FEEDBACK REGULATION OF A DC MOTOR VIA INTERCONNECTION AND DAMPING ASSIGNMENT

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Abstract: In this work, we consider the application of the Interconnection and Damping Assignment (IDA) control methodology, recently proposed in the literature, to the asymptotic position regulation problem of a brushed DC motor driving a mechanical load. To achieve this objective the electromechanical system is firstly transformed into the general port controlled Hamiltonian form and, then, the IDA design procedure is applied to synthesize the stabilizing control law. The Hamiltonian structure of the closed loop system is preserved and the asymptotic stability of the mechanical position is verified by digital simulations. *Copyright ©2005 IFAC*

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1. INTRODUCTION

Energy shaping control methods have attracted a lot of interest recently. A common outstanding feature of these methods, in the face of the stabilization problem, is seeing physical systems as the interconnection of simpler sub-systems or components either storing or dissipating energy (Ortega *et al.*, 1998). This viewpoint allows us to obtain physical system models in two alternative structures (Lagrangian or Hamiltonian) (Meisel, 1969; Ortega *et al.*, 1998; van der Schaft, 2000). Energy shaping methods pursue to preserve the physical structure (Lagrangian or Hamiltonian) in the closed-loop. This characteristic has the advantage that the closed-loop energy function can be used as Lyapunov (or storage) function for stability analysis purposes.

Whilst the controller design methods that shape the potential energy have been known for years (Ortega *et al.*, 1998), the challenging problem of shaping the total energy of electromechanical systems has been addressed more recently (Bloch *et al.*, 2000; Hamberg, 2000; Ortega *et al.*, 2002a). To this end, it is required that the overall energy function has a minimum at the desired equilibrium point. This is guaranteed in mechanical systems by firstly modifying the inertia matrix (in the kinetic energy) and then by shaping the potential energy. The Interconnection and Damping Assignment (IDA) passivity based control approach is used to achieve this goal in Ortega *et al.* (2002a), for mechanical systems in the general Port-Controlled Hamiltonian (PCH) form (Ortega *et al.*, 2002b). It is important to emphasize that in the work by Chang *et al.* (2002), the complete equivalence of the PCH/IDA methodology to that of the controlled Lagrangian methodology is proven . In particular, the Lagrangian form of the gyroscopic terms corresponding to the Poisson structure modification is identified.

On the other hand, the total energy problem for generalized electromechanical systems has been addressed in Rodriguez and Ortega (2003) to solve the asymptotic position regulation problem of fully actuated systems with linear magnetic material consisting of inductances, permanent magnets and one mechanical coordinate. This objective is achieved by modifying the interconnection structure of the system and adding gyroscopic terms to the energy function so that the minimum of the total closed-loop system is only determined by the mechanical potential energy. In this paper we address the problem of achieving asymptotic position regulation, by following the PCH/IDA methodology reported in Rodriguez and Ortega (2003), of a brushed DC motor driving a mechanical load.

2. IDA CONTROL OF ELECTROMECHANICAL SYSTEMS IN PCH FORM

For the seek of clarity, we revisit the modelling procedure explained in Rodriguez and Ortega (2003), which is based on the energy conversion methodology detailed in Meisel (1969), to represent the generalized electromechanical system. It consists of n_e windings with linear magnetic materials and it is also assumed that all parameters are constant and known. Then, by applying the Gauss's law and the Ampere's law, the following affine relationship arises

$$\lambda = L(\theta)i + \mu(\theta) \tag{1}$$

between the flux linkage $\lambda \in \Re^{n_e}$ and the current vector $i \in \Re^{n_e}$, with $\theta \in \Re$ the mechanical angular position, and $L(\theta) = L^T(\theta) > 0$ the $n_e \times n_e$ multi-port inductance matrix. The vector $\mu(\theta)$ represents the flux linkages.

By assuming fully actuated electrical coordinates, the voltage equilibrium equation yields

$$\dot{\lambda} + Ri = u \tag{2}$$

where $u \in \Re^{n_e}$ is the vector of voltages applied to the windings, $R = R^T > 0$ is the matrix of electrical resistance of the windings.

Coupling between the electrical and mechanical subsystems is established through the torque of electrical origin (see Meisel (1969) for further details)

$$\tau(i,\theta) = \frac{1}{2}i^T\nabla_\theta L(\theta)i + i^T\nabla_\theta\mu(\theta)$$

$$= \frac{1}{2}i^T\frac{\partial L(\theta)}{\partial\theta}i + i^T\frac{\partial\mu(\theta)}{\partial\theta} \tag{3}$$

In the sequel, we will use the notation $\nabla_w H(z,w) := \partial H(z,w)/\partial w$.

To complete the model, the latter equation is replaced in the mechanical dynamics

$$J\ddot{\theta} = -r_m\dot{\theta} + \tau(i,\theta) - \nabla_\theta V(\theta) \tag{4}$$

where $J > 0$ is the rotational inertia of the mechanical subsystem, $r_m \geq 0$ is the viscous friction coefficient, and $V(\theta)$ is the potential energy function.

To apply the IDA control design approach, it is needed to express the model (1)-(4) in PCH form

$$\dot{x} = [\mathcal{J}(x) - \mathcal{R}(x)]\nabla_x H(x) + g(x)u \tag{5}$$

where $x \in \Re^n$ is the state space, $\mathcal{J}(x) = -\mathcal{J}^T(x)$, $g(x)$ are the internal and external *interconnection* structures, respectively, and $\mathcal{R}(x) = \mathcal{R}^T(x) \geq 0$ is the *damping* structure (van der Schaft, 2000).

To achieve this objective, the total energy function is introduced (see Ortega *et al.* (2002b))

$$H(x) = \frac{1}{2}[\lambda - \mu(\theta)]^T L(\theta)^{-1}[\lambda - \mu(\theta)] + V(\theta) + \frac{1}{2}\frac{p^2}{J} \tag{6}$$

where $p = J\dot{\theta}$ is the mechanical momentum; and the state vector $x = [\lambda^T, \theta, p]^T$ is defined.

Thus, (1)-(4) can be rewritten in the compact form

$$\begin{bmatrix}\dot{\lambda}\\ \dot{\theta}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}-R & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & -r_m\end{bmatrix}\begin{bmatrix}\nabla_\lambda H(x)\\ \nabla_\theta H(x)\\ \nabla_p H(x)\end{bmatrix} + \begin{bmatrix}I\\ 0\\ 0\end{bmatrix}u \tag{7}$$

and comparing with the PCH model (5) we identify

$$\mathcal{J} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}, g = \begin{bmatrix}I\\ 0\\ 0\end{bmatrix}, \mathcal{R} = \begin{bmatrix}R & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & r_m\end{bmatrix}$$

The control problem is the asymptotic regulation of θ to a constant position $\theta_* \in \Theta \subset \Re$. The equilibria of the electromechanical system (7) are of the form

$$x_* = [\lambda_*, \theta_*, 0]^T \tag{8}$$

where $\lambda_* = L(\theta_*)i_* + \mu(\theta_*)$ and i_* is the solution of (3),(4) for $\theta = \theta_*$, that is

$$\frac{1}{2}i_*^T\nabla_\theta L(\theta_*)i_* + i_*^T\nabla_\theta\mu(\theta_*) - \nabla_\theta V(\theta_*) = 0 \tag{9}$$

The corresponding control is $u_* = Ri_*$. Thus, all equilibria correspond to nonzero current, hence to nonzero electrical energy.

2.1 *IDA control synthesis*

In order to assign the equilibria of the closed-loop system via a selection of the potential energy only, Rodriguez and Ortega (2003) have chosen an energy function of the form

$$H_d(x) = \frac{1}{2}[\lambda - \mu_d(\theta, p)]^T L(\theta)^{-1}[\lambda - \mu_d(\theta, p)] + V_d(\theta) + \frac{1}{2}\frac{p^2}{J} \quad (10)$$

where $\mu_d(\theta, p)$ is fixed such that $\lambda_* = \mu_d(\theta_*, 0)$. Thus, the equilibria will coincide with the extrema of $V_d(\theta)$, and it is only needed to select a function with a unique isolated minimum at θ_*.

Then, to assign the proposed energy function preserving the PCH structure, the original interconnection and damping structures are modified to take the form

$$\begin{bmatrix} \dot{\lambda} \\ \dot{\theta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -R & \alpha(x) & \beta(x) \\ -\alpha^T(x) & 0 & 1 \\ -\beta^T(x) & -1 & -r_a(p) \end{bmatrix} \begin{bmatrix} \nabla_\lambda H_d(x) \\ \nabla_\theta H_d(x) \\ \nabla_p H_d(x) \end{bmatrix} \quad (11)$$

where $\alpha(x), \beta(x)$ $r_a(p) > 0$ are the *free parameters* to be used for assigning the desired energy function.

By following the proof of the Proposition 1 in Rodriguez and Ortega (2003), it can be shown that matching the first n_e rows of the desired dynamics (11) with the corresponding rows of the original dynamics (7), we can obtain the control law

$$\begin{aligned} u = Ri_d - \alpha\Big\{\frac{1}{2}[i^T\nabla_\theta L(\theta)i - i_d^T\nabla_\theta L(\theta)i_d]\Big\} \\ +\alpha\{(i - i_d)^T[L(\theta)\nabla_\theta i_d + \nabla_\theta\mu_\theta] - \nabla_\theta V_d(\theta)\} \\ +\beta\left[\frac{p}{J} - (i - i_d)^T L(\theta)\nabla_p i_d\right] \end{aligned} \quad (12)$$

defining

$$i_d := L(\theta)^{-1}[\mu_d(\theta, p) - \mu(\theta)] \quad (13)$$

and proposing the $\alpha(x)$, $\beta(x)$ functions

$$\alpha = -L(\theta)\nabla_p i_d \quad (14)$$

$$\beta = L(\theta)[\nabla_\theta i_d + r_a(p)\nabla_p i_d] \quad (15)$$

respectively. This suitable choice of the parameter β yields

$$\frac{1}{2}i_d^T\nabla_\theta i_d + i_d^T\nabla_\theta\mu(\theta) + \nabla_\theta V_d(\theta) - \nabla_\theta V(\theta) - [r_m - r_a(p)]\frac{p}{j} = 0 \quad (16)$$

which is a quadratic *algebraic* equation in i_d. That is, by leaving the inductance matrix unchanged in (10), it is possible to transform the matching conditions into simple algebraic equations, instead of partial differential equations, as in the case of mechanical systems. Furthermore, it is proved in Rodriguez and Ortega (2003) that the electromechanical system (7) in closed-loop with the control law (12), and $V_d(\theta)$ a positive definite function, has an asymptotically stable equilibrium point at (8). Under these conditions, the original dynamics (7) in closed loop with (12) matches the desired dynamics (11) in the set

$$\mathcal{D} := \Big\{(\lambda, \theta, p) \in \Re^{n_e+2} \mid \nabla_\theta V(\theta) - \nabla_\theta V_d(\theta) + [r_m - r_a(p)]\frac{p}{j} \in \mathcal{T}\Big\} \quad (17)$$

where $\mathcal{T} := [\tau_m, \tau_M] \subset \Re$ is the interval of admissible torques.

3. IDA CONTROL OF A BRUSHED DC MOTOR

We consider in this section the application of the IDA control methodology above for the asymptotic regulation of the angular position of a brushed DC motor driving a mechanical load. The system model is taken from Dawson *et al.* (1998) by considering that most electromechanical systems can be separated into three different parts:

- a dynamic mechanical subsystem, including in this case a position dependent load and the motor rotor;
- a dynamic electrical subsystem which includes all relevant electrical effects;
- a static relationship representing the conversion of electrical energy into mechanical energy.

The mechanical and electrical subsystem dynamics are respectively represented by the equations

$$\begin{aligned} M\ddot{q} + B\dot{q} + N\sin(q) = i \\ L\frac{di}{dt} = v - Ri - K_B\dot{q} \end{aligned} \quad (18)$$

where

- M constant lumped inertia
- N constant lumped load term
- B friction coefficient
- *q(t)* angular load position
- *i(t)* electrical current.

- L rotor inductance
- R rotor resistance
- K_B back-emf coefficient
- $v(t)$ input control voltage.

Considering the generalized electromechanical system (1)-(4) and specializing it to this case, we can identify $n_e = 1$, $L(\theta) = L \in \Re$, $V(\theta) = N_o(1 - \cos(\theta))$ and $\mu(\theta) = K_b\theta = \tau_L\theta$; where $N_o := N\tau_L$, τ_L is the constant torque coefficient, and we can also obtain the relationships

$$\lambda = Li + \tau_L\theta \tag{19}$$

$$\dot{\lambda} = L\frac{di}{dt} + \tau_L\dot{\theta} \tag{20}$$

$$L\frac{di}{dt} + \tau_L\dot{\theta} + Ri = u \tag{21}$$

$$\tau(i,\theta) = \tau_L i \tag{22}$$

$$J\ddot{\theta} + r_m\dot{\theta} + N_o\sin(\theta) = \tau_L i \tag{23}$$

Note that the equation (23) coincides with the mechanical subsystem equation in (18) for $J = M\tau_L$, $r_m = B\tau_L$, $N_o = N\tau_L$; whilst (21) coincides with the electrical subsystem equation in (18) for $\tau_L = K_B$.

To transform the brushed DC motor system model into the PCH form, we propose the total energy function

$$H(x) = \frac{1}{2L}(\lambda - \tau_L\theta)^2 + N_o(1 - \cos(\theta)) + \frac{p^2}{2J} \tag{24}$$

Then, by considering the new state vector $x = [\lambda, \theta, p]^T$ proposed in Rodriguez and Ortega (2003), the brushed DC motor model can be written

$$\begin{aligned}\dot{\lambda} &= -Ri + u\\ \dot{\theta} &= \frac{p}{J}\\ \dot{p} &= J\ddot{\theta}\end{aligned} \tag{25}$$

which can be rewritten in the compact form

$$\begin{bmatrix}\dot{\lambda}\\ \dot{\theta}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}-R & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & -r_m\end{bmatrix}\begin{bmatrix}\nabla_\lambda H(x)\\ \nabla_\theta H(x)\\ \nabla_p H(x)\end{bmatrix} + \begin{bmatrix}1\\0\\0\end{bmatrix}u \tag{26}$$

Thus, we can identify the matrices of the system (5)

$$\mathcal{J} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}, g = \begin{bmatrix}1\\0\\0\end{bmatrix}, \mathcal{R} = \begin{bmatrix}R & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & r_m\end{bmatrix}$$

Note that, the equation $\dot{p}$ in (25) is equivalent to $J\ddot{\theta} + r_m\dot{\theta} + N_o\sin(\theta) = \tau_L i$.

By following the design procedure explained in the previous section, we choose the functions $V_d(\theta)$ and $r_a(p)$ as suggested in Rodriguez and Ortega (2003)

$$\begin{aligned}V_d(\theta) &= K_p\frac{(\theta - \theta_*)^2}{\sqrt{(1 + (\theta - \theta_*)^2)}}; \quad K_p > 0\\ r_a(p) &= r_{a1} + r_{a2}\frac{p^2}{1 + p^2}; \quad r_{a1}, r_{a2} > 0\end{aligned} \tag{27}$$

Concerning the brushed DC motor equations, we can identify

$$\begin{aligned}L(\theta) = L \quad &\Rightarrow \nabla_\theta L(\theta) = 0\\ \mu(\theta) = \tau_L\theta \quad &\Rightarrow \nabla_\theta\mu(\theta) = \tau_L\\ \nabla_\theta V_d(\theta) &= K_p\bar{\theta}\frac{(2 + \bar{\theta})}{\sqrt{1 + \bar{\theta}^2}}; \quad (28)\\ V(\theta) = N_0(1 - \cos(\theta)) \quad &\Rightarrow \nabla_\theta V(\theta) = N_o\sin(\theta)\end{aligned}$$

Then, by defining $\bar{\theta} := \theta - \theta_*$ and replacing these functions into (16), we can obtain, after some further manipulations,

$$\begin{aligned}i_d = \frac{1}{\tau_L}\Big[N_o\sin(\theta) + \{r_m - r_{a1} - r_{a2}\frac{p^2}{1 + p^2}\}\frac{p}{J}\Big]\\ -\frac{K_p\bar{\theta}}{\tau_L}\Big[\frac{2 + \bar{\theta}^2}{\sqrt{(1 + \bar{\theta}^2)^3}}\Big]\end{aligned} \tag{29}$$

The equation (29) can be replaced into (14) and (15) to yield

$$\begin{aligned}\alpha(x) &= \frac{-L}{\tau_l J}\left(r_m - r_{a1} - r_{a2}\left[\frac{p^2(3 + p^2)}{(1 + p^2)^2}\right]\right)\\ \beta(x) &= \frac{1}{\tau_L}\left[N_o\cos(\theta) + K_p\left[\frac{-2 + \bar{\theta}^2}{\sqrt{(1 + \bar{\theta}^2)^5}}\right]\right]\\ &+ \frac{L}{\tau_L J}\left[r_{a1} + r_{a2}\frac{p^2}{1 + p^2}\right] \times\\ &\left[r_m - r_{a1} - r_{a2}\left(\frac{p^2(3 + p^2)}{(1 + p^2)^2}\right)\right]\end{aligned}$$

Finally, by substituting the corresponding equations for α, β and i_d into (12), we obtain the stabilizing feedback control law

$$\begin{aligned}u = Ri_d - \alpha(i - i_d)\Bigg[\frac{L}{\tau_L}\Bigg(N_o\cos(\theta) + \tau_L\\ + K_p\Big[\frac{-2 + \bar{\theta}^2}{\sqrt{(1 + \bar{\theta}^2)^5}}\Big]\Bigg)\\ - K_p\bar{\theta}\Big[\frac{2 + \bar{\theta}^2}{\sqrt{(1 + \bar{\theta}^2)^3}}\Big]\Bigg]\\ + \beta\Bigg[\frac{p}{J} - (i - i_d)L\frac{1}{\tau_L J}\Bigg(r_m - r_{a1}\\ - r_{a2}\Big(\frac{p^2(3 + p^2)}{(1 + p^2)^2}\Big)\Bigg)\Bigg]\end{aligned} \tag{30}$$

This systematic design procedure allows to deal with an important class of electromechanical system. An adaptive partial state feedback control design method has been proposed in Rodríguez *et al.* (2003) for asymptotic position regulation of electromechanical systems, when only measurement of the electrical coordinates and of the mechanical position are available.

Digital simulations were carried out to evaluate the performance of the closed loop system under control of the feedback law (30). The system parameters used in simulations were: $M = 0.005242\ K_g - \frac{m^2}{rad}$, $N = 2.2839\ K_g - \frac{m}{seg^2}$, $B = 0.018\ N - \frac{seg}{rad}$, $K_B = 0.90\ N - \frac{m}{A}$, $\mathrm{R} = 5\ \Omega$, $L = 25 \times 10^{-3}\ H$, y $K_p = 0.8$ for an equilibrium position $\bar{X}_1 = 1.5707\ rad$, corresponding to an equilibrium voltage value $\bar{U} = 11.4195\ voltios$ and a current value $\bar{X}_3 = 2.2839\ amp$. Figure 1 shows the controlled state variables response and the input control voltage. This figure verifies the asymptotic behavior of the controlled state variables of the brushed DC motor.

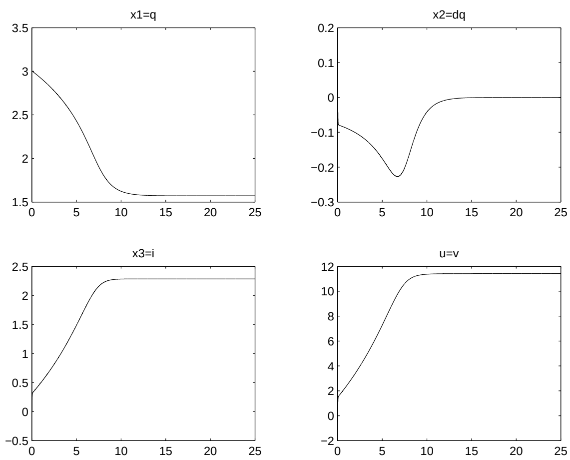

Fig. 1. Controlled state response of the brushed DC motor

4. CONCLUSIONS

We have applied the systematic PCH/IDA control methodology to a brushed DC motor driving a mechanical load for achieving asymptotic stability of the mechanical position. It was shown that modifying the interconnection and damping structures, and with a suitable choice of the free parameters, the matching conditions become simple algebraic equations. Digital simulations demonstrated the asymptotic stability of the controlled response of the brushed DC motor. The output feedback control of this system under parametric uncertainty is an interesting problem to be addressed.

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