ADAPTIVE PARAMETER SELCETOIN OF QUANTUM-BEHAVED PARTICLE SWARM OPTIMIZATION ON GLOBAL LEBVEL

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Abstract: In this paper, we formulate the dynamics and philosophy of Quantum-behaved Particle Swarm Optimization (QPSO) Algorithm, and suggest a parameter control method based on the whole population level. After that we introduce a diversity-guided model into the QPSO to make the PSO system an open evolutionary particle swarm and therefore propose the Adaptive Quantum-behaved Particle Swarm Optimization Algorithm (AQPSO). We compare the performance of APSO algorithm with those of SPSO and original QPQSO by test the algorithms on several benchmark functions. The experiments results show that APSO algorithm outperforms due to its strong global search ability.

Keywords: Adaptive algorithm, Global Optimisation, Intelligence, Multidimensional, Non-linear system, Optimisation Problem.

1. INTRODUCTION

Particle Swarm Optimisation (PSO), motivated by the collective behaviours of bird and other social organisms, is a novel evolutionary optimisation strategy introduced by J. Kennedy and R. Eberhart in 1995 (Kennedy, *et al*, 1995). It has already shown to be comparable in performance with traditional optimization algorithms such as simulated annealing (SA) and the genetic algorithm (GA) (Angeline, 1998; Eberhart, 1998; Krink, 2001; Vesterstrom 2001).

Since its origin in 1995, many revised versions of PSO have been proposed to improve its performance. In 1998, Shi and Eberhart introduced inertia weight *w* into evolution equation to accelerate the convergence speed (Shi and Eberhart, 1998) and therefore proposed the so-called Standard PSO (SPSO). In 1999, Clerc employed Constriction Factor *K* to guarantee convergence of the algorithm and release the limitation of velocity (Clerc, 1999). Ozcan in 1999 and Clerc in 2002 did trajectory analysis of PSO respectively (Ozcan et al, 1999; Clerc et al, 2002), and their works provide the golden rule of parameter selection. Other achievements in the field include Neighbourhood Topology Structure (Kennedy, 1999), Selection Operator in PSO (Angeline, 1998), Binary Version of PSO (Kennedy et al, 1997) and so forth.

In 2004, Jun Sun *et al.* introduce quantum theory into PSO and propose a Quantum-behaved PSO based on Delta potential well (QDPSO) algorithm (Sun et al, 2004). The experiment results indicate that the QDPSO works better than standard PSO on several benchmark functions and it is a promising algorithm due to its characteristic of global convergence.

In this paper, we suggest an Adaptive Quantum-Behaved Particle Swarm Optimisation algorithm based on Diversity-Guided Model, the parameter control of which is adaptive on global level and are able to overcome the problem of premature convergence efficiently.

The rest part of the paper is arranged as follows. In Section 2 and Section 3, we formulate the philosophy and the parameter control of QPSO. The Diversity-Guided model of QPSO is proposed in Section 4. The experiment results are shown in Section 5. And some conclusion remark is made in Section 6.

2. PHILOSOPHY OF QPSO

2.1 Dynamics of Standard PSO

In the Standard PSO model, each individual is treated as a volume-less particle in the D-dimensional space, with the position vector and velocity vector of particle *i* represented as $\vec{x}_i(t)=(x_{i1}(t),x_{i2}(t),\cdots,x_{iD}(t))$ and $\vec{v}_i(t)=(v_{i1}(t),v_{i2}(t),\cdots,v_{iD}(t))$. The particles move according to the following equation:

$$\vec{v}_i(t+1)=w*\vec{v}_i(t)+\varphi_1(\vec{p}_i-\vec{x}_i(t))+\varphi_2(\vec{p}_g-\vec{x}_i(t)) \quad (1a)$$

$$\vec{x}_i(t+1)=\vec{x}_i(t)+\vec{v}_i(t+1) \quad (1b)$$

where φ_1 and φ_2 are random numbers whose upper limits are parameters of the algorithm. Parameter *w* is the inertia weight introduced to accelerate the convergence speed of the PSO. Vector $\vec{p}_i=(p_{i1},p_{i2},\cdots,p_{iD})$ is the best previous position (the position giving the best fitness value) of particle *i* called ***pbest***, and vector $\vec{p}_g=(p_{g1},p_{g2},\cdots,p_{gD})$ is the position of the best particle among all the particles in the population and called ***gbest***.

In essence, the traditional model of PSO system is of linear system, under the circumstance that ***pbest*** and ***gbest*** are fixed as well as all random numbers are considered constant. Trajectory analyses (Clerc *et al*, 2002) show that, whatever model is employed in the PSO algorithm, each particle in the PSO system converges to its local point $\vec{p}=(p_1,p_2,\cdots p_D)$, one and only local attractor of each particle, of which the coordinates are

$$p_d=(\varphi_1 p_{id}+\varphi_2 p_{gd})/(\varphi_1+\varphi_2) \quad (2)$$

so that the *pbests* of all particles will converges to an exclusive *gbest* with $t\to\infty$.

However, a social organism is a system far more complex than that formulated by equation (1), and even the thinking mode of an individual of the social organism is so intricate that a linear evolvement equation is not sufficient to depict it at all. In practice, the evolution of man's thinking is uncertain to a great extent somewhat like a particle having quantum behaviour.

2.2 Quantum Model of PSO

In the quantum model of a PSO, the state of a particle is depicted by wavefunction $\Psi(\vec{x},t)$, instead of $\vec{x}$ and $\vec{v}$. The dynamic behaviour of the particle is widely divergent from that of the particle in traditional PSO systems in that the exact values of $\vec{x}$ and $\vec{v}$ cannot be determined simultaneously. We can only learn the probability of the particle's appearing in position $\vec{x}$ from probability density function $|\psi(\vec{x},t)|^2$, the form of which depends on the potential field the particle lies in.

In (Jun Sun *et al*, 2004), employed Delta potential well with the centre on point $\vec{p}=(p_1,p_2,\cdots p_D)$ to constrain the quantum particles in PSO in order that the particle can converge to their local $\vec{p}$ without explosion. The wavefunction of the particle in Delta potential well is as follows

$$\psi(x)=\frac{1}{\sqrt{L}}\exp(-\|p-x\|/L) \quad (3)$$

And the probability density function is

$$Q(x)=|\psi(x)|^2=\frac{1}{L}\exp(-2\|p-x\|/L) \quad (4)$$

The parameter *L* depending on energy intension of the potential well specifies the search scope of a particle and is called "Creativity" or "Imagination" of the particle in this paper.

In quantum-behaved PSO, search space and solution space of the problem are two spaces of different quality. Wavefunction or probability function of position depicts the state of the particle in quantized search space, not informing us of any certain information about the position of a particle that is vital to evaluate the fitness of a particle. Therefore, state transformation between two spaces is absolutely necessary. In terms of quantum mechanics, the transformation from quantum state to classical state is called collapse, which in nature is the measurement of a particle's position.

Monte Carlo Method, a stochastic simulation, can realize the process of measurement on computers. And the position can be given by

$$x(t)=p\pm\frac{L}{2}\ln(1/u) \quad (5)$$

In (Sun *et al*, 2004), the parameter *L* is evaluated by

$$L(t+1)=2*\alpha*|p-x(t)| \quad (6)$$

Thus the iterative equation of Quantum PSO is

$$x(t+1)=p\pm\alpha*|p-x(t)|*\ln(1/u) \quad (7)$$

which replaces Equation (1) in QDPSO algorithm.

2.3 Quantized Knowledge Seeking Model of Particles

In quantum PSO, at every iteration, each particle records its ***pbest*** and compares its ***pbest*** with those of all other particles in its neighbourhood or population to get the ***gbest***. Then its Learning Inclination Point ***p*** can be given by Equation (2) after random numbers φ_1 and φ_2 is generated. And a Delta potential well is established at point p to simulate the tendentiousness of the particle. To execute the next knowledge-seeking step, parameter *L* must be evaluated. We call parameter *L* "Creativity" or "Imagination", for it characterize the knowledge-seeking scope of the particle, and the larger the value of L, the more likely the particle find out new knowledge. In QDPSO, the Creativity of the particle is evaluated by the gap between the particle's current position and its LIP, as shown in equation (6). At last, by state transformation from search space to solution space, the new position can be got by Equation (7). If the new position is better knowledge than *pbes*t, *pbest* will be replaced by the new knowledge.

3. AN APPROACH GLOBAL PARAMTER CONTROL

As mentioned above, parameter *L*, "Creativity" or "Imagination" of a particle, is the only parameter in QPSO algorithm. The control method of *L* is vital to convergence rate and performance of the algorithm. In (Sun et al, 2004a), *L* is evaluated by the gap between the particles current position and LIP. However, this control method has two drawbacks: (1) evaluating the "Creativity" of individual by the individual's LIP is illogical. (2) The parameter control method is based on local and individual level, for LIP is a volatile local point. It results in unstable and uneven convergence speed of an individual particle, and therefore probable premature of the algorithm when population size is small.

In this section, we propose a novel method of parameter control based on global level.

Although various thoughts exist in human society, there must be a mainstream thought accepted by majority. The mainstream thought can be used to assess individual's creativity. If the deviation of individual's thought from the mainstream thought is great, the individual is generally more creative and imaginative, and therefore capable of discovering new knowledge; On the contrary, if the deviation is small, the individual is lack of its own judgment and apt to drift with mainstream thought tide. Such an individual has a narrow knowledge-seeking scope and poor creativity.

In our revised QPSO, we employ a mainstream thought point to evaluate parameter *L,* the creativity of a particle. The Mainstream Thought Point or Mean Best Position (*mbest*) is defined as the center-of –gravity *gbest* position of the particle swarm. That is

$$\mathrm{mbest} = \sum_{i=1}^{M} p_i \Big/ M = \left(\sum_{i=1}^{M} p_{i1} \Big/ M,\ \ \sum_{i=1}^{M} p_{i2} \Big/ M,\ \ \cdots,\ \ \sum_{i=1}^{M} p_{id} \Big/ M \right) \qquad (8)$$

where M is the population size and p_i is the ***gbest*** position of particle *i*. The value of L is given by

$$L(t+1) = 2 * \beta * \left| \mathrm{mbest} - x(t) \right| \qquad (9)$$

where β is called Creativity Coefficient. Thus Equation (10) is rewritten as

$$x(t+1) = p \pm \beta * \left| \mathrm{mbest} - x(t) \right| * \ln(1/u) \qquad (10)$$

4. DIVERSITY-GUIDED MODEL OF QPSO

As we know, a major problem with PSO and other evolutionary algorithms in multi-modal optimization is premature convergence, which results in great performance loss and sub-optimal solutions. In a PSO system, with the fast information flow between particles due to its collectiveness, diversity of the particle swarms declines rapidly, leaving the PSO algorithm with great difficulties of escaping local optima. Therefore, the collectiveness of particles leads to low diversity with fitness stagnation as an overall result. In QPSO, although the search space of an individual particle at each iteration is the whole feasible solution space of the problem, diversity loss of the whole population is also inevitable due to the collectiveness.

Recently, R. Ursem has proposed a model called Diversity-Guided Evolutionary Algorithm (DGEA) (Ursem, 2001), which applies diversity-decreasing operators (selection, recombination) and diversity-increasing operators (mutation) to alternate between two modes based on a distance-to-average-point measure. The performance of the DGEA clearly shows its potential in multi-modal optimisation.

In 2002, Riget *et al* (Riget *et al*, 2002) adopt the idea from Usrem into the basic PSO model with the decreasing and increasing diversity operators used to control the population. This modified model of PSO uses a diversity measure to have the algorithm alternate between exploring and exploiting behaviour. They introduce two phases: attraction and repulsion. The swarm alternate between these phases according to its diversity and the improved PSO algorithm is called Attraction and Repulsion PSO (ARPSO) algorithm.

Inspired by works undertaken by Ursem and Riget *et al*, we introduce the Diversity-Guided model in Quantum-behaved PSO. As Riget did, we also define two phases of particle swarm: attraction and repulsion. It can be demonstrated that when the Creativity parameter satisfies $\beta \le 1$, the particles will be bound to converge to its local LIP *p*, and some particles will depart from *p* when β>1; the larger the β, the more particles will explode. Consequently, the two phases is distinguished by the parameter β and defined as

Attraction Phase: $\beta=\beta_a$, where $\beta_a \le 1$;

Repulsion Phase: $\beta=\beta_r$, where $\beta_r > 1$.

In attraction phase ($\beta=\beta_a$) the swarm is contracting, and consequently the diversity decreases. When the diversity drops below a lower bound, d_{low}, we switch to the repulsion phase ($\beta=\beta_r$), in which the swarm expands. Finally, when the diversity reaches a higher bound, we switch back to the attraction phase. The result of this is a QPSO algorithm that alternates between phases of exploiting and exploring-attraction and repulsion-low diversity and high diversity, according to the diversity of the swarm measured by

$$\text{diversity}(S) = \frac{1}{|S| \cdot |L|} \cdot \sum_{i=1}^{|S|} \sqrt{\sum_{j=1}^{D} (p_{ij} - \overline{p_j})^2} \qquad (11)$$

where S is the swarm, $|S| = M$ is the population size, $|L|$ is the length of longest the diagonal in the search space, D is the dimensionality of the problem, p_{ij} is the j'th value of the i'th particle (pbest) and $\overline{p_j}$ is the j'th value of the average point $\overline{p}$ (mbest).

The Quantum-behaved PSO algorithm with attraction and repulsion phases is called Adaptive Quantum-Behaved Particle Swarm Optimisation (APSO) algorithm, which is described as following.

APSO ALGORITHM

```
Initialise population: random x_i
Do
   find out mbest using equation (8)
   Measure the diversity of the swarm by equation (11)
   If (diversity<dlow) beta=betaa;
   If (diversity>dhigh) beta=betar;
   for i=1 to population size M
     If f(x_i)<f(p_i) then p_i=x_i
     p_g=min(p_i)
     for d=1 to dimension D
        fi_1=rand(0,1), fi_2=rand(0,1)
        p=(fi_1*p_id+fi_2*p_gd)/(fi_1+fi_2)
        u=rand(0,1)
        if rand(0,1)>0.5
           x_id=p-beta*abs(mbest_d-x_id)*(ln(1/u)
        else
           x_id=p+beta*abs(mbest_d-x_id)*(ln(1/u)
Until termination criterion is met
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5. EXPERIMENT RESULTS

To test the performance of AQPSO, seven benchmark functions are used here for comparison with SPSO in QPSO. The first function is Sphere function described by

$$f(x)_1 = \sum_{i=1}^{n} x_i^2 \qquad (12)$$

The second function is the Rosenbrock function described by

$$f(x)_2 = \sum_{i=1}^{n} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2) \qquad (13)$$

The third function is the generalized Rastrigrin function described by

$$f(x)_3 = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10) \qquad (14)$$

The fourth function is generalized Griewank function described by

$$f(x)_4 = \frac{1}{4000} \sum_{i=1}^{n} (x_i - 100)^2 - \prod_{i=1}^{n} \cos\left(\frac{(x_i - 100)}{\sqrt{i}}\right) + 1 \qquad (15)$$

The fifth function is De Jong's function (no noise) described by

$$f(x)_5 = \sum_{i=1}^{n} i \cdot x_i^4 \qquad (16)$$

The sixth function is Rosenbrock variant function described by

$$f(x)_6 = 100(x_1^2 - x_1)^2 + (1 - x_1)^2 \qquad (17)$$

The seventh function is Shaffer's function described by

$$f(x)_7 = 0.5 + \frac{(\sin\sqrt{x^2 + y^2})^2 - 0.5}{(1.0 + 0.001(x^2 + y^2))^2} \qquad (18)$$

These functions are all minimization problems with minimum value zero.

In all experiments, the initial range of the population listed in table 1 is asymmetry as used in (Sun *et at*, 2004). The fitness value is set as function value and the neighbourhood of a particle is the whole population. We had 50 trial runs for every instance and recorded mean best fitness. In order to investigate the scalability of the algorithm, different population sizes *M* are used for each function with different dimensions. The population sizes are 20, 40 and 80. Generation is set as 1000, 1500 and 2000 generations corresponding to the dimensions 10, 20 and 30 for first five functions, respectively, and the dimension of the last two functions is 2.

We also test performance of the QPSO1, in which the creative coefficient *beta* decreases from 1.0 to 0.5 linearly when the algorithm is running. In the experiments to test APSO, we set β_a to be 0.7, β_r to be 2.0, d_{low} to be 5.0×10^{-6}, and d_{high} to be 0.25.

The best fitness values for 50 runs of each function in table 1 to table 7. The value in column of SPSO and QDPSO in from Table 2 to Table 8 is taken from (Sun *et al*, 2004; Shi, 1998).

Table 1

Function	Asymmetric Initialization Range
f_1	(50, 100)
f_2	(15, 30)
f_3	(2.56, 5.12)
f_4	(300, 600)
f_5	(30, 100)
f_6	(30, 100)
f_7	(30, 100)

Table 2. THE MEAN FITNESS VALUE FOR SPHERE FUNCTION

M	D	G_{max}	SPSO	QDPSO	QPSO1	AQPSO
20	10	1000	1e-20	1e-25	1e-31	1e-38
	20	1500	1e-11	1e-15	1e-20	1e-21
	30	2000	1e-06	1e-08	1e-11	1e-14
40	10	1000	1e-23	1e-41	1e-62	1e-64
	20	1500	1e-14	1e-23	1e-32	1e-35

	30	2000	1e-10	1e-14	1e-23	1e-28
80	10	1000	1e-28	1e-61	1e-82	1e-88
	20	1500	1e-17	1e-32	1e-50	1e-58
	30	2000	1e-12	1e-19	1e-38	1e-38

Table 3. THE MEAN FITNESS VALUE FOR ROSENBROCK FUNCTION

M	D	G_{max}	SPSO	QDPSO	QPSO1	AQPSO
20	10	1000	96.1715	14.2221	13.8377	10.2356
	20	1500	214.6764	175.3186	116.0543	91.5467
	30	2000	316.4468	242.3770	187.1783	121.3245
40	10	1000	70.2139	15.8623	12.9653	10.3456
	20	1500	180.9671	112.4612	87.9421	79.2357
	30	2000	299.7061	76.4273	75.6933	67.4569
80	10	1000	36.2954	36.3405	11.8327	9.5639
	20	1500	87.2802	23.5443	19.7310	17.7201
	30	2000	205.5596	71.9221	58.5165	51.6923

Table 4. THE MEAN FITNESS VALUE FOR RASTRIGRIN FUNCTION

M	D	G_{max}	SPSO	QDPSO	RQPSO1	RQPSO2
20	10	1000	5.5572	4.9698	4.5712	4.5178
	20	1500	22.8892	17.0789	16.0244	14.7239
	30	2000	47.2941	48.6199	35.2052	31.6284
40	10	1000	3.5623	2.0328	2.0489	2.0138
	20	1500	16.3504	10.9453	10.2717	9.1689
	30	2000	38.5250	21.3712	23.4756	20.3481
80	10	1000	2.5379	0.9232	0.8871	0.7165
	20	1500	13.4263	6.9554	7.2781	6.2659
	30	2000	29.3063	18.130	19.9324	17.3468

Table 5. THE MEAN FITNESS VALUE FOR GRIEWANK FUNCTION

M	D	G_{max}	SPSO	QDPSO	QPSO1	AQPSO
20	10	1000	0.0919	0.1003	0.0078	0.0059
	20	1500	0.0303	0.0086	0.0002	0.0005
	30	2000	0.0182	0.0544	0.0011	0.0015
40	10	1000	0.0862	0.0484	0.0009	0.0008
	20	1500	0.0286	0.0004	0.0002	0.0008
	30	2000	0.0127	0.0009	0.0001	0.0000
80	10	1000	0.0760	0.0000	0.0000	0.0000
	20	1500	0.0288	0.0000	0.0000	0.0000
	30	2000	0.0128	0.0000	0.0000	0.0000

Table 6. THE MEAN FITNESS VALUE FOR DE JONG'S FUNCTION

M	D	G_{max}	SPSO	QDPSO	RQPSO1	RQPSO2
20	10	1000	0.0000	0.0000	0.0000	0.0000
	20	1500	0.0000	0.0000	0.0000	0.0000
	30	2000	0.0000	0.0000	0.0000	0.0000
40	10	1000	0.0000	0.0000	0.0000	0.0000
	20	1500	0.0000	0.0000	0.0000	0.0000
	30	2000	0.0000	0.0000	0.0000	0.0000
	10	1000	0.0000	0.0000	0.0000	0.0000
	20	1500	0.0000	0.0000	0.0000	0.0000
	30	2000	0.0000	0.0000	0.0000	0.0000

Table 7. THE MEAN FITNESS VALUE FOR ROSENBROCK VARIANG FUNCTION

M	D	G_{max}	SPSO	QDPSO	RQPSO1	RQPSO2
20	2	2000	0.0000	0.0000	0.0000	0.0000
40	2	2000	0.0000	0.0000	0.0000	0.0000
80	2	2000	0.0000	0.0000	0.0000	0.0000

Table 8. THE MEAN FITNESS VALUE FOR SHAFFER'S FUNCTION

M	D	G_{max}	SPSO	QDPSO	RQPSO1	RQPSO2
20	2	2000	0.0012	0.0051	7.9437-e4	3.8735-e5
40	2	2000	0.0006	0.0018	1.5385-e5	6.7258-e6
80	2	2000	0.0002	0.0004	8.5111-e7	3.8259-e7

The numerical results show that the revised AQPSO works better than QDPSO, SPSO and QPSO. And the results of experiment on Shaffer's function in table 8 are exciting. The function has sub-optima 0.0097. It is shown that particles of SPSO system are able to escape trap of the sub-optima more frequently than that of QDPSO, for the reason that the particles in SPSO search along relatively continuous trajectories, while those in QDPSO fly discretely so that they may miss the narrow zone where the optimal may lies. But the particles of QPSO1 and APSO can escape the narrow trap more readily than SPSO and QDPSO. It demonstrates, with new adaptive approach of parameter control, the AQPSO has better global search ability.

6. CONCLUSION

In this paper, based on the Quantum-behaved PSO, we formulate the philosophy of quantum-behaved PSO, which is not discussed in detail in (Sun et al, 2004). And then, we set forth an adaptive approach of parameter control and propose AQPSO algorithm. The AQPSO outperforms QDPSO and SPSO on all benchmark functions employed by this paper, as the experiment result shows. In the AQPSO, the evaluation of parameter L depends on a global position, Mean Best Position (*mbest*), which is relatively stable as the population is evolving, and parameter β alternate between two phases (attraction and repulsion). In this model, the PSO system is an open system instead, and consequently, the global search ability of the algorithm is enhanced.

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