

# GUIDANCE-BASED PATH FOLLOWING FOR WHEELED MOBILE ROBOTS

Morten Breivik<sup>\*,1</sup> Thor I. Fossen<sup>\*,†</sup>

<sup>\*</sup>Centre for Ships and Ocean Structures (CESOS)

<sup>†</sup>Department of Engineering Cybernetics (ITK)

Norwegian University of Science and Technology (NTNU)

NO-7491 Trondheim, Norway

E-mails: morten.breivik@ieee.org, fossen@ieee.org

**Abstract:** This paper addresses the problem of path following for wheeled mobile robots (WMRs) by utilizing a novel guidance-based approach. The guidance laws are developed at an ideal, dynamics-independent level, entailing generically valid laws not influenced by particular peculiarities relating to any given dynamics case. Hence, the resulting guidance-based framework is equally applicable to any planar dynamic system. A nonlinear model-based controller is then designed for an underactuated WMR to enable it to comply with the guidance commands, yielding a UGAS/ULES closed loop system. Simulation results demonstrating the capability of the proposed guidance and control scheme are successfully carried out for a unicycle-type WMR. *Copyright © 2005 IFAC*

**Keywords:** Guidance-based path following, Nonlinear model-based control, Wheeled mobile robots, Underactuated systems, Cascaded systems

## 1. INTRODUCTION

When considering the task of traversing a given geometric path, traditional trajectory tracking schemes mix the positional and temporal requirements into one single assignment even in the cases where the geometric path is specified by a path planner and, as such, is completely known in advance. In this context, two weaknesses are apparent. Firstly, most of these schemes do not take advantage of geometric information in the sense that they lapse into plain servosystem tracking of the position. This fact degrades the transient convergence behaviour of the position significantly, and makes it unnatural. Secondly, if the original time parametrization of the path for some reason becomes dynamically infeasible, it must be awkwardly reparametrized to avoid an unstable system as a result of growing positional errors.

As an alternative to using traditional trajectory tracking for solving such problems, this paper presents a *guidance-based path following* scheme which takes advantage of geometric information of the path whenever available. The approach also separates the positional and temporal requirements into two independent assignments. Specifically, the proposed approach is illustrated through the task of forcing wheeled mobile robots to follow a desired geometric path at land.

### 1.1 Previous Work

A lot of research has been carried out in the field of path following, especially in the field of wheeled mobile robots (WMRs). Relevant results are reported in (Samson 1992), (Aicardi *et al.* 1995), (Fierro and Lewis 1995), (Aguilar *et al.* 1998), (del Río *et al.* 1999) and (Soetanto *et al.* 2003). A treatment of one of the most frequently used path following schemes can be found in a paper by Claude Samson (Samson 1992).

---

<sup>1</sup> Supported by the Research Council of Norway through the Centre for Ships and Ocean Structures at NTNU.

The author considers a strategy of projecting the position of the actual WMR onto the desired geometric path, effectively ensuring that an imagined, virtual vehicle exists on the position of the path which is closest to the real WMR at all times. This is achieved by applying the so-called Serret-Frenet equations, yielding the kinematics associated with the Serret-Frenet (SF) frame, which is the path tangential frame at the exact point of projection. The path following problem is then solved in the error space of this frame. However, there is a catch to this approach. Consider a path parametrized by its arc length  $s$  along the path. For every point along such a path, there exists an associated tangent circle with radius  $r(s) = 1/c(s)$  where  $c(s)$  is the curvature of the path. This circle is known as the osculating circle. If at any time the WMR is located at the origin of the osculating circle, the projected point on the path will move infinitely fast. Hence, the SF kinematics contain a singularity at such a point. Samson solves this by restricting the position of the WMR to be contained inside a tube surrounding the path, with radius less than a minimum radius derived from the maximum path curvature. Such a restriction is obstructing, especially from a theoretical point-of-view, and effectively excludes the derivation of any global path following results. Also, due to the way in which the guidance laws are chosen, only a convergence result can be concluded since Barbalat's lemma has to be applied.

In (Soetanto *et al.* 2003), a substantial improvement to the path following scheme from (Samson 1992) is made. Instead of considering a SF frame attached to the exact point on the path which is closest to the WMR at all times, the origin of the SF frame is made to dynamically evolve according to a suitably defined function of time, thus removing the singularity previously associated with the SF kinematic equations. Consequently, a global result can be claimed. However, since the authors have to resort to Barbalat's lemma in the stability analysis, only a convergence result can be concluded. In addition, the approach is restricted to arc-length parametrized paths, which are generally not trivial to deduce.

### 1.2 Main Contribution

This paper presents a guidance-based path following scheme which is singularity-free for all regular paths. Furthermore, the paths are not required to be arc-length parametrized as in (Samson 1992) and (Soetanto *et al.* 2003). The proposed scheme is lucidly exposed, and has an intuitive physical interpretation. Finally, a closed loop system represented by a guided and controlled unicycle-type WMR is rendered UGAS/ULES by applying a nonlinear model-based backstepping-type control law.

## 2. PROBLEM STATEMENT

The primary objective in guidance-based path following is to ensure that a vehicle converges to and follows a desired geometric path, without any temporal requirements. The secondary objective is to ensure that the vehicle complies with a desired dynamic behaviour while traversing the path. By using the convenient task classification scheme of (Skjetne 2005), the path following problem can thus be expressed by the following two task objectives:

**Geometric Task:** Make the position of the vehicle converge to and follow a desired geometric path.

**Dynamic Task:** Make the speed of the vehicle converge to and track a desired speed assignment.

The ability to accurately maneuver a vehicle along a given path is of primary importance for most applications, and the path following concept ensures such a prioritization.

## 3. GUIDANCE SYSTEM DESIGN

The main purpose of this section is to develop the necessary guidance laws required for solving the planar guidance-based path following problem in question. Throughout the section we will consistently employ the notion of an *ideal particle*, which is to be interpreted as a planar position variable that has no dynamics, i.e. it can instantly attain any assigned motion behaviour. The guidance laws developed for this particle can subsequently be extended to any desirable dynamics case at a later stage since they are generically valid.

### 3.1 Assumptions

The following assumptions are made throughout the paper:

- A.1** The desired geometric path is regularly parametrized, i.e.  $\mathbf{p}_d(\theta) \in \mathcal{C}^1 \forall \theta \in \mathbb{R}$ .
- A.2** The speed of the ideal particle is lower-bounded, i.e.  $U_d(t) \in [U_{d,\min}, \infty) \forall t \geq 0$ . Note that it is non-negative by definition.
- A.3** The guidance variable is positive and upper-bounded, i.e.  $\Delta(t) \in (0, \Delta_{\max}] \forall t \geq 0$ .

### 3.2 Principles of Guidance in the Plane

Denote the inertial position and velocity vectors of the ideal particle by  $\mathbf{p} = [x, y]^\top \in \mathbb{R}^2$  and  $\mathbf{v} = \dot{\mathbf{p}} = [\dot{x}, \dot{y}]^\top \in \mathbb{R}^2$ , respectively. The velocity vector has two characteristics; size and orientation. Denote the size by  $U = |\mathbf{v}|_2 = (\mathbf{v}^\top \mathbf{v})^{\frac{1}{2}}$  (the speed) and the orientation by  $\chi = \arctan(\frac{\dot{y}}{\dot{x}})$  (the azimuth angle). Since it is assumed that both  $U$  and  $\chi$  can attain any desirable value instantaneously, they are rewritten

as  $U_d$  and  $\chi_d$ . The inertial position of a point belonging to the geometric path, which is continuously parametrized by a scalar variable  $\theta \in \mathbb{R}$ , is denoted by  $\mathbf{p}_d(\theta) \in \mathbb{R}^2$ . The desired geometric path can consequently be expressed by the set:

$$\mathcal{P} = \{\mathbf{p} \in \mathbb{R}^2 \mid \mathbf{p} = \mathbf{p}_d(\theta) \forall \theta \in \mathbb{R}\}, \quad (1)$$

where  $\mathcal{P} \subset \mathbb{R}^2$ . For a given  $\theta$ , define a local reference frame at  $\mathbf{p}_d(\theta)$  and name it the Path Parallel (PP) frame. The PP frame is rotated an angle:

$$\chi_t(\theta) = \arctan\left(\frac{y'_d(\theta)}{x'_d(\theta)}\right) \quad (2)$$

relative to the inertial frame, where the notation  $x'_d(\theta) = \frac{dx_d}{d\theta}(\theta)$  has been utilized. Consequently, the  $x$ -axis of the PP frame is aligned with the tangent vector to the path at  $\mathbf{p}_d(\theta)$ . The error vector between  $\mathbf{p}$  and  $\mathbf{p}_d(\theta)$  expressed in the PP frame is given by:

$$\boldsymbol{\varepsilon} = \mathbf{R}_t^\top (\mathbf{p} - \mathbf{p}_d(\theta)), \quad (3)$$

where:

$$\mathbf{R}_t(\chi_t) = \begin{bmatrix} \cos \chi_t & -\sin \chi_t \\ \sin \chi_t & \cos \chi_t \end{bmatrix} \quad (4)$$

is the rotation matrix from the inertial frame to the PP frame,  $\mathbf{R}_t \in SO(2)$ . The error vector  $\boldsymbol{\varepsilon} = [s, e]^\top \in \mathbb{R}^2$  consists of the *along-track error*  $s$  and the *cross-track error*  $e$ , see Figure 1. The along-track error represents the longitudinal distance to  $\mathbf{p}_d(\theta)$  along the tangent vector at  $\mathbf{p}_d(\theta)$ , while the cross-track error represents the lateral distance to the tangent vector at  $\mathbf{p}_d(\theta)$ . A most convenient error space in which to operate is thus represented by  $\boldsymbol{\varepsilon}$ . Also, recognize the concept of the *off-track error*, represented by  $|\boldsymbol{\varepsilon}|_2 = \sqrt{\boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon}} = \sqrt{s^2 + e^2}$ .

Define the positive definite and radially unbounded Lyapunov function candidate (LFC):

$$V_\boldsymbol{\varepsilon} = \frac{1}{2} \boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon} = \frac{1}{2} (s^2 + e^2), \quad (5)$$

and differentiate it with respect to time along the trajectories of  $\boldsymbol{\varepsilon}$  to obtain:

$$\dot{V}_\boldsymbol{\varepsilon} = s(U_d \cos(\chi_d - \chi_t) - U_{PP}) + eU_d \sin(\chi_d - \chi_t). \quad (6)$$

We can clearly consider the path tangential speed  $U_{PP}$  as a virtual input for stabilizing  $s$ . Consequently, by choosing  $U_{PP}$  as:

$$U_{PP} = U_d \cos(\chi_d - \chi_t) + \gamma s, \quad (7)$$

where  $\gamma > 0$  becomes a constant gain parameter in the guidance law, we achieve:

$$\dot{V}_\boldsymbol{\varepsilon} = -\gamma s^2 + eU_d \sin(\chi_d - \chi_t). \quad (8)$$

From (8) we see that  $(\chi_d - \chi_t)$  can be considered a virtual input for stabilizing  $e$ . Denote this angular difference by  $\chi_r = \chi_d - \chi_t$ , i.e. the relative angle between the desired azimuth angle and the azimuth angle of the path tangential. Obviously, such a variable should depend on the cross-track error itself, such that

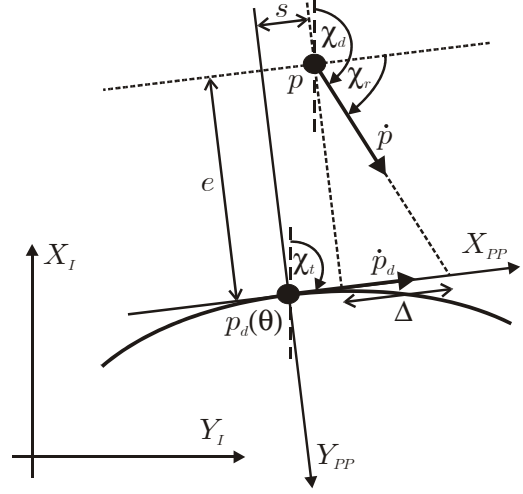


Fig. 1. The geometric relationship between all the relevant parameters and variables utilized in the proposed guidance-based path following scheme.

$\chi_r = \chi_r(e)$ . An attractive choice for  $\chi_r(e)$  could be the physically motivated:

$$\chi_r(e) = \arctan\left(-\frac{e}{\Delta}\right), \quad (9)$$

where  $\Delta > 0$  becomes a time-varying guidance variable utilized to shape the convergence behaviour towards the path tangential, i.e.  $\Delta = \Delta(t)$  satisfying A.3. It is often referred to as the *lookahead distance* in literature dealing with path following along straight lines (Papoulias 1991), and the physical interpretation can be derived from Figure 1. Note that other sigmoidal shaping functions are also possible candidates for  $\chi_r(e)$ , for instance the tanh function. The desired azimuth angle is thus given by:

$$\chi_d(\theta, e) = \chi_t(\theta) + \chi_r(e) \quad (10)$$

with  $\chi_t(\theta)$  as in (2) and  $\chi_r(e)$  as in (9). Also, since  $\theta$  is the actual path parametrization variable that we control for guidance purposes, we need to state the relationship between  $\theta$  and  $U_{PP}$ :

$$\begin{aligned} \dot{\theta} &= \frac{U_{PP}}{\sqrt{x_d'^2 + y_d'^2}} \\ &= \frac{U_d \cos \chi_r + \gamma s}{\sqrt{x_d'^2 + y_d'^2}}, \end{aligned} \quad (11)$$

which is non-singular for all paths satisfying assumption A.1. Hence, the derivative of the LFC finally becomes:

$$\begin{aligned} \dot{V}_\boldsymbol{\varepsilon} &= -\gamma s^2 + eU_d \sin \chi_r \\ &= -\gamma s^2 - U_d \frac{e^2}{\sqrt{e^2 + \Delta^2}}, \end{aligned} \quad (12)$$

which is negative definite under assumptions A.2 and A.3, hence the LFC is a Lyapunov function. The last transition is made by utilizing trigonometric relationships from Figure 1. Note that the speed by definition cannot be negative.

Elaborating on these results, we find that the error system can be represented by the states  $\varepsilon$  and  $\theta$ . Moreover, it is non-autonomous since  $U_d$  and  $\Delta$  can be time-varying. However, it can be rendered autonomous by reformulating its time dependence through the introduction of an extra state:

$$\dot{l} = 1, l_0 = t_0 \geq 0, \quad (13)$$

see e.g. (Teel *et al.* 2002). Hence, the new and extended system can be represented by the state vector  $\mathbf{x} = [\varepsilon^\top, \theta, l]^\top \in \mathbb{R}^2 \times \mathbb{R} \times \mathbb{R}_{\geq 0}$ , with the dynamics:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}). \quad (14)$$

The time variable for this new system is denoted  $t$  with initial time  $t = 0$ , such that  $l(t) = t + t_0$ . We can now utilize set-stability analysis for time-invariant systems in order to be able to conclude on the task objectives in the problem statement. Hence, define the closed, but non-compact set:

$$\mathcal{G} = \{\mathbf{x} \in \mathbb{R}^2 \times \mathbb{R} \times \mathbb{R}_{\geq 0} \mid \varepsilon = \mathbf{0}\}, \quad (15)$$

which represents the dynamics of the extended system when the ideal particle has converged to the path. Also, let:

$$|\mathbf{x}|_{\mathcal{G}} = \inf \{\|\mathbf{x} - \mathbf{y}\| \mid \mathbf{y} \in \mathcal{G}\} \quad (16)$$

$$= \|(\varepsilon, 0, 0)\|_2 \quad (17)$$

$$= (\varepsilon^\top \varepsilon)^{\frac{1}{2}} \quad (18)$$

represent a function measuring the distance from  $\mathbf{x}$  to the set  $\mathcal{G}$ , i.e. the previously mentioned off-track error. As can be seen from (16), it is apparent that  $|\mathbf{x}|_{\mathcal{G}} \rightarrow 0 \Leftrightarrow \|\varepsilon\|_2 \rightarrow 0 \Leftrightarrow \|\mathbf{p} - \mathbf{p}_d(\theta)\|_2 \rightarrow 0$ , since  $\mathbf{R}_t \in SO(2) \Leftrightarrow \mathbf{R}_t^\top \mathbf{R}_t = \mathbf{1}$ . The goal is consequently to make  $|\mathbf{x}|_{\mathcal{G}}$  converge to zero since it is equivalent to solving the geometric task of the guidance-based path following problem. The following proposition can now be stated:

*Proposition 1.* The error set  $\mathcal{G}$  is rendered uniformly globally asymptotically and locally exponentially stable (UGAS/ULES) under assumptions A.1-A.3 if  $\chi_r$  is equal to (9) and  $\theta$  is updated by (11).

**PROOF.** Since the set  $\mathcal{G}$  is closed, but not bounded, we initially have to make sure that the system (14) is forward complete (Teel *et al.* 2002), i.e. that for each  $\mathbf{x}_0$  the solution  $\mathbf{x}(t, \mathbf{x}_0)$  is defined on  $[0, \infty)$ . This means that the solution cannot escape to infinity in finite time. We already know that  $l$  by definition cannot escape in finite time. Also, (5) and (12) shows that neither can  $\varepsilon$ . Consequently, (11) tells us that  $\theta$  cannot escape in finite time either, under assumptions A.1 and A.2. The system is therefore forward complete. We also know that  $\forall \mathbf{x}_0 \in \mathcal{G}$  the solution  $\mathbf{x}(t, \mathbf{x}_0) \in \mathcal{G} \forall t \geq 0$  because  $\varepsilon_0 = \mathbf{0} \Rightarrow \dot{\varepsilon} = \mathbf{0}$ . This renders  $\mathcal{G}$  forward invariant for (14) since the system is already shown to be forward complete. Having established that (14) is forward complete and that

$\mathcal{G}$  is forward invariant, and considering the fact that  $V_\varepsilon = \frac{1}{2} \varepsilon^\top \varepsilon = \frac{1}{2} |\mathbf{x}|_{\mathcal{G}}^2$ , we can derive our stability results simply by considering the properties of  $V_\varepsilon$ , see e.g. (Skjetne 2005). Hence, by standard Lyapunov arguments the error set  $\mathcal{G}$  is rendered UGAS since the Lyapunov function (5) is positive definite and radially unbounded, while its derivative with respect to time (12) is negative definite under assumptions A.1-A.3 when (9) and (11) are satisfied. Furthermore,  $\dot{V}_\varepsilon = -\gamma s^2 - \frac{U_d}{\Delta} e^2 \leq -\gamma s^2 - \frac{U_{d,\min}}{\Delta_{\max}} e^2$  for the error dynamics at  $\varepsilon = \mathbf{0}$ , which proves ULES.

By stabilizing the error set  $\mathcal{G}$ , we have achieved the geometric task. The dynamic task is fulfilled by assigning a desired speed which satisfies assumption A.2 to the ideal particle. In total, we have now solved the guidance-based path following problem for the planar case of interest.

Note that by choosing the speed of the ideal particle equal to:

$$U_d = \kappa \sqrt{e^2 + \Delta^2}, \quad (19)$$

where  $\kappa > 0$  is a constant gain parameter, we obtain:

$$\dot{V}_\varepsilon = -\gamma s^2 - \kappa e^2, \quad (20)$$

which results in the following proposition:

*Proposition 2.* The error set  $\mathcal{G}$  is rendered uniformly globally exponentially stable (UGES) under assumptions A.1 and A.3 if  $\chi_r$  is equal to (9),  $\theta$  is updated by (11) and  $U_d$  satisfies (19).

**PROOF.** The first part of the proof is identical to that of Proposition 1. Hence, we conclude by standard Lyapunov arguments that the error set  $\mathcal{G}$  is rendered UGES.

Although very powerful, this result is clearly not achievable by physical systems since these exhibit natural limitations on their maximum attainable speed. In this regard, Proposition 1 states the best possible stability property a planar physical system like a wheeled mobile robot can hold.

#### 4. CONTROL SYSTEM DESIGN

The kinematic model of an underactuated WMR is given by:

$$\begin{aligned} \dot{x} &= u \cos(\psi) \\ \dot{y} &= u \sin(\psi) \\ \dot{\psi} &= r \end{aligned} \quad (21)$$

where  $\mathbf{p} = [x, y]^\top \in \mathbb{R}^2$  denotes the position of the center of mass of the WMR relative to an inertial reference frame,  $\psi \in \mathbb{R}$  denotes the yaw angle (heading angle) of the vehicle,  $u \in \mathbb{R}$  denotes the surge

speed (longitudinal speed), and  $r \in \mathbb{R}$  denotes the yaw rate. The WMR is considered to be underactuated in the sway direction (lateral direction), i.e. it has no actuator capability to generate a force in this direction. Furthermore, the vehicle travels on a surface without slipping. Consequently, it experiences no sway speed (lateral speed).

A simplified kinetic model of the underactuated WMR under consideration can be expressed as:

$$\mathbf{M}\dot{\boldsymbol{\xi}} = \boldsymbol{\tau} \quad (22)$$

where  $\mathbf{M} = \text{diag}(m, I_z) \in \mathbb{R}^{2 \times 2}$  represents the inertia matrix of a vehicle with mass  $m$  and moment of inertia  $I_z$ .  $\boldsymbol{\xi} = [u, r]^\top \in \mathbb{R}^2$  is the body-fixed vector composed of the surge speed  $u$  and yaw rate  $r$ , and  $\boldsymbol{\tau} = [\tau_1, \tau_2]^\top$  represents the body-fixed force and moment input vector.

Define the projection vector  $\mathbf{h}$ :

$$\mathbf{h} = [0, 1]^\top, \quad (23)$$

then the error variables  $z_1 \in \mathbb{R}$  and  $\mathbf{z}_2 \in \mathbb{R}^2$  according to:

$$z_1 = \psi - \psi_d \quad (24)$$

$$\mathbf{z}_2 = [z_{2,1}, z_{2,2}]^\top = \boldsymbol{\xi} - \boldsymbol{\alpha} \quad (25)$$

where  $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]^\top \in \mathbb{R}^2$  is a vector of stabilizing functions to be specified later. Hence, the error system for the controlled WMR can be represented by the state  $\mathbf{z} = [z_1, \mathbf{z}_2^\top]^\top \in \mathbb{R}^3$ .

Define the positive definite and radially unbounded Control Lyapunov Function (CLF):

$$V_{\mathbf{z}} = \frac{1}{2}k_1 z_1^2 + \frac{1}{2}\mathbf{z}_2^\top \mathbf{M} \mathbf{z}_2, \quad (26)$$

where  $k_1 > 0$  and  $\mathbf{M} = \mathbf{M}^\top > 0$ . Differentiating  $V_{\mathbf{z}}$  with respect to time along the  $\mathbf{z}$ -dynamics yields:

$$\begin{aligned} \dot{V}_{\mathbf{z}} &= k_1 z_1 \dot{z}_1 + \mathbf{z}_2^\top \mathbf{M} \dot{\mathbf{z}}_2 \\ &= k_1 z_1 (\mathbf{h}^\top \boldsymbol{\xi} - \dot{\psi}_d) + \mathbf{z}_2^\top (\boldsymbol{\tau} - \mathbf{M} \dot{\boldsymbol{\alpha}}) \\ &= k_1 z_1 (\alpha_2 - \dot{\psi}_d) + \mathbf{z}_2^\top (\mathbf{h} k_1 z_1 + \boldsymbol{\tau} - \mathbf{M} \dot{\boldsymbol{\alpha}}), \end{aligned}$$

which motivates the choice of the stabilizing function  $\alpha_2$  as:

$$\alpha_2 = \dot{\psi}_d - z_1, \quad (27)$$

and the control law  $\boldsymbol{\tau}$  as:

$$\boldsymbol{\tau} = \mathbf{M} \dot{\boldsymbol{\alpha}} - \mathbf{h} k_1 z_1 - \mathbf{K}_2 \mathbf{z}_2, \quad (28)$$

where  $\mathbf{K}_2 = \text{diag}(k_{2,1}, k_{2,2}) > 0$ , giving:

$$\dot{V}_{\mathbf{z}} = -k_1 z_1^2 - \mathbf{z}_2^\top \mathbf{K}_2 \mathbf{z}_2, \quad (29)$$

which is negative definite, hence the CLF is a Lyapunov function. Since we want the surge speed of the WMR to track a given speed assignment, we choose  $\alpha_1 = u_d$ . The main result of the control design is then summarized by the following proposition:

*Proposition 3.* For continuous reference signals  $\psi_d, \dot{\psi}_d, \ddot{\psi}_d \in \mathcal{L}_\infty$  and  $u_d, \dot{u}_d \in \mathcal{L}_\infty$ , the origin  $\mathbf{z} = \mathbf{0}$  becomes uniformly globally exponentially stable (UGES) by choosing the control input as (28).

**PROOF.** We conclude by standard Lyapunov arguments that the origin  $\mathbf{z} = \mathbf{0}$  is UGES since the Lyapunov function (26) is positive definite and radially unbounded, while its time derivative (29) becomes quadratically negative definite when adhering to (28).

## 5. CLOSED LOOP BEHAVIOUR

Since we consider a WMR that travels forward and does not slip, the interpretation of the yaw angle  $\psi$  from the control section becomes the same as for the azimuth angle  $\chi$  from the guidance section. Hence, we will proceed by only employing the symbol  $\psi$ .

The  $\varepsilon$ -dynamics of a controlled WMR is given by:

$$\dot{s} = -\dot{\psi}_t e - \gamma s \quad (30)$$

and:

$$\begin{aligned} \dot{e} &= \dot{\psi}_t s + U \sin(\psi - \psi_t) \\ &= \dot{\psi}_t s + U \left( \frac{\Delta \sin z_1 - e \cos z_1}{\sqrt{e^2 + \Delta^2}} \right), \end{aligned} \quad (31)$$

since  $\psi - \psi_t = z_1 + \psi_r$ , and where  $U = u = z_{2,1} + u_d \geq 0$  is the speed of the WMR. The  $\theta$ -dynamics is given by:

$$\dot{\theta} = \frac{U \cos(z_1 + \psi_r) + \gamma s}{\sqrt{x_d^2 + y_d^2}}. \quad (32)$$

Hence, the closed loop system of a guided and controlled WMR can be considered as the cascaded interconnection of the undisturbed control subsystem perturbing the guidance subsystem. The following theorem can now be stated:

*Theorem 4.* The control and guidance cascade is rendered UGAS/ULES under assumptions A.1-A.3 if the desired yaw angle  $\psi_d$  is given by (10), the control law is chosen as (28), and the path parametrization variable  $\theta$  is updated by (32).

**PROOF.** [Indication] The undisturbed control subsystem was shown to be UGES in Proposition 3 by standard Lyapunov analysis, while the unperturbed guidance subsystem was shown to be UGAS/ULES by set-stability analysis in Proposition 1. It is then straightforward to carry out the rest of the proof by resorting to cascade theory as in e.g. (Lor a 2004), however the details are omitted due to space limitations.

## 6. CASE STUDY: A UNICYCLE-TYPE WMR

Consider a unicycle-type WMR which employs two fore-mounted driving wheels as active actuators, with an aft-mounted and passive free-rotating support wheel. The interaction between the wheels and the ground is assumed to be purely of a rolling nature, i.e. slipping never occurs. This corresponds to the underactuated WMR considered in this paper. A numerical simulation is performed to illustrate the performance of the proposed guidance and control scheme applied to the unicycle-type WMR. The model parameters are chosen as  $m = 5 \text{ kg}$  and  $I_z = 2.5 \text{ kgm}^2$ . The desired path is a circle with a radius of  $5 \text{ m}$  located at the origin of the inertial reference frame. The initial states of the WMR are chosen to be  $\mathbf{p}_0 = [3 \text{ (m)}, 8 \text{ (m)}, 0.785 \text{ (rad)}]^\top$  and  $\boldsymbol{\xi}_0 = [0.5 \text{ (m/s)}, 0 \text{ (rad/s)}]^\top$ . Hence, the vehicle starts out with a heading of 45 degrees, i.e. moving away from the desired path. The initial cross-track error is approximately  $3.5 \text{ m}$ . The WMR is supposed to keep its surge speed constant at  $0.5 \text{ m/s}$  during the run. The controller gains are chosen as  $k_1 = k_{2,1} = k_{2,2} = 1$ . The gain parameter is set to  $\gamma = 100$ , while the lookahead distance is chosen to be  $\Delta = 1 \text{ m}$ . Figure 2 shows that the WMR converges beautifully to the path, while Figure 3 illustrates the behaviour of the cross-track error.

## 7. CONCLUSIONS

This paper has presented a novel guidance-based path following approach for wheeled mobile robots. The guidance laws have been developed at an ideal, dynamics-independent level, entailing generically valid laws not influenced by particular peculiarities relating to any given dynamics case. Hence, the resulting guidance-based framework is equally applicable to any planar dynamic system. Also, a nonlinear model-based controller have been designed for an underactuated WMR to enable it to comply with the guidance commands, yielding a UGAS/ULES closed loop system. Simulation results for a unicycle-type WMR have successfully demonstrated the capability of the proposed guidance and control scheme.

## REFERENCES

- Aguilar, L. E., P. Souères, M. Courdresses and S. Fleury (1998). Robust path-following control with exponential stability for mobile robots. In: *Proceedings of the IEEE ICRA, Leuven, Belgium*.
- Aicardi, M., G. Casalino, A. Bicchi and A. Balestrino (1995). Closed loop steering of unicycle-like vehicles via Lyapunov techniques. *IEEE Robotics and Automation Magazine* **2**(1), 27–35.
- del Río, F. D., G. Jiménez, J. L. Sevillano, S. Vicente and A. Civit Balcells (1999). A generalization of

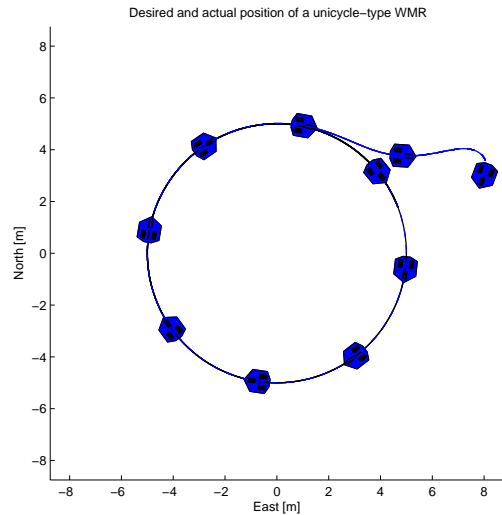


Fig. 2. The wheeled mobile robot converges naturally to the desired path.

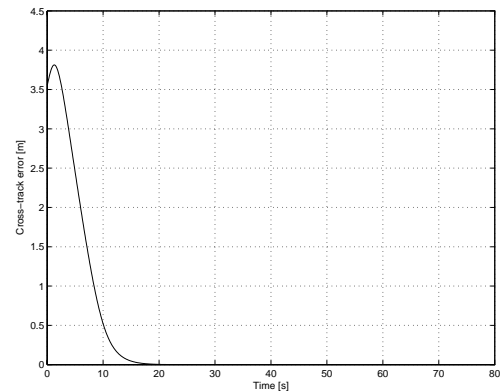


Fig. 3. The cross-track error converges to zero.

- path following for mobile robots. In: *Proceedings of the IEEE ICRA, Detroit, Michigan, USA*.
- Fierro, R. and F. L. Lewis (1995). Control of a non-holonomic mobile robot: Backstepping kinematics into dynamics. In: *Proceedings of the 34th IEEE CDC, New Orleans, LA, USA*.
- Loría, A. (2004). Cascaded nonlinear time-varying systems: Analysis and design. Lecture notes, Minicourse at ECTS, France.
- Papoulias, F. A. (1991). Bifurcation analysis of line of sight vehicle guidance using sliding modes. *International Journal of Bifurcation and Chaos* **1**(4), 849–865.
- Samson, C. (1992). Path following and time-varying feedback stabilization of a wheeled mobile robot. In: *Proceedings of the ICARCV'92, Singapore*.
- Skjetne, R. (2005). The Maneuvering Problem. PhD thesis. Norwegian University of Science and Technology, Trondheim, Norway.
- Soetanto, D., L. Lapierre and A. Pascoal (2003). Adaptive, non-singular path-following control of dynamic wheeled robots. In: *Proceedings of the 42nd IEEE CDC, Maui, Hawaii, USA*.
- Teel, A., E. Panteley and A. Loría (2002). Integral characterization of uniform asymptotic and exponential stability with applications. *Mathematics of Control, Signals, and Systems* **15**, 177–201.