COMPUTER-GENERATED EXAMS [1]

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Abstract: In this work, we present a practical way to generate individual exercises for students. To this end, a generic problem is modified by randomly selecting the unknowns and parameter values. This is implemented in two ways: one is to develop a program that generates the exam as a PDF file, and the other one is to profit of a freely available software to examine the students via internet. Several examples are presented to show how this idea can be applied effectively in control education, although it can be applied in other areas of knowledge.

Keywords: Educational aids, Computer-aided instruction, Control education, Frequency response.

1. INTRODUCTION

The most important part of the teaching process is to ensure that the student has acquired the required knowledge, and is able to apply it. To this end, it is a common practice to challenge the students with problems given as assignments, and measure their degree of mastering of the material by means of exams.

In the traditional way, all the students are challenged with the same set of problems. However, this has many drawbacks, the main one being that students will tend to copy, so the grades will not necessarily reflect their degree of competence.

The ideal situation would be to give different sets of problems to each student. However, the preparation and evaluation normally imply too much work for the instructor, so this option would rarely be considered.

In this paper, we show that this option can be applied in a practical way. This main idea is to design a *generic problem* and then modify it by either

- randomly choosing values for some of the parameters, or
- randomly *varying* the problem (e.g. by modifying the list of variables to be considered as unknowns).

This approach has been used to develop a software denominated WIMS [WIMS4], which also may ask for the answer and grade it, all of this via internet.

This system allows the student to practice as many times as she/he wishes, and to obtain his/her grade at the end of an evaluation. Nevertheless, this approach displays some drawbacks. The most important is that it does not allow to see the exposition of the problem and its development by the student, which can be important for the professor. Also, not all students may have access to computers connected to the internet (this is particularly true in countries under development). For these reasons, in some cases it is preferable to have a written version of the exams.

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The purpose of this document is to present two ways to generate exams and/or assignments using a personal computer. The considered alternatives are:

- using the WIMS software to present and grade control problems
- using a computer-algebra system (CAS) to generate printable documents and their corresponding answer sheet.

The material appears in three parts. In the first one, the proposed approach is explained. It is illustrated by the development of an examination on the analysis of linear systems. In the second part, the process to develop a program to generate the examinations is detailed. We show an example of typical examination with the corresponding answers sheet. Finally, we present some examples on the use of WIMS for control teaching.

2. PROPOSED APPROACH

In this section some guidelines to develop programs to generate random examinations are presented. To facilitate their understanding, a hypothetical examination on linear systems will be used as example.

Basically, the approach consists on the following steps:

(1) Define the problems to solve, leaving some data as parameters, and establishing the ranges of acceptable values for each parameter.
(2) Based om this information, write a program that generates a set of random parameter values and writes them in a particular format.
(3) Generate the examinations using the typesetting program LaTeX [LAT04]

Now these steps are detailed.

2.1 Development of generic problems

2.1.1. Design The design of good generic problems is a work that requires good knowledge of the subject and some creativity.

The first step is to define in general terms an exercise that forces the student to show its degree of understanding of the subject. As an example, consider the following problems.

P1 Given the Bode plot of a system, find the transfer function that generates it.
P2 Given the matrices A, B, C of a linear system, calculate its controllability and observability matrices, and determine if it is controlable and/or observable.

The first problem requires a good degree of conceptual knowledge. The second one, only requires to follow a procedure.

2.1.2. Parametric ranks The next step consists in determining suitable parameter ranges. This is one of the most delicate parts of the design process, since a bad choice of them can lead to generate problems with illogical values (e.g. negative resistances or frequencies), or in which is impossible to generate a correct answer.

For example, in problem P1, three restrictions were considered:

(1) The range of the graph must include the cut-off frequencies.
(2) The poles must be easily visible in the graph.
(3) The range for the poles must be sufficiently wide to generate an important number of different problems, and sufficiently small so the scale is not too small to properly distinguish the poles.

After some tests, it was decided to challenge the students with second-order systems, whose poles would be of the form $p_i = a\,10^b$, with a, b integers, $a \in [1, 9]$, $b \in [-2, 2]$. These restrictions allow to reasonably distinguish the poles in the graphs, simultaneously that obtain a good number of possible different examinations ($(9 \cdot 5)^2 = 2025$).

2.2 Programming

2.2.1. Selection of packages. Once the parametric problem and parameter ranges are defined, it one can proceed to programming it. In order to present the problem P1 to the student, is required to generate semilogarithmic plots, whereas for the P2 problem is necessary to display mathematical equations. In all the cases, it is necessary to generate random values for the parameters.

In this work, we used three packages (under the operating system Linux):

MAXIMA 5.9.0 Package of symbolic computations [MAX04] used to generate the random parameters. It has an own language of programming, that includes the possibility of calling to other programs, and saving its output directly in LaTeXformat. For these reasons it was used to write the general program.
GNUPLOT 3.7 Plotting program [GNU04]. It is called by the main program to generate graphs in a suitable format to be processed by LaTeX.
LaTeX 2$_\varepsilon$ Document processor. It takes the information generated by MAXIMA and GNUPLOT and generates a file in PDF (Portable

Document Format) with the questions, and another one with the answers.

The interaction of these packages is depicted in the following diagram.

$$\begin{array}{ccccccc}
\textbf{maxima} & \longrightarrow & \texttt{*.tex} & \longrightarrow & \text{\LaTeX} & \longrightarrow & \texttt{*.pdf} \\
\downarrow & & & & \uparrow & & \\
\texttt{*.out} & \longrightarrow & \textbf{gnuplot} & \longrightarrow & \texttt{*.eps} & &
\end{array}$$

All this software is freely available for different platforms (windows, mac and linux, among others). Altough commercial software can also be considered, an advantage of the choice of free software is that it is possible to provide to the students copies of this software and the developed programs so that they practice in their own computers if they wish so.

3. PROBLEM GENERATION USING WIMS

In this section we present how to use the WIMS software to generate control problems. This software is nothing but a web interface to several packages with different capabilities (e.g. numeric, symbolic, plotting) that allows the creation of exercises, examinations and even virtual classrooms, where the students can consult their grades, use discussion forums, and so on. More details can be found in Gang (1999).

To show how this software can be used in control teaching, two simple exercises were developed. Following the procedure mentioned before, we start by defining the *generic* problems. In this case, we will consider the following one:

P3 Given a transfer function, identify its corresponding Bode plot among several possibilities.

Figure 1 show an actual implementation of this problem in the WIMS system. The program, which is called *graphic Bode*, generates a random transfer function (corresponding to a second-order system) and four Bode plots (magnitude only). Among these, only one corresponds to the transfer function. The student must identify it and click on it. After this, the system verifies if the answer is right and grades it accordingly. The interested reader may try it online at `http:\\control.cicese.mx\wims` (keyword: bode).

4. DISCUSSION

In this paper we have presented two alternatives to generate individual assignments for students. Both are based on modifying a generic problem by randomly varying its parameters and/or unknowns. Their main difference is that one is well suited to present the problems on a PC via internet, while the other generates printable documents which are graded in a traditional way, except for the answer sheet, which gives the individual answers corresponding to each candidate.

The main advantages of using the WIMS option are

- The grades have a good *reliability* Gang (2004)
- The student can practice as many times she/he wishes.
- Besides that the WIMS software is actively being improved, since most of the capabilities rely on third-party software, its capabilities increase with them.
- It is possible to generate interactive documents with it, in which the text may include examples that may be changed on-line.
- The students receive their grades by the end of the session
- Knowing that a problem will be used in an exam motivates the students to learn the *methodology* to solve it.

The main advantages of the printed version are that

- the supervisor is able to evaluate not only the answer, but the procedure to obtain it as well.
- cheating in examinations becomes very hard, since copying is no longer possible.
- The possibilities of communication between students are reduced with respect to the computer version (due to instant messenger program).
- The supervisor can control the material used for the test, while in the computer version the student can have access to electronic books and additional software.

Although the initial effort on the part of the instructor is greater under this scheme, this is compensated with the advantages of having an almost endless source of problems, along with their solutions. In addition, the professor can limit himself to the conceptual development of the exercises, interact with students for the selection of parametric ranks, leaving them the programming task. In addition, the exercises designed for WIMS can be modified to generate a printed version and viceversa.

We have been using both techniques in the last three years in lectures of linear algebra, optimization, and control theory. According to our experience, we found the following benefits.

For students: it is a source of solved problems for training. In addition, the online version

- grades them in real time,
- motivates them to work individually.

For educators:

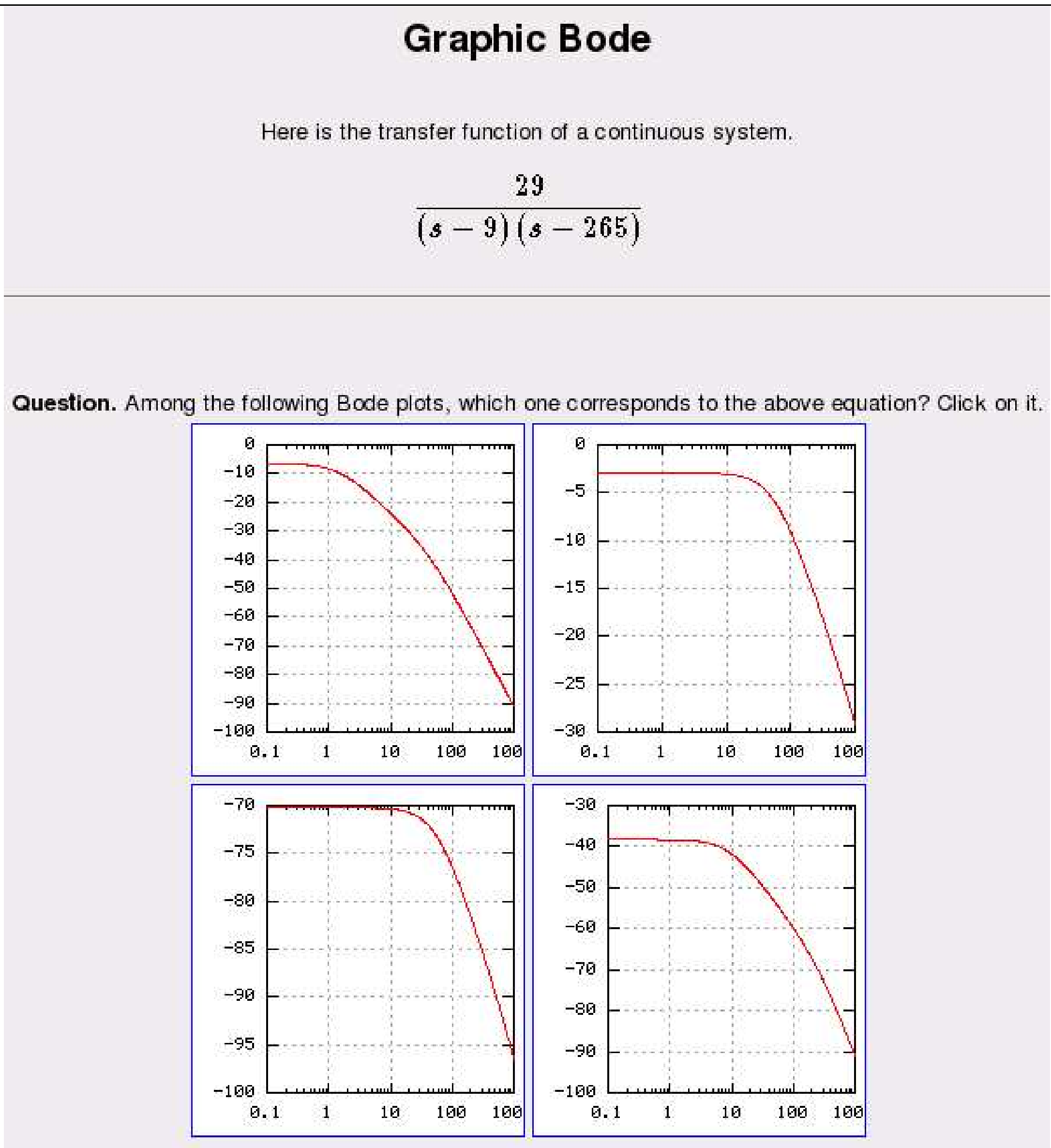

Fig. 1. Screenshot of one computer generated exercise using wims.

- online version
  - · can be used as a teaching tool,
  - · reduces the problem of cheating in exams,
  - · provides extensive statistical data regarding grades and solving times.
  - · easy to use.
- printed version
  - · eliminates cheating while applying exams in a more conventional way.

The main drawback of the printed version is that its use requieres a certain level of computer skills and more work to put different exercises together in an exam.

With the online version the only problem detected so far is that with some exercises students try to figure out a rule to obtain the right answer rather than applying the theory to find it out. For this reason we have chosen to do not give the right answer in case of failure (which is an option of the WIMS system).

Programming under WIMS may be more difficult depending on the desired exercise. However, for the teacher as an end user it is much simpler to use than the printed version approach.

Finally, it is worth to mention that the same approaches can be used in other areas of the knowledge as engineering, physics, mathematics, biology, chemistry and economy, to mention some.

Appendix A. EXAMPLE OF COMPUTER-GENERATED EXAMS

To illustrate the random variations that can be obtained, in this section we include two different exams that were generated using the program described in this article, as well as the answer

sheet. The interested reader can download the source code from
`http://control.cicese.mx/downloads/genexos`

Linear Control Exam (**E.1**)

(1) Consider the system given by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & 1 \end{pmatrix}$$

Find the controllability and observability matrices. Indicate if the system is controlable and/or observable.

(2) In figure A.1 have Bode diagrams of the magnitude and phase, respectively. Find the transfer function and the poles of the system.

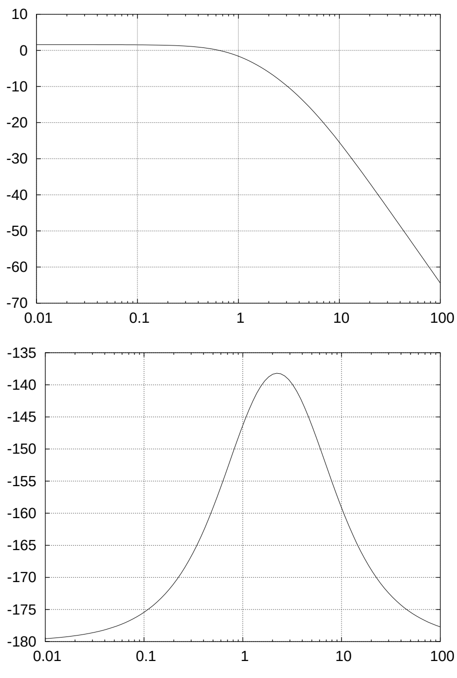

Fig. A.1. Magnitude and phase.

Linear Control Exam (**E.2**)

(1) Consider the system given by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where

$$A = \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Find the controllability and observability matrices. Indicate if the system is controlable and/or observable.

(2) In figure A.2 have Bode diagrams of the magnitude and phase, respectively. Find the transfer function and the poles of the system.

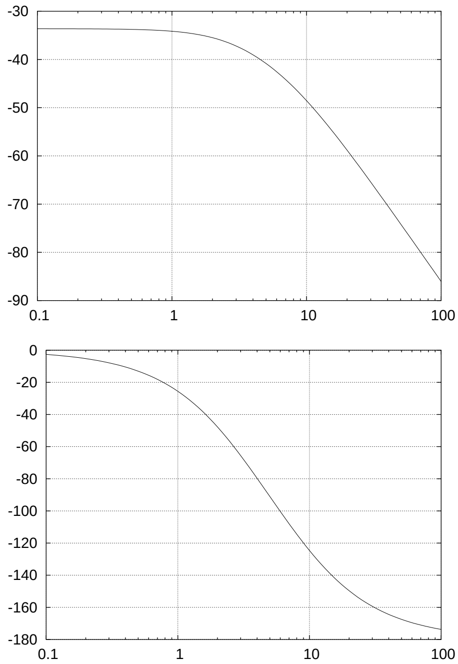

Fig. A.2. Magnitude and phase.

Solution for **E.1**

(1) Controllability matrix :

$$\begin{pmatrix} -1 & -1 \\ -1 & 0 \end{pmatrix}$$

rank:

$$2$$

Observability matrix:

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$$

rank:

$$2$$

(2) **Transfer function and poles:**

$$f(s) = -\frac{6}{s^2 + 4\,s - 5}$$

$$f(s) = -\frac{6}{(s-1)\,(s+5)}$$

Solution for **E.2**

(1) Controllability matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

rank:

$$2$$

Observability matrix:

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

rank:

$$2$$

(2) **Transfer function and poles:**

$$f(s) = \frac{0.5}{s^2 + 11\,s + 24}$$

$$f(s) = \frac{0.5}{(s+3)\;(s+8)}$$

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