A GENERALIZATION OF THE OGY CONTROL TO CONTINUOUS-TIME SYSTEMS USING FLOQUET THEORY

Noboru Sakamoto *

* *Nagoya University, Nagoya, Japan*

Abstract: In this paper, a framework to generalize the OGY control to continuous-time systems is proposed. The framework is based on Floquet theory of linear periodic differential equations and provides a practical method to stabilize unstable periodic orbits (UPOs) and a stability analysis of the closed loop systems. An example of controlling the circular restricted three-body system known as halo orbits is illustrated. It is also reported that stabilization of UPOs can be effective by using the maximum principle to select a nominal orbit.

Keywords: Nonlinear control, Chaotic behaviour, Periodic motion, Stabilization

1. INTRODUCTION

In recent years, much attention has been paid to controlling chaotic systems. Control of chaos refers to a design process of small perturbations in the system in order to realize a desirable behavior. Ott, Grebogi and Yorke (OGY) were the first to present a successful approach for controlling chaos(Ott *et al.*, 1990). Their key idea is as follows. A chaotic attractor densely contains an infinite number of unstable periodic orbits (UPOs). So by making small perturbations around an UPO when states are close to the UPO, the state can be shifted to a stable manifold if it exists around the UPO. An alternative way to control of chaos is delayed feedback(Pyragas, 1992). The delayed feedback scheme uses a time-delay element in the control loop and is useful since it does not require much knowledge on system dynamics. However, the understanding of mechanism in delayed feedback control is still limited(Just *et al.*, 1997; Nakajima, 1997).

A large amount of research has been devoted to improve the OGY scheme (see, for example Boccaletti *et al.* (2000) and Andrievskii and Fradkov (2003)). However, the OGY scheme is for discrete-time systems and to apply it to continuous-time systems, a discretization process is needed. The most common discretization for the OGY method is taking the Poincaré section. And then, a matrix that describes the behavior around the fixed point is numerically calculated to design an OGY controller. It would be easy to imagine that the number of sample points and the calculation task increase exponentially according to the system dimension. Also, there is no rigorous stability relations in controlled systems between discrete behavior on the Poincaré section and original continuous behavior around UPOs.

In this paper, we propose a method of designing an OGY controller for continuous-time systems without taking the Poincaré section and of ensuring the stability of the continuous-time controlled systems. The method is based on Floquet theory of linear periodic differential equations. The discretization process is carried out by using the Monodromy matrix. In this framework, stability analysis for the continuous-time system controlled by the OGY method becomes less involved. Also, this framework does not require exact closed UPOs, which are very difficult to obtain in general. Instead, the proposed stabilization method allows almost periodic orbits (APOs), which are the orbits that return closely to initial states after the period T. The performance of OGY controllers greatly depends on how accurate APOs can be found. For this problem, we propose a method using the maximum principle in control theory. That is, we modify an APO by giving a feedforward input obtained from the maximum principle so that the modified orbit is closer to a closed one while maintaining minimum control effort.

Given a target UPO (an APO in practice), the proposed method gives a sufficient condition for stabilization and an error estimate of a controlled orbit since it is based on the OGY-scheme while it is difficult to get sufficient conditions from the delayed feedback scheme. The control result of the circular restricted three-body problem (halo orbit stabilization) is illustrated, which is an example of higher dimensional systems and of practical interest from space observation programs.

2. PROBLEM FORMULATION

To apply the OGY control to continuous time systems, one has to find a periodic orbit that corresponds to a fixed point in the standard OGY scheme that employs the Poincaré section. It is, however, very difficult to find a closed orbit in general nonlinear systems. Also, it requires many sample points to get matrices describing linear dynamics on the Poincaré section with accuracy, especially for higher dimensional systems. Moreover, there is no rigorous relation between the closed loop stability of a discrete-time system on the Poincaré section and that of the corresponding orbit of the original continuous-time system. With these difficulties in mind we formulate the problem to be solved to generalize the OGY control scheme for continuous-time systems.

Let us consider the following nonlinear system

$$\dot{x}(t) = f(x(t)) + G(x)u(t), \quad x(t) \in \mathbb{R}^n, \quad u(t) \in \mathbb{R}^m. \tag{1}$$

Definition 1. (Almost periodic orbits). We say that (1) with no control ($u = 0$) has an almost periodic orbit (APO) with period T and accuracy α if there exists an initial state x_0 and a time T such that for the orbit $x(t)$

$$\frac{|x(T) - x_0|}{\max_{[0,T]} |x(t) - x_0|} = \alpha.$$

If the orbit is stationary, that is, $x(t) \equiv x_0$, we define $\alpha = 0$.

We note that the condition is only for time duration $[0, T]$ and a periodic orbit is an almost periodic with accuracy 0. It is convenient for analysis of UPOs to be able to handle APOs since, in practice, UPOs are numerically searched for and closed UPOs cannot be expected.

Assumption 2. There exists an almost periodic orbit with period T and sufficient accuracy $\alpha \ll 1$ in the system (1) with $u = 0$ for an initial state x_0 satisfying $f(x_0) \neq 0$. —

The problem we wish to solve is the following.

Problem. (Stabilization of an APO). When the system (1) with $u = 0$ has an APO with accuracy $\alpha \ll 1$ and period T for an initial state x_0, design a control law u such that the following holds; the controlled system satisfies $x(iT) \in U$ (for all $i \in \mathbb{N}$) for a bounded neighborhood U of x_0 and the control makes $\min_{i\in\mathbb{N}} \alpha_i$ as small as possible, where,

$$\alpha_i := \frac{|x(iT) - x_0|}{\max_{[iT,(i+1)T]} |x(t) - x_0|} \quad \text{for } i \in \mathbb{N}.$$

We also require that if the APO is actually a periodic orbit, then u must stabilize it in the usual sense, that is, the neighborhood U can be taken arbitrarily small. In addition, when a control law is designed, obtain an estimate of the smallest radius of possible U. —

We note that the stability of our problem is not in the usual sense of the standard stability theory, because an APO is not necessarily a closed orbit and we cannot expect the standard stability of the orbit.

The purpose of this paper is to design an OGY-type controller for the above problem.

Definition 3. (OGY-type control). We call a controller for continuous-time systems OGY-type when it is zero-order held, that is, during the interval $[iT, (i+1)T]$ input is constant $u(iT)$. The control scheme that has this structure will be called continuous-time OGY control.

3. STABILIZATION OF APOS USING FLOQUET THEORY

We design an OGY-type stabilizing control in two steps. First, we construct a nominal periodic orbit around which linear stabilization is carried out using Floquet theory of linear periodic systems. Next, we analyze the stability of the controlled APO, in other words, an estimate of the error between the nominal and controlled orbits will be obtained based on the Lyapunov stability theory.

3.1 Construction of a nominal orbit and linear stabilization

By Assumption 2, let $x(t)$ $(0 \leqslant t \leqslant T)$ be an APO of (1). We extend the domain of the orbit to $[0, \infty)$ by repeating the value of $x(t)$ in $[0, T]$ over $t \geqslant T$ and name it x^* to get a nominal periodic orbit that is defined for all $t \geqslant 0$. That is,

$$x^*(t) = \begin{cases} x(t) & (0 \leqslant t < T) \\ x(t-T) & (T \leqslant t < 2T) \\ \cdots & \cdots \\ x(t-nT) & (nT \leqslant t < (n+1)T) \\ \cdots & \cdots \end{cases} \tag{2}$$

Note that $x^*(t)$ is not necessarily continuous.

We next linearize (1) around $x^*(t)$. Let $y(t) \in \mathbb{R}^n$ be an error from the nominal orbit; $x(t) = x^*(t) + y(t)$. Then, the first order equation for y is

$$\begin{aligned} \dot{y}(t) =& A(t)y(t) + B(t)u(t), \qquad (3)\\ A(t) =& \frac{\partial f}{\partial x}(x^*(t)) \in \mathbb{R}^{n\times n}, \\ B(t) =& G(x^*(t)) \in \mathbb{R}^{n\times m} \end{aligned}$$

Equation (3) is a linear periodic equation that is not necessarily continuous in t. For stabilization of (1), it is necessary to stabilize (3), and therefore we design an OGY controller for (3).

The homogeneous equation of (3)

$$\dot{y}(t) = A(t)y(t)$$

has a fundamental matrix $\Phi(t)$, $\Phi(0) = I$. Using $\Phi(T)$, which is called the Monodromy matrix, we can construct a discrete-time equation of (3)

$$y_{k+1} = \Phi(T)y_k + B(T)u_k, \tag{4}$$

where y_k and u_k are the state and control at time kT, respectively. Equation (4) corresponds to a linearized system in the OGY scheme, the main difference is that the system in the OGY is derived from a discrete-time system but (4) is from a continuous-time system. Therefore, our pole placement feedback must be the one that guarantees the stability of (3). To this end, Floquet theory will be employed(Coddington and Levinson, 1955; Kabamba, 1986). It should be also noted that unlike existing techniques that use the Poincaré section for discretization, we have discretized the system with period T. A similar approach was taken to get a linearized equation in Epureanu and Dowell (2000) without stability argument.

Proposition 4. The necessary and sufficient condition for (3) to be stabilized by continuous-time OGY state feedback is that the pair

$$\left(\Phi(T),\ \int_0^T \Phi(T)\Phi^{-1}(\tau)B(\tau)d\tau\right)$$

is a stabilizable pair.

Proof. (Outline) The closed loop system of (3) with state feedback of the OGY-form

$$u(t) = Ky(iT),\ K \in \mathbb{R}^{m\times n} \quad (i = 0, 1, 2, \cdots)$$

is

$$\dot{y}(t) = A(t)y(t) + B(t)Ky(iT). \tag{5}$$

It can be shown that the Monodromy matrix of (5) is

$$\Theta(T) = \int_0^T \Phi(T)\Phi^{-1}(\tau)B(\tau)d\tau K + \Phi(T).$$

Therefore, from Floquet theory, stability of (5) is equivalent to the fact that the eigenvalues of $\Theta(T)$ lie inside the unit circle. This completes the proof. □

3.2 Stability analysis — estimation of the error bound

In this subsection, we show that the stabilizing control for (3) in Proposition 4 is a solution to the problem in §2, that is, we show that under state feedback in Proposition 4, the orbit of the nonlinear system (1) stays close to the nominal orbit in the previous subsection. To achieve this end, we shall clarify the following.

The nominal orbit used in the design of the state feedback is periodic but not necessarily continuous and it does not satisfy (1) for $[T, \infty)$. Also the matrix $A(t)$ in (3) is periodic but not necessarily continuous. The effect of these discontinuities need to be examined in order not to cause problems during control process. Moreover, in the previous subsection, higher order terms of the system (1) are neglected. Since the controlled system is not a standard nonlinear system due to the above reason, a careful analysis should be given to see the behavior of controlled orbit.

We first show the following.

Proposition 5. Let $x^*(t)$ $(t \geqslant 0)$ be the nominal orbit (2) with the discrepancy $\Delta := x(T) - x_0$. Let $x_u(t)$ denote a solution of (1) under an arbitrary input u with initial state $x_u(0)$. Then, the error $y = x_u - x^*$ satisfies

$$\dot{y} = A(t)y + B(t)u + g'(t, u, y) + \Delta \sum_{i=1}^{\infty} \delta(t - iT) \tag{6a}$$

$$g'(t, u, y) = G(x^* + y)u - G(x^*)u + O(|y|^2), \tag{6b}$$

where $A(t)$ and $B(t)$ are defined in (3) and δ is the Dirac's delta function. If, moreover, the state feedback $u(t, y)$ that is piecewise continuous in t and smooth in y satisfying $u(t,0) = 0$ are assumed, then y satisfies

$$\dot{y} = A(t)y + B(t)u + g(t,y) + \Delta \sum_{i=1}^{\infty} \delta(t - iT) \tag{7a}$$

$$g(t,y) = O(|y|^2). \tag{7b}$$

Lemma 6. If the condition in Proposition 4 is satisfied, there exists a Lyapunov function $V(t,y)$ for the closed loop system (5) such that for suitable positive constants a_1, a_2, a_3, the following inequalities hold.

$$a_1|x|^2 \leqslant V(t,x) \leqslant a_2|x|^2 \tag{8}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x) \leqslant -|x|^2 \tag{9}$$

$$\left|\frac{\partial V}{\partial t}\right| \leqslant a_3|x| \tag{10}$$

Theorem 7. Let $D_r = \{y \in \mathbb{R}^n \,|\, |y| < r\}$. Assume that the condition of Proposition 4 holds true. Assume also that $g(t,y)$ in (7b) satisfies

$$|g(t,y)| \leqslant d \quad \text{on} \quad [0,\infty) \times D_r \tag{11}$$

for a positive constant d. If a constant μ satisfies

$$a_3 d + \frac{a_2 a_3}{a_1} \frac{\Delta}{T} < \mu < \sqrt{\frac{a_1}{a_2}} r \tag{12}$$

for the positive constants a_1, a_2 and a_3 in Lemma 6, then, under the OGY-state feedback in Proposition 4 with initial state $|y(0)| \leqslant \sqrt{a_1/a_2} r$, there exists a $t_1 \geqslant 0$ such that

$$|y(t)| \leqslant |y(0)| \sqrt{\frac{a_2}{a_1}} \times \exp\left\{-\frac{1}{2}\left(\frac{1}{a_2} - \frac{a_3}{\mu a_2} - \frac{a_3}{\mu a_1}\frac{\Delta}{T}\right) t\right\} \quad \text{for} \quad t_1 > t \geqslant 0,$$

$$|y(t)| \leqslant \sqrt{\frac{a_2}{a_1}} \mu \quad \text{for} \quad t \geqslant t_1.$$

Proof. We calculate the derivative of $V(t,y)$ in Lemma 6 along the closed loop system (7a) with the OGY-state feedback $u = Ky(iT)$. Using (9),

$$\begin{aligned}\dot{V} &= -|y|^2 + \frac{\partial V}{\partial y} g(t,y) + \Delta \frac{\partial V}{\partial y} \sum_{i=1}^{\infty} \delta(t - iT) \\ &\leqslant -|y|^2 + a_3|y||g(t,y)| + a_3\Delta|y| \sum_{i=1}^{\infty} \delta(t - iT),\end{aligned}$$

where we have used (10). Limiting y in $D_r \cap \{y \in \mathbb{R}^n \,|\, |y| \geqslant \mu\}$ and using (11), we have

$$\dot{V} \leqslant -\left(1 - \frac{a_3 d}{\mu}\right)|y|^2 + \frac{a_3\Delta}{\mu} \sum_{i=1}^{\infty} \delta(t - iT).$$

Since $1 - \frac{a_3 d}{\mu} > 0$ by assumption (12), we use (8) to get

$$\dot{V} \leqslant -\left(\frac{1}{a_2} - \frac{a_3 d}{a_2 \mu} - \frac{a_3 \Delta}{a_1 \mu} \sum_{i=1}^{\infty} \delta(t - iT)\right) V(t,y).$$

By integrating the above inequality over $[0,t]$, $t = t' + iT$ $(0 \leqslant t < T)$, we get

$$\begin{aligned}&V(t,y(t)) \\ &\leqslant V(0,y(0)) \exp\left\{-\left(\frac{1}{a_2} - \frac{a_3 d}{a_2 \mu} - \frac{a_3 \Delta}{a_1 \mu} i\right)\right\} \\ &\leqslant V(0,y(0)) \exp\left\{-\left(\frac{1}{a_2} - \frac{a_3 d}{a_2 \mu} - \frac{a_3}{a_1 \mu}\frac{\Delta}{T}\right) t\right\},\end{aligned}$$

which is valid as long as $|y(t)| \geqslant \mu$. Since the exponential function on the right side is decreasing by assumption (12), if $|y(t)| \geqslant \mu$ for all $t \geqslant 0$, we have a contradiction. Thus, there exists a $t_1 \geqslant 0$ such that $|y(t_1)| = \mu$, for which the first estimate of the theorem follows. Since $\dot{V}(t_1, y(t_1)) \leqslant 0$, $V(t,y(t)) \leqslant a_2\mu^2$ for all $t \geqslant t_1$ from (8). Therefore, we get $|y(t)| \leqslant \sqrt{a_2/a_1}\mu$ for $t \geqslant t_1$ from (8).□

4. NOMINAL ORBIT SELECTION WITH THE MAXIMUM PRINCIPLE

In general, it is very difficult to find UPOs in nonlinear systems as closed orbits. Therefore, we have proposed a stabilization method around APOs with accuracy $\alpha \ll 1$ by the OGY method. Since our purpose is the stabilization of an UPO, it may be possible to find better APOs for feedback design by modifying them with small control inputs. That is, the stabilization around an APO thus obtained may require less control effort.

From Assumption 2, the solution of (1) with initial state x_0 returns closely to x_0 after time T. This implies that it may be possible to get an orbit $x(t)$ of (1) with smaller $\Delta = |x(T) - x_0|$ under small control inputs. The standard method to do this is the maximum principle by Pontryagin. Let us consider the following cost functional

$$J = (x(T)-x_0)^T Q (x(T)-x_0) + \int_0^T u(t)^T R u(t) dt,$$

with positive definite matrices Q and $R > 0$. The optimal trajectory x minimizing J is given by a solution of the following two-point boundary condition problem

$$\dot{x} = f(x) + G(x)u^*(x,p) \tag{13}$$

$$\dot{p} = -\frac{\partial^T f}{\partial x}(x(t))p - \left.\frac{\partial^T G(x)u}{\partial x} p\right|_{u=u^*(x,p)}$$

$$x(0) = x_0, \quad p(T) = 2Q(x(T) - x_0),$$

and the optimal input is given by

$$u^*(x,p) = -\frac{1}{2} R^{-1} G^T(x) p.$$

If the initial state x_0 gives an APO with $\alpha \ll 1$, then, choosing $Q \gg R$, the optimal control problem will give a small control input that renders Δ

smaller and the optimal trajectory, the solution of (13), can be used as a nominal orbit $x^*(t)$ in the OGY stabilization design in §3.1. When a nominal orbit is obtained using the optimal input u^*, we change $g'(t,u,y)$ and $g(t,y)$ in (6b) and (7b) to

$$\begin{aligned}\tilde{g}'(t,u,y) =& G(x^*+y)u - G(x^*)u - G(x^*)u^* \\ &+ O(|y|^2) \\ \tilde{g}(t,y) =& -G(x^*)u^* + O(|y|^2),\end{aligned}$$

respectively. In the following section, we will show a case in which a modified nominal orbit with the maximal principle gives the OGY-stabilization with smaller control inputs than the nominal orbit that naturally exists in the given nonlinear dynamics $\dot{x} = f(x)$ (see, Fig. 3). Theorem 7 shows that it can happen when the input u^* is relatively small compared to the reduction of Δ by u^*.

The control input u^* by the maximum principle is also efficient as *a feedforward control* although stability cannot be guaranteed and robustness for noise and parameter variations is poor.

5. AN APPLICATION RESULT

To verify the effectiveness of the proposed method of stabilization and selection of a nominal orbit, we show an application result for the three-body problem known as halo orbit.

Halo orbits are particular solutions of the circular restricted three-body problem and of great interest for space mission programs. For example, a halo orbit about Sun-Earth L1 point was selected in the Genesis mission as the platform for collecting solar wind samples. However, since halo orbits are unstable, stationkeeping control is required. A number of research papers have been published on stationkeeping control including approximation methods of halo orbits(Dunham and Roberts, 2001; Richardson, 1980; Wiesel and Shelton, 1983).

In this section, we illustrate that it is possible to design an autonomous stabilization control for stationkeeping in halo orbits.

The equation of the circular restricted three-body problem with control inputs is

$$\begin{cases} \ddot{x} - 2\dot{y} - x = -\dfrac{\partial U}{\partial x} + u_x \\ \ddot{y} + 2\dot{x} - y = -\dfrac{\partial U}{\partial y} + u_y \\ \ddot{z} = -\dfrac{\partial U}{\partial z} + u_z, \end{cases}$$

$$U(x,y,x) = -\frac{1-\mu}{\sqrt{(x+\mu)^2+y^2+z^2}} - \frac{\mu}{\sqrt{(x-1+\mu)^2+y^2+z^2}}.$$

The initial values below give rise to a halo orbit with period $T = 3.0597$(Simó *et al.*, 1987) and

$$\mu = 3.040357143 \times 10^{-6} \quad y(0) = \dot{x}(0) = \dot{z}(0) = 0,$$

$$\begin{pmatrix} x(0) \\ z(0) \\ \dot{y}(0) \end{pmatrix} = \begin{pmatrix} 0.988836754421 \\ 0.000783384715 \\ 0.008932446653 \end{pmatrix}.$$

We chose a nominal orbit using the maximum principle. For a cost function with $Q = 10^{18}I$, $R = I$, we have obtained a nominal orbit with the error of the initial and terminal states

$$x_0 - x^*(T) = \big(0,\ 6.78 \times 10^{-21},\ 0,\ -2.03 \times 10^{-20},\ 0,\ 3.38 \times 10^{-21}\big)^T.$$

The Monodromy matrix is calculated by solving the following differential equation.

$$\dot{\Phi}(t) = A(t)\Phi(t), \quad \Phi(0) = I_6$$

where
$$A(t) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -U_{11} & -U_{12} & -U_{13} & 0 & 2 & 0 \\ -U_{21} & -U_{22} & -U_{23} & -2 & 0 & 0 \\ -U_{31} & -U_{32} & -U_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$U_{ij} = \left.\frac{\partial^2 U}{\partial x_i \partial x_j}\right|_{x=x^*}, \quad i,j = 1,2,3.$$

The poles of $\Phi(T)$ are (1740, 5.76e-004, 0.997±7.475e-002i, 1.000±2.73e-005i), indicating the instability of the nominal orbit. From

$$\int_0^T \Phi(T)\Phi^{-1}(\tau)Bd\tau = \big[\, 188.9\ -63.8\ -4.3\ 433.8\ -202.4\ -23.8 \,\big]^T,$$

the pair $\left(\Phi(T),\ \int_0^T \Phi(T)\Phi^{-1}(\tau)B(\tau)d\tau\right)$ in Proposition 4 is stabilizable. We designed K so that the closed loop poles are placed at (5.76e-004, 0.499±3.74e-002i, 0.500±1.81e-004i, 1.00). In the case of the halo orbit stabilization, we do not control the motion of the z direction because it is known to be stable.

Fig. 1 and Fig. 2 are the uncontrolled motion for time duration $4T$ and the closed loop motion for $30T$, respectively, showing that the stabilization is successful. Also, Fig. 3 shows the comparison result of inputs. The plots of $\circ$ and $*$ are the values of inputs designed by the nominal orbits selected with and without using the maximum principle, respectively. Both inputs stabilize the nominal orbits, but the value of input designed with the maximum principle is smaller ($* = 1.1211 \times 10^{-9}$ and $\circ = 1.8890 \times 10^{-10}$ at $t = 30T$). This shows that the nominal orbit modified by the maximum principle is closer to the natural equilibrium orbit and therefore, the control effort to shift the unstable poles inside the unit circle is less.

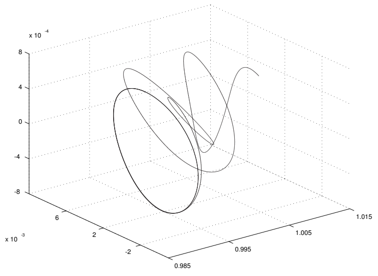

Fig. 1. unstable halo orbit for $[0, 4T]$

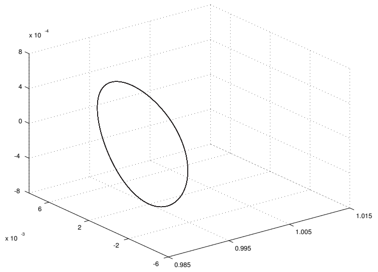

Fig. 2. stabilized orbit for $[0, 30T]$

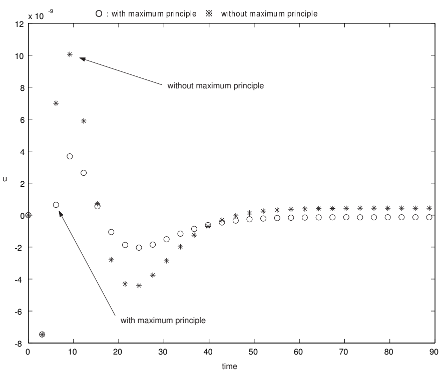

Fig. 3. comparison of inputs

6. CONCLUSIONS

In this paper, we have attempted to generalize the OGY-scheme of controlling chaotic nonlinear discrete-time systems to continuous-time systems. Using a linearized equation around an UPO and its Monodromy matrix, we discretize the system and construct OGY control. In contrast to the existing method to apply the OGY control to continuous-time systems, we do not use the Poincaré section for discretization and gave an analytical proof for stability, in the sense of continuous-time systems, of the closed loop system.

It is difficult, in general, to find exact closed UPOs and our approach does not require them. Instead, we proposed a method of stabilizing almost periodic orbits based on Floquet theory and finding better APOs for control using the maximum principle in control theory. As a matter of fact, it is impossible without exact closed orbits to asymptotically stabilize the system around UPOs. In section §3.2, we gave an analysis to estimate the bound of the controlled behavior using Lyapunov stability theory. Also, the proposed framework provides a design procedure of stabilizing control for a given target orbit, whereas the delayed feedback scheme is not well-developed as a design technique. The example of halo orbit stabilization shows the effectiveness of the proposed method, which is an example of higher dimensional systems that are numerically difficult to stabilize by the standard OGY-control using the Poincaré section.

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