

A MODIFIED OPTIMAL VELOCITY MODEL FOR VEHICLE FOLLOWING

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Abstract: This paper presents a modified optimal velocity model (MOVM) for vehicle following. It makes use of time to collision (TTC) in order to reduce the reference speed when relative speed against the leading vehicle is negative. Simulation results have shown an anticipation behavior which leads to smoother maneuvers. The driver risk perception of rear-end collision is also enhanced.
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1. INTRODUCTION

Microscopic traffic models use different approaches for car following in order to describe the dynamics of individual vehicles with interaction with neighboring vehicles (Pipes, 1953; Chowdhury, *et al.*, 2000; Helbing, 2001; Nagel, *et al.*, 2003). Most of them are based on the fact that each driver reacts with a certain sensitivity to a stimulus via the acceleration input after a specific time delay. The stimulus may be a change in the relative speed or the driver visual angle. Others models use safety distance or collision avoidance based

approaches and Psycho-spacing or Neural-Fuzzy (Rekersbrink, 1994).

Few years ago, Bando, *et al.*, (1995) modified the model by making the driver desired speed called "optimal velocity model" (OVM) as a function of the spacing with the preceding vehicle. With few parameters, this model is able to describe and to interpret several traffic flow situations.

In this paper, we propose a modified optimal velocity model that takes into account longitudinal driver behavior. It introduces the driver perception of the risk of rear-end collision. It is a function of both the spacing and the relative speed. This

model has also the ability to perform smoother maneuver thanks to anticipation.

The remainder of the paper is organized as follows: section 2 gives a short review of longitudinal driver behavior and models. Section 3 presents the modified optimal velocity model. Simulation results are presented in section 4.

2. REVIEW OF LONGITUDINAL DRIVER BEHAVIOR AND MODELS

When controlling his car, the driver behavior changes according to traffic situation, neighboring vehicles and infrastructure characteristics. Leutzback, (1988) has first proposed a psychophysical spacing models with perceptual thresholds for situations classification. Driver's parameters such as the desired speed, safety need and reaction time are used in order to determine the drivers' level of perception for four different driving situations

- **Free driving:** This situation occurs generally in free flow traffic, the driver is uninfluenced by the others vehicles. The driver desired speed is rather constant and is determined as a compromise between need of safety (road geometry and adhesion, visibility, ...) and travel time minimization.
- **Approaching:** The relative speed is positive and the driver is closing the front vehicle. He has thus to slow down and to adjust his speed to the speed of the preceding vehicle. Meanwhile the driver leads a headway distance or time according to his desire of safety.
- **Braking:** when headway distance or time to collision is under a minimal value, the driver initiates a braking maneuver until stoppage or recovering of the desired safety level.
- **Car-following:** The driver follows the leading vehicle and tries to regulate his speed and to maintain a desired headway.

Different types of variables and threshold can be used for transition from one situation to another.

During free driving, the simplest model of how the driver tries to approach the desired speed is the use of a relaxation time τ

$$\frac{dv_f(t)}{dt} = \frac{v_{des}(t) - v_f(t)}{\tau} \quad (1)$$

For car following, Pipes model uses as desired speed the speed of the preceding vehicle (Pipes, 1953). This is motivated by the fact that speed of all vehicles are equal in steady state. Chandler, *et al.*, (1958) added a time delay Δt such that the vehicle acceleration becomes

$$\frac{dv_f(t)}{dt} = \frac{v_p(t - \Delta t) - v_f(t - \Delta t)}{\tau} \quad (2)$$

Observing that the clustering effect of traffic can not be reproduced, Gazis, *et al.*, (1959) presented several model refinements where the relaxation time is made dependant of the headway distance ($s(t) = x_p(t) - x_f(t)$) and the preceding vehicle speed. The relative speed term ($v_p(t - \Delta t) - v_f(t - \Delta t)$) is called the stimulus and the multiplicative term is the sensitivity. This final model is known as the General Motor Nonlinear (GM) model. The model expression is

$$\frac{dv_f(t)}{dt} = \alpha \frac{v_p(t)^\beta}{s(t - \Delta t)^\gamma} (\dot{s}(t - \Delta t)) \quad (3)$$

The exponents β and m are first proposed to be integers but they are now allowed to be real values as shown by May and Keller, (1967), they are however difficult to determine from real data. This equation can be integrated and a speed-density relation for homogenous flow can be obtained.

Few years ago, Bando, *et al.*, (1994) modified model (1) by making the driver desired speed called "optimal velocity" as a function of spacing with the preceding vehicle. With very few parameters, this model is able to describe and interpret many traffic flow situations (Bando, *et al.*, 1994) and (Bando, *et al.*, 1995).

$$\frac{dv_f(t)}{dt} = \frac{v_{ref}(s) - v_f(t)}{\tau} \quad (4)$$

where $v_{ref}(s)$ is the spacing-dependent optimal velocity that the driver attends to achieve with a relaxation time τ . This optimal velocity has to vanish when the spacing goes to zero and is bounded by the free speed when spacing goes to infinity. This optimal velocity can thus be chosen as an increasing but saturating function of spacing. Assuming that the flow density is locally equal to the spacing inverse, this gives a direct link to fundamental diagram. The optimal speed is generally of the form

$$v_{ref}(s) = V_1 + V_2 \tanh(C_1 s - C_2) \quad (5)$$

where parameters V_1 , V_2 , C_1 and C_2 are calibrated empirically using real data measurements.

In the following section, a enhancement of this model is proposed. Some additional requirements are added in order to enhance the braking reactivity of the model.

3. THE PROPOSED MODEL

The proposed model introduces a weighting factor of the optimal speed that depends on the ratio of the relative speed to spacing, that is the opposite of the inverse of time to collision (TTC). Time to collision has been used in previous works, Lee, (1976) and Van der Horst, (1990) proposed that the driver based his judgment for start braking

and to control braking on TTC. In our case, the objective of this weighting factor is to make the model more reactive on braking. This reactivity is based on the excess of follower speed in comparison to that of the leader. It also modulates this reactivity according to the actual spacing with the leading vehicle. This factor has to fulfill the following requirements

- it should maintain the reference speed v_{ref} unchanged when the relative speed is positive
- it has to be decreasing for negative decreasing relative speed and has to go toward zero when the relative speed goes to $-\infty$.

The optimal velocity equation (5) is thus changed to $v_{ref}^{new}(s, \dot{s})$ given by

$$v_{ref}^{new}(s, \dot{s}) = v_{ref}(s) \times w(s, \dot{s}) \quad (6)$$

where the weighting factor is set to

$$w(s, \dot{s}) = \left[A \left(1 + \tanh B \left(\frac{\dot{s}}{s} + C \right) \right) \right] \quad (7)$$

Obviously, the parameter A is necessarily equal to $\frac{1}{2}$, this can be obtained by the limit $(\frac{\dot{s}}{s} \rightarrow +\infty)$. The first requirement leads to the condition

$$\tanh(BC) = 1 \quad (8)$$

The second requirement is fulfilled assuming that B is positive. Defining a minimum time to collision value T_{min} under which the introduced factor is near zero gives a supplementary equation that permits in combination with equation (8) to determine uniquely parameters B and C . This equation is

$$\frac{1}{2} \left(1 + \tanh B \left(\frac{-1}{T_{min}} + C \right) \right) = \varepsilon \quad (9)$$

where ε is positive small.

4. SIMULATION RESULTS

In all simulations, the parameter values used in equation (5) are adapted from Helbing and Tilch, (1998), where it was established that for a city traffic $V_1 = 6.75\text{m/s}$, $V_2 = 7.91\text{m/s}$, $C_1 = 0.13\text{m}^{-1}$, $C_2 = 1.57$. The shape of the optimal velocity $v_{ref}(s)$ is shown on figure 1. The free speed is of 14.66m/s . We choose the relaxation time as $\tau = 1/1.8\text{s}$.

Letting $T_{min} = 1\text{s}$ and $\varepsilon = 0.0067$, we achieved $B = 5\text{s}$ and $C = \frac{1}{2}\text{s}^{-1}$. The shape of the weighting factor $w(s, \dot{s})$ for different values of the spacing $s(t)$ is shown in figure 2. The weighting increases with $\dot{s}(t)$, and becomes smoother and approaches 1 when the spacing increases. The combination of the weighting factor and the optimal velocity leads to the mesh-plot shown in figure 3.

In the presented simulation results, the vehicle with "optimal velocity model" and the suggested modified strategy are called OVM and MOVm vehicle respectively.

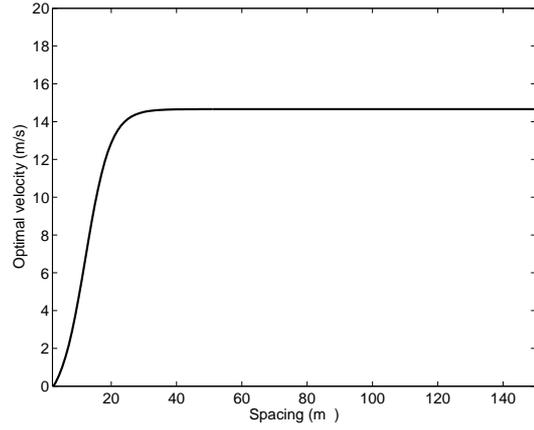


Fig. 1. Initial optimal velocity as function of spacing

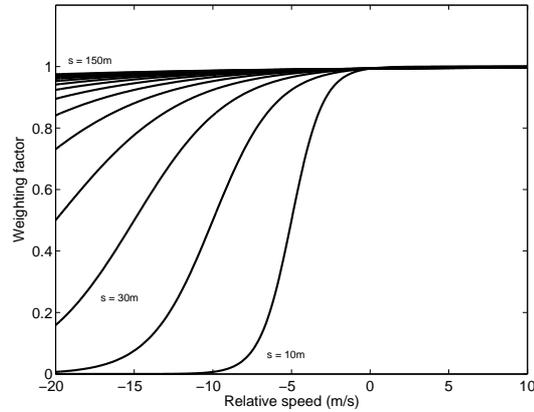


Fig. 2. Weighting factor function of relative speed and increasing spacing

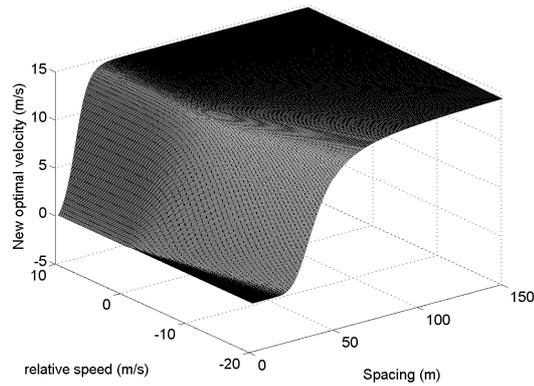


Fig. 3. Modified optimal velocity

4.1 Stopping at traffic signal

In this first simulation, the leader is stopped at a traffic signal. Initially the follower is located at a distance of 60m upstream and with a speed of 10m/s . This initial distance is chosen such as the follower vehicle first reacts by acceleration in order to reach the optimal free speed and thus it brakes a short time after in order to stop. Figure 4-a shows the time evolution of the speed of both OVM and MOVm vehicles. It

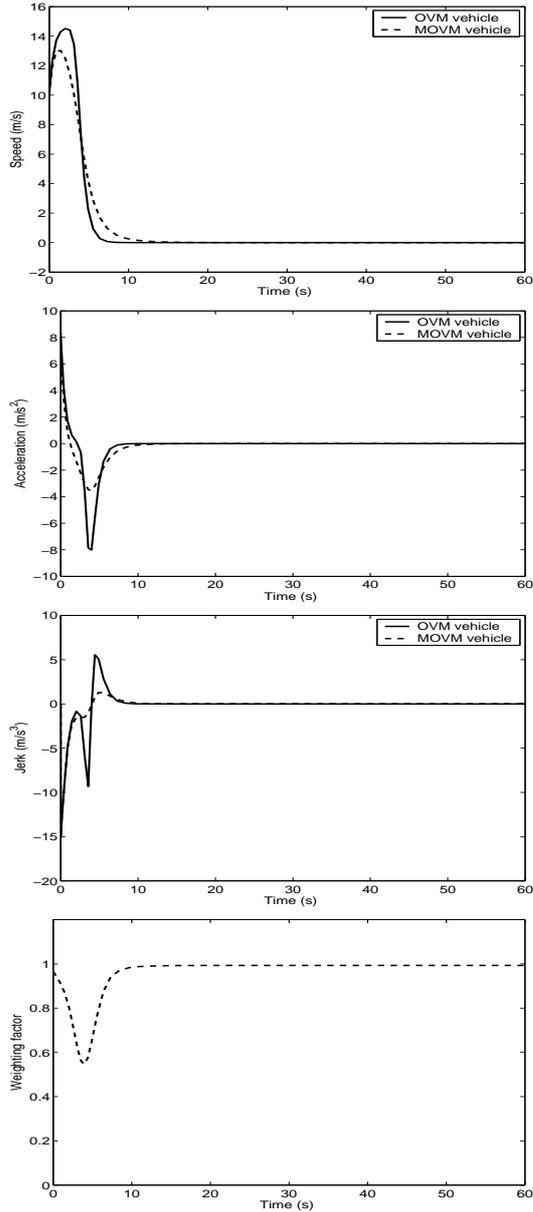


Fig. 4. Speed (a), acceleration (b), jerk (c) and weighting factor (d) for OVM and MOV vehicles when approaching a standing vehicle

can be noticed that the MOV vehicle velocity maximum is under that of the OVM vehicle. Its velocity begins to decrease before the OVM vehicle velocity reaches its maximum. This is due to the weighting factor which reduces the optimal velocity by a factor of 0.45 (figure 4-d). As argued in section 2, this model enhances the reactivity of the braking phase. In addition, the acceleration profile is smoother with smaller peak values (figure 4-b). Comfort of vehicle passengers is better preserved (figure 4-c).

The initial range is decreased to 20m while the initial speed remains unchanged, figure 5-a shows that the MOV vehicle no more accelerates while the OVM one still have an acceleration phase.

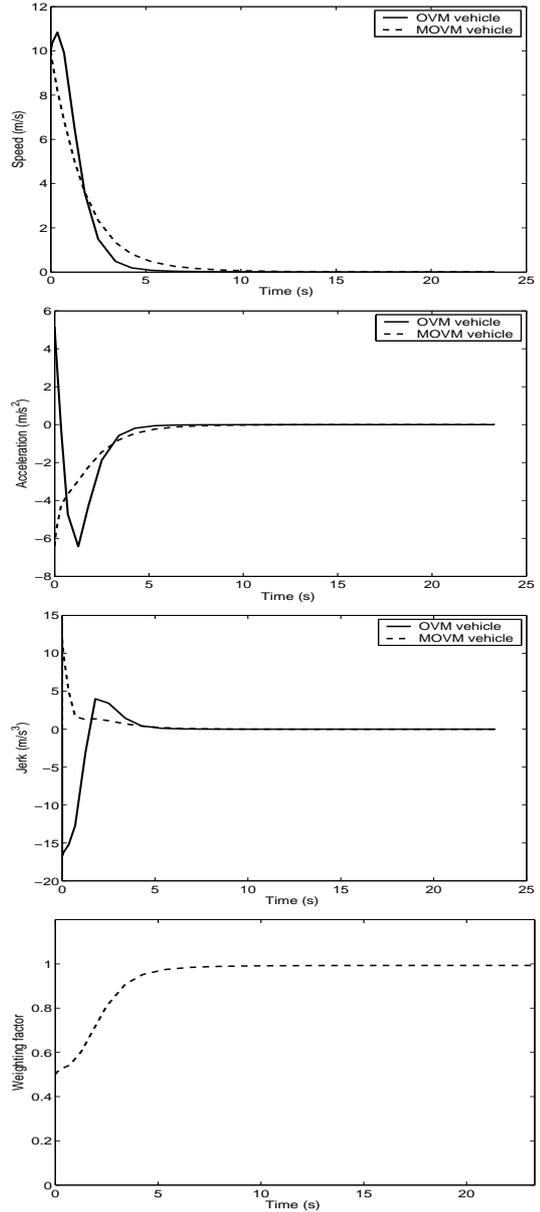


Fig. 5. Speed (a), acceleration (b), jerk (c) and weighting factor (d) for OVM and MOV vehicles when approaching a standing vehicle. Initial spacing reduced to 20m.

b and 5-c are more realistic. The weighting factor commands a speed reduction of 50% (figure 5-d).

It is also well known that a short relaxation time τ leads to an overshooting of the vehicle velocity. The modified optimal velocity model, reduces this overshoot as we can see it from figure 6 for which $\tau = 1s$ and the leading vehicle is moving at a velocity of 5m/s.

4.2 Behavior in stop-and-go traffic

Initially the vehicles are at free speed with a spacing of 40m. The leading vehicle performs an acceleration/deceleration profile given by $\dot{v}_p = -0.48 + 4 \sin(0.3t)$ until standing. The vehicle starts again

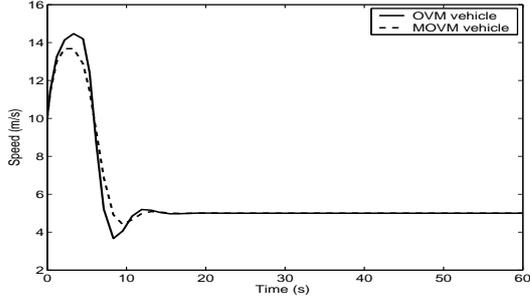


Fig. 6. Velocity of the follower for reduced relaxation time.

when the acceleration becomes positive. The maneuver is simulated during 60sec. Results are shown on figure 7. The speed, the acceleration and the jerk of MOVm vehicle are smoother and suffer less variations. This limitation is possible due to the weighting factor. The MOVm vehicle presents smoother acceleration during transition phases.

4.3 Rear-end collision risk

While TTC, defined by $\frac{-s}{\dot{s}}$, is a visual information to the driver of the time to collision, its temporal derivative is used by the driver as a visual information of the risk of future rear end collision. This visual risk is given by

$$R = -1 + \frac{s\ddot{s}}{\dot{s}^2} \quad (10)$$

First of all lets us notice that when R is zero, TTC is constant. In this case, if t_c is this constant TTC value, thus the following differential equation holds

$$\ddot{s} = \frac{-1}{t_c} \dot{s} \quad (11)$$

This means that, in order to ensure a constant time to collision, the follower has to adopt an acceleration profile such that the relative velocity exponentially converges toward zero.

On the other hand, field test and driving simulator tests results showed that a threshold on R of -0.5 can be fixed in order to know if the deceleration of the driver is sufficient or not. On the basis of several behavior of the leading vehicle, we investigate the improvement of the risk perception introduced by the new factor.

The leading vehicle is assumed to be at stand. The previous formula of R reduces to

$$R_m = -1 + \frac{s}{\tau v_f} \left(1 - \frac{v_{ref}(s)}{v_f} w(s, -v_f) \right) \quad (12)$$

The risk associated to the optimal velocity model is given by

$$R_v(s, v_f) = -1 + \frac{s}{\tau v_f} \left(1 - \frac{v_{ref}(s)}{v_f} \right) \quad (13)$$

When the speed of the follower equals the reference speed, R_m and R_v are both equal to -1.

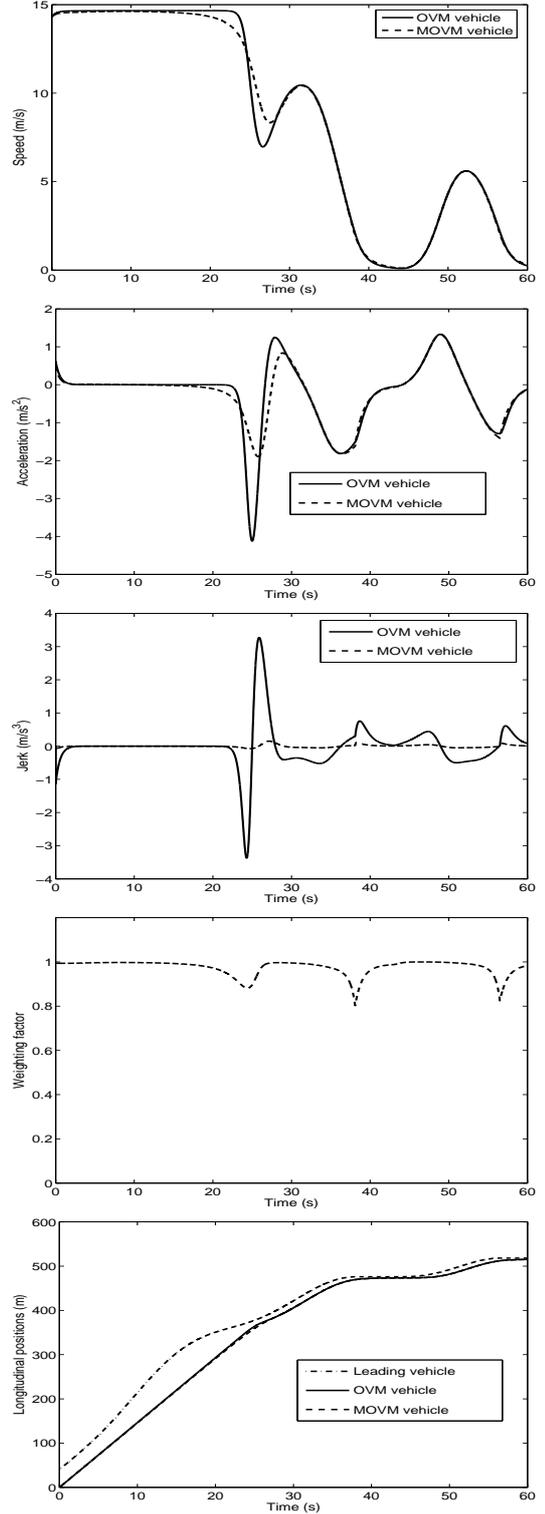


Fig. 7. Velocity (a), acceleration (b), jerk (c), weighting factor (d) and longitudinal positions (e) for OVM and MOVm vehicles in stop-and-go traffic. Sinusoidal acceleration of the leader

The optimal velocity model automatically tries to reduce the risk factor. In fact if for example the speed of the vehicle exceeds the reference speed by 10%, then R_v becomes

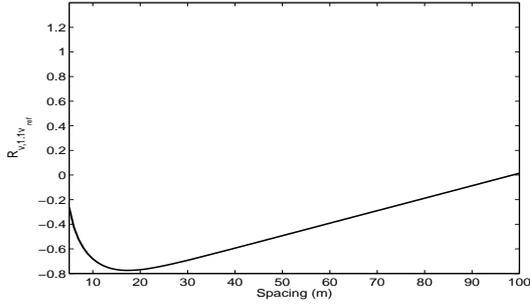


Fig. 8. OVM risk for speed higher than the reference one by 10 %

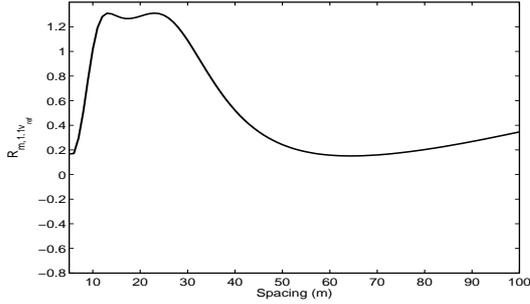


Fig. 9. MOVm risk for speed higher than the reference one by 10%

$$R_v(s, 1.1v_{ref}) = -1 + \frac{0.1s}{\tau(1.1)^2 v_{ref}(s)} \quad (14)$$

Figure 8 shows the plot of $R_v(s, 1.1v_{ref})$ as a function of the spacing. According to the cited results, the deceleration produced by the model is not sufficient between 28m and 12m of spacing.

When the speed is still exceeding by 10%, the risk associated with MOVm is

$$R_m(s, 1.1v_{ref}) = -1 + \frac{s}{\tau 1.1v_{ref}(s)} \left(1 - \frac{1}{1.1} w(s, -1.1v_{ref}(s)) \right) \quad (15)$$

Figure 9 gives now the risk associated to the proposed MOVm when the speed is still exceeding the reference one by 10% the nominal optimal speed. One can notice that both R_v and R_m first decrease. R_v goes under -0.5 at a spacing of 50m. This means that the driver do not decelerate enough. On the other hand, the MOVm reacts better as R_m remains always positive, which means that the drivers reacts well and makes the time-to-collision increasing.

5. CONCLUSION

In this paper a modified optimal velocity model has been proposed. The use of a multiplicative factor on the reference speed makes the model more reactive. All the simulated maneuvers perform smoother. On the other hand, the driver perception of the risk of rear end collision is enhanced.

Future work will focus of the calibration and the validation of the model on the basis of data which will be collected on the Paris corridor during peak hours congested traffic. Afterward the model will be integrated to the semi-macroscopic simulation model METACOR (Elloumi, *et al.*, 1994).

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