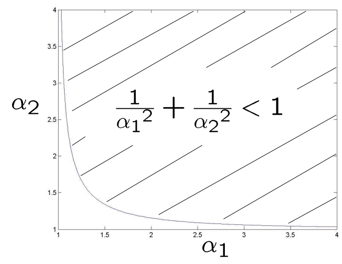

Fig. 1. Failure region

which is independent of the history of S_k. In this case $\eta_{i,k+1} = \Pr[I_{i,k+1} = 1 | P_{i,k}] = \Pr[I_{i,k+1}] = \pi_i$. So (8) reduces to

$$\mathbb{E}[P_{i,k+1}] = \mathbb{E}[A_i P_{i,k} A_i^T + Q_i - \pi_i A_i P_{i,k} C_i^T \times \left(C_i P_{i,k} C_i^T + R_i\right)^{-1} C_i P_{i,k} A_i^T] \quad (9)$$

Now this equation is exactly the same as the one analyzed in (Sinopoli *et al.*, 2004) for packet based networks. The following two results easily follow.

Proposition 2. Consider the system in Eq. (1). Let $(A_i, \sqrt{Q_i})$ be controllable, (A_i, C_i) be detectable and A_i be unstable for all i. The sensor motion is governed by an i.i.d. distribution with Prob$[S_k = i] = \pi_i$. Now if

$$\sum_{i=1}^{N} \frac{1}{\alpha_i^2} < N - 1 \quad (10)$$

where α_i is the spectral radius of A_i, then a single sensor fails to solve the dynamic coverage problem.

PROOF.

$$\sum_{i=1}^{N} \frac{1}{\alpha_i^2} < N - 1 \Longrightarrow \sum_{i=1}^{N} \left(1 - \frac{1}{\alpha_i^2}\right) > 1.$$

Therefore for any steady state probability vector π there exists an i s.t. $\pi_i < 1 - 1/{\alpha_i}^2$. Now from (Sinopoli *et al.*, 2004) we know that $\lim_{k\to\infty} \mathbb{E}[P_{i,k}]$ is unbounded for some initial condition $P_{i,0} \geq 0$. Thus a single sensor can not solve the dynamic sensor coverage problem. □

It can be seen that Eq. (10) is a measure of how fast the systems evolve. In Fig. 1 the region above the curve is where a single sensor fails to solve the dynamic coverage problem for 2 systems. It should be noted that if one system is evolving very slowly then the sensor can tolerate very fast dynamics of the other system before it fails. In such a scenario the sensor distributes its time, spending relatively large amount of time observing the fast system.

We now give some conditions under which its possible to solve the dynamic sensor coverage problem by employing a single sensor. Before that we need to carry over a few terms from (Sinopoli *et al.*, 2004).

For real symmetric Y, define $\Psi_i(Y, Z)$ as

$$\Psi_i(Y,Z) = \begin{bmatrix} Y & \sqrt{\pi}(YA_i + ZC_i) & \sqrt{1-\pi}YA_i \\ \sqrt{\pi}(A_i'Y + C_i'Z') & Y & 0 \\ \sqrt{1-\pi}A_i'Y & 0 & Y \end{bmatrix}$$

and π_i^u as

$$\pi_i^u = \arg\min_{\pi} \big(\exists 0 \le Y \le I, Z \mid \Psi_i(Y,Z) > 0\big).$$

Proposition 3. Let sensor motion be an i.i.d. process with distribution π. If $\sum_{i=1}^{N} \pi_i^u < 1$ and if π lies in the convex hull of the N points, a_i, $i = 1, \cdots, N$, defined as

$$a_i = \left[\pi_1^u \; \cdots \; \pi_{i-1}^u \; 1-\sum_{k\neq i}\pi_k^u \; \pi_{i+1}^u \; \cdots \; \pi_N^u\right]^T,$$

then the dynamic coverage problem is solved.

PROOF. Since π lies in the convex hull of the above points, therefore there exist $\beta_i \ge 0$, $\sum_i \beta_i = 1$, s.t.

$$\begin{aligned} \pi_j &= \pi_j^u \sum_{i\neq j} \beta_i + \beta_j(1 - \sum_{i\neq j} \pi_i^u) \\ &= \pi_j^u(1-\beta_j) + \beta_j(1 - \sum_{i\neq j} \pi_i^u) \\ &> \pi_j^u(1-\beta_j) + \beta_j \pi_j^u \\ &= \pi_j^u \end{aligned}$$

Now it was shown in (Sinopoli *et al.*, 2004) that if $\pi_i > \pi_i^u$ then $\mathbb{E}[P_{i,k}]$ remains bounded as $k \to \infty$ for all initial conditions $P_{i,0} \in \mathbb{S}_{n_i}$, and hence the result follows. □

4. S_K VARIES ACCORDING TO AN ERGODIC MARKOV PROCESS

In this section we will let S_k be a discrete time discrete state DTDS Markov process with transition probability matrix $\mathcal{T}$. $\mathcal{T}_{ij}$ is the probability that the sensor will be at location j at time $k+1$ given that it is in location i at time k. If $\pi_{i,k}$ is the probability of finding the sensor in location i at time k, then the column vector π_k follows the recursion

$$\pi_{k+1}^T = \pi_k^T \mathcal{T}$$

This kind of model is better for sensor motion because there may be physical constraints on the motion of the sensor. For example the sensor may not be able to move between two systems located far away in space in one time interval. Such restrictions can be imposed by making the corresponding transition probability between such states equal to zero.

Markov chains have been used earlier for search and surveillance problems in the operations research community (Jeffcoat, 2004).

Under the ergodicity assumption we know that the Markov chain S_k has a unique steady state distribution and $\lim_{k\to\infty} \pi_k = \pi$ for all initial probability distributions. (Brémaud, 1999)

For the analysis of the Markov chain case we define the following relations for $X \in \mathbb{S}_n$.

$$h(X) \triangleq AXA' + Q \quad (11)$$

$$f(X) \triangleq AXC'(CXC' + R)^{-1}CXA' \quad (12)$$

$$g(X) \triangleq h(X) - f(X) \quad (13)$$

In the rest of this paper $h_i(X)$, $g_i(X)$ and $f_i(X)$ will refer to the same functional forms as described above but with parameters of system i. For example $h_i(X) = A_i X A_i' + Q_i$ for $i \in \{1, 2, \cdots, N\}$. At this point we would like to remind the reader that under the estimation scheme described in section 2. the recursion of the error covariance matrix of location i can be written in terms of h_i and g_i as,

$$P_{i,k+1} = \begin{cases} h_i(P_{i,k}) & S_k \neq i \\ g_i(P_{i,k}) & S_k = i \end{cases} \quad (14)$$

We now present some preliminary results required to prove our main Theorem.

Lemma 4. If $X \ge Y$, then $g(X) \ge g(Y)$ and $h(X) \ge h(Y)$.

PROOF. See (Sinopoli *et al.*, 2004). □

Lemma 5. If $U \in \mathbb{S}_n^+$ and $V \in \mathbb{S}_n$, then $\exists$ a scalar $t \ge 0$ such that $tU - V \in \mathbb{S}_n$.

PROOF. By Weyl's Theorem (Horn and Johnson, 1985), $t \ge 0$

$$\begin{aligned} \lambda_{min}(tU - V) &\ge \lambda_{min}(tU) + \lambda_{min}(-V), \\ &= t\lambda_{min}(U) - \lambda_{max}(V), \end{aligned}$$

where λ_{min} is the minimum eigenvalue and λ_{max} is the maximum eigenvalue So any $t \ge \frac{\lambda_{max}(V)}{\lambda_{min}(U)}$ proves the Lemma. Such a t always exists because $\lambda_{min}(U) > 0$. □

Lemma 6. $g(X) \ge Q, \forall X \ge 0$ and if C is invertible then, $g(X) \le AC^{-1}RC'^{-1}A' + Q, \forall X \ge 0$.

PROOF. Clearly $g(X) \ge g(0) = Q$. For any $X \ge 0$, as $C^{-1}RC'^{-1} \in \mathbb{S}_n^+$, by Lemma 5, $\exists t \ge 0$ such that

$$
\begin{aligned}
X &\le tC^{-1}RC'^{-1},\\
g(X) &\overset{a}{\le} g(tC^{-1}RC'^{-1}),\\
&= t/(t+1)AC^{-1}RC'^{-1}A' + Q,\\
&\le AC^{-1}RC'^{-1}A' + Q,
\end{aligned}
$$

by using Lemma 4 in a. □

Lemma 7. (a) If A is unstable then

$$\lim_{k\to\infty} h^k(X_0) = \infty,\ \forall X_0 \in \mathbb{S}_n.$$

(b) If the spectral radius of A, $\alpha < 1$ and the pair $(A, \sqrt{Q})$ is observable, then the Lyapunov difference equation $X_{k+1} = h(X_k)$ converges to a unique positive semidefinite solution $T > 0$ as $k \to \infty$. In other words the following infinite sum

$$\lim_{k\to\infty} \left[A^k X_0 A'^k + \sum_{m=0}^{k-1} A^m Q A'^m \right]$$

is a finite positive definite matrix $T > 0$ for all $X_0 \ge 0$, where $T = h(T)$.

PROOF. See (Gajić and Qureshi, 1995) □

The following probabilities will be useful in our analysis. The derivation is relatively simple and we omit the proofs due to space constraints. $\mathcal{T}_{ii}$ is the ith diagonal entry of the transition probability matrix and π_i is the steady state probability of finding the sensor at location i.

$$
\begin{aligned}
\rho_{i,hh} &= \Pr[S_{k+1} \ne i | S_k \ne i] = \frac{1-\pi_i(2-\mathcal{T}_{ii})}{1-\pi_i}\\
\rho_{i,hg} &= \Pr[S_{k+1} = i | S_k \ne i] = 1 - \rho_{i,hh}\\
\rho_{i,gg} &= \Pr[S_{k+1} = i | S_k = i] = \mathcal{T}_{ii}\\
\rho_{i,gh} &= \Pr[S_{k+1} \ne i | S_k = i] = 1 - \rho_{i,gg} \qquad (15)
\end{aligned}
$$

Theorem 8. (a) Let (A_i, C_i) be detectable and $(A_i, \sqrt{Q_i})$ be observable, and if the sensor motion is described by an ergodic Markov chain S_k then the sensor fails to solve the Dynamic Coverage problem if at least one of the following conditions hold.

$$\rho_{i,hh} = \frac{1-\pi_i(2-\mathcal{T}_{ii})}{1-\pi_i} > \frac{1}{{\alpha_i}^2}, i \in 1, 2 \cdots N$$

where α_i is the spectral of A_i.

(b) If in addition all C_is are invertible then the sensor solves the Dynamic Coverage problem, if all the following conditions hold

$$\rho_{i,hh} = \frac{1-\pi_i(2-\mathcal{T}_{ii})}{1-\pi_i} < \frac{1}{{\alpha_i}^2}, i \in 1, 2 \cdots N$$

PROOF. For simplicity we prove this result for the case when the initial probability distribution of the sensor is the same as the steady state distribution. In practice if one knows the transition probability matrix of a Markov chain, implementing such a constraint is easy.

Table 4: Illustration of how to find the lower bound

Probabilities	Values	Lower bounds
$(1-\pi_i)\rho_{i,hh}^2$	$h_i^3(P_{i,0})$	$h_i^3(P_{i,0})$
$\pi_i\rho_{i,gh}\rho_{i,hh}$	$h_i^2 g_i(P_{i,0})$	$h_i^2(Q_i)$
$(1-\pi_i)\rho_{i,hg}\rho_{i,gh}$	$h_i g_i h_i(P_{i,0})$	$h_i(Q_i)$
$\pi_i\rho_{i,gg}\rho_{i,gh}$	$h_i g_i^2(P_{i,0})$	$h_i(Q_i)$
$(1-\pi_i)\rho_{i,hh}\rho_{i,hg}$	$g_i h_i^2(P_{i,0})$	Q_i
$\pi_i\rho_{i,gh}\rho_{i,hg}$	$g_i h_i g_i(P_{i,0})$	Q_i
$(1-\pi_i)\rho_{i,hg}\rho_{i,gg}$	$g_i^2 h_i(P_{i,0})$	Q_i
$\pi_i\rho_{i,gg}^2$	$g_i^3(P_{i,0})$	Q_i

(a) $P_{i,k+1}$ can take 2^{k+1} different values with different probabilities for a given value of $P_{i,0}$ depending on the values of $S_1, S_2 \cdots S_{k+1}$. From Lemma 6 we know that $g_i(X) \ge Q_i$, and from Lemma 4 we know that h_i is an increasing function. Therefore

$$
\begin{aligned}
\mathbb{E}[P_{i,k}] \ge \pi_i Q_i &+ \frac{(1-\pi_i)}{\rho_{i,hh}} \rho_{i,hh}^k h_i^k(P_{i,0})\\
&+ \frac{\pi_i \rho_{i,gh}}{\rho_{i,hh}} \sum_{m=0}^{k-2} \rho_{i,hh}^{m+1} h_i^{m+1}(Q_i) \quad (16)
\end{aligned}
$$

To illustrate how we obtain the above inequality we consider the case when $k = 3$, in table 4. The right hand side of the above equation is the inner product of the 1st and 3rd rows of the table. Using Lemma 7 the sensor would fail to solve the dynamic coverage problem, if the following condition holds for at least one system i.

$$\rho_{i,hh} = \frac{1-\pi_i(2-\mathcal{T}_{ii})}{1-\pi_i} > \frac{1}{{\alpha_i}^2}$$

(b) If C_is are invertible then we can find an upper bound using Lemma 6

$$
\begin{aligned}
\mathbb{E}[P_{i,k}] \le \pi_i M_i &+ \frac{(1-\pi_i)}{\rho_{i,hh}} \rho_{i,hh}^k h_i^k(P_{i,0})\\
&+ \frac{\pi_i \rho_{i,gh}}{\rho_{i,hh}} \sum_{m=0}^{k-2} \rho_{i,hh}^{m+1} h_i^{m+1}(M_i) \quad (17)
\end{aligned}
$$

where $M_i = A_i C_i^{-1} R_i C_i'^{-1} A_i' + Q_i$ Now the first term on the right hand side is finite. From Lemma 7 the second term is finite as $k \to \infty$ if $\rho_{i,hh}{\alpha_i}^2 < 1$. The third term after summing the geometric series can be rewritten as

$$
\begin{aligned}
&\frac{\pi_i \rho_{i,gh}}{\rho_{i,hh}} \left[\sum_{m=1}^{k-1} \tilde{A}_i^m M_i (\tilde{A}_i^m)' \right.\\
&\left. + \frac{\rho_{i,hh}}{1-\rho_{i,hh}} \sum_{m=0}^{k-2} \tilde{A}_i^m Q_i (\tilde{A}_i^m)' (1 - \rho_{i,hh}^{k-1-m}) \right]
\end{aligned}
$$

where $\tilde{A}_i = \sqrt{\rho_{i,hh}} A_i$. Again using Lemma 7 we know that this term is finite as $k \to \infty$ if $\rho_{i,hh}{\alpha_i}^2 < 1$. □

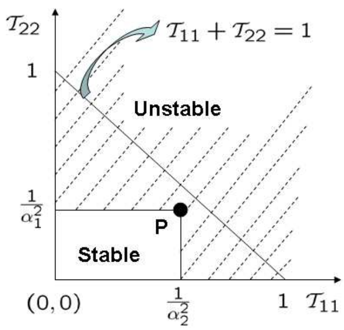

Fig. 2. S_k is a markov process, $N = 2$.

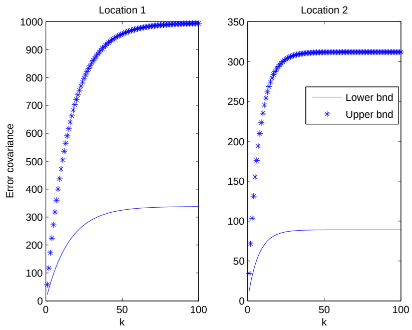

Fig. 3. Bounds on error covariance.

Note that for the $N = 2$ case, $\pi_1 = (1 - \mathcal{T}_{22})/(2 - \mathcal{T}_{22} - \mathcal{T}_{11})$ and $\pi_2 = 1 - \pi_1$, where $\mathcal{T}_{ii} \in (0, 1)$ for ergodicity. It can be easily verified using (15) that $\rho_{1,hh} = \mathcal{T}_{22}$ and $\rho_{2,hh} = \mathcal{T}_{11}$. Therefore the instability region from Theorem 8 is the shaded region in Fig. 2. Now a 2 state Markov chain is an i.i.d. distribution for the case when $\mathcal{T}_{11} + \mathcal{T}_{22} = 1$. Now we can see from Fig. 2 that if $1/\alpha_1^2 + 1/\alpha_2^2 < 1$, then the point P lies below the line and thus the dynamic coverage problem cannot be solved by an i.i.d. sensor motion algorithm. This shows that Proposition 2 is a special case of Theorem 8.

Example 1. Consider 2 scalar systems with parameters $A_1 = 1.25$, $C_1 = 0.2$, $R_1 = 2.5$, $Q_1 = 20$, $A_2 = 1.7$, $C_2 = 0.4$, $R_2 = 2$ and $Q_2 = 10$. The quantity $1/\alpha_1^2 + 1/\alpha_2^2 = 0.986 < 1$, therefore an i.i.d. sensor motion strategy will not be able solve the dynamic coverage problem, but a Markov chain strategy with the following transition probability matrix

$$\mathcal{T} = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

solves the coverage problem with the expected error covariance contained between the lower and upper bounds as shown in Fig. 3.

5. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper we define the dynamic sensor coverage problem. We have considered a simple case in which N spatially separated linear systems whose dynamics are decoupled have to be observed by a single mobile sensor. Due to the finite range of the sensor, it can make measurements for a particular system, only if it happens to be at that system. We have modeled the motion of the sensor as an iid process and as an ergodic Markov chain.

There are several avenues of research that this paper opens up. The most immediate one is the introduction of feedback. It should be noted that even though this paper gives success and failure bounds on probabilities for random sensor motion algorithms, it does not talk about how to change the motion algorithm based on the uncertainty profile in the space. The question of *"Where to move?"* based on the confidence in estimates, requires further analysis. Constructive proceedures for an appropriate transition probability matrix, respecting physical motion constraints between spacially separate locations need to be developed.

Other research directions that we are currently pursuing are solving the coverage problem when the dynamics of the environment are coupled and dependent at different locations and the dynamic coverage problem with multiple sensors.

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