THERMAL COMFORT BASED PREDICTIVE CONTROLLERS FOR BUILDING HEATING SYSTEMS

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Abstract:
This work is focused on indoor thermal comfort control problem in buildings equipped with HVAC (Heating Ventilation and Air Conditioning) systems. The occupants thermal comfort is addressed here by a comfort zone in the psychrometric chart and the PMV (Predict Mean Vote) index. In this context, three control algorithms are proposed by using only-one-actuator system associated to a heating equipment. The methods are based on the model predictive control scheme and on the improvement of indices related to occupants thermal comfort sensation. Simulation results - obtained by using the weather data file for the city of Curitiba, Brazil - are presented to validate the proposed methodology in terms of room air temperature, relative humidity and PMV control.

Keywords: predictive control, thermal comfort, HVAC systems, non-linear control.

1. INTRODUCTION

Energy efficiency in buildings is nowadays an important issue due to the growth of energy costs, energy consumption and environmental impacts. However, there is a trade-off between energy consumption and indoor thermal comfort, which relevance has been progressively attracting the attention of industrial and academic researches since early 70's. In fact, people spend most of their lifetime in indoor environments and the lack of indoor comfort has a direct effect on their productivity and satisfaction. Therefore, the aim is to save energy while maintaining the occupants' thermal comfort.

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As it will be discussed further in this paper, thermal comfort in buildings is a concept that is difficult to define. Over the last decades, a large number of thermal comfort indices have been established for indoor climate analysis and HVAC control system design (see (ASHRAE, 1993)) and the most disseminated one is the PMV (Predict Mean Vote), proposed in (Fanger, 1974). Such index considers environmental variables and individual factors and the closer to zero is its value, the better is the occupants' thermal comfort sensation.

However, a majority of HVAC control systems are still considered as temperature control problems, for instance (Astrom *et al.*, 1993), but there are some solutions proposed in the literature that searches to improve the building occupants' thermal comfort. These approaches can be divided into two groups: the one that deals with temperature and relative humidity signals and the other one that uses the PMV concept.

Some works related to the first approach are recalled in the following. In (Dumur *et al.*, 1997), a strategy to anticipate future changes on the temperature set-point value is proposed in order to keep this signal as close as possible to the set-point. Such a strategy, first tested in PID's is then proposed for a Generalized Predictive Controller. In (Oliveira *et al.*, 2003), it is noticed that in the thermal comfort context, for the thermal comfort sensation, it might be enough just setting a temperature band value instead of having a temperature regulation control in a precise preset value (Fanger, 1974). Such a characteristic is then explored by a fuzzy logic type control law.

Some works related to the second mentioned approach are described in the following. An idea in this context is to assume a PMV sensor, that is, the PMV is a measured controlled variable that is a part of an ordinary closed-loop structure. In (Kolokotsa *et al.*, 2001), a fuzzy control is used and, in (Gouda *et al.*, 2001), a PID and a fuzzy controller are proposed and compared. A different proposal, but still in the PMV-context, are presented in (Hamdi and Lachiver, 1998) (Yonezawa *et al.*, 2000). In these works, a fuzzy logic expert system defines set-points for temperature and air velocity signals of a multivariable controller. It is shown in (Hamdi and Lachiver, 1998) that a constant temperature indoor signal is not sufficient to satisfactorily provide thermal comfort.

The present paper proposes three control schemes for improving the thermal comfort by using the two previous cited approaches. A characteristic of these schemes are the assumption of a SIMO (Single Input, Multiple Outputs) building system, where indoor temperature and relative humidity are measured variables and the single manipulated variable is the electrical power applied to a heating device. All these three schemes use constrained model predictive control (MPC) fundamentals (Clarke, 1994). The first control law assures that the temperature signal lies within a comfort bound while optimizes the relative humidity. The second one computes the optimal value for the heating power based on cost function using temperature and relative humidity optimization. The third one is a PMV-based-Predictive control since it calculates the control signal that optimizes the PMV index in terms of thermal comfort.

The paper is organized as follows. In the next section, concepts related to thermal comfort are reviewed. In Section 3, the three proposed control laws are presented, including the PMV prediction equation. In Section 4, some simulation results are presented. These results are obtained by using TRY (Test Reference Year) weather data file for a cold week in the city of Curitiba, Brazil. Finally, in Section 5, the conclusions are addressed.

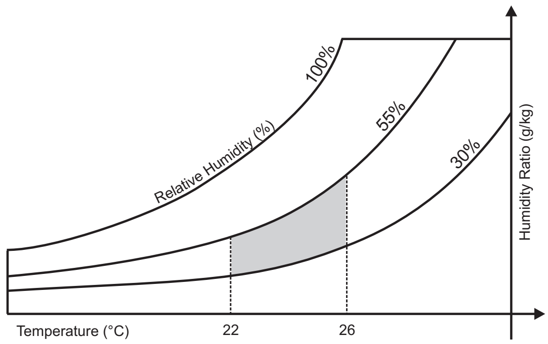

Fig. 1. Psychrometric chart with a comfort zone

2. THERMAL COMFORT

Definition and control of indoor conditions for reaching thermal comfort in buildings are hard to be established. As thermal satisfaction depends on many parameters - most of them controllable - research works on thermal comfort have been conducted and some comfort indices have been proposed over the last fifty years. An example is the thermal comfort index called effective temperature, which is computed by using the indoor temperature and relative humidity signals and have been adopted by ASHRAE (ASHRAE, 1993) for decades.

Thermal comfort can also be identified by a comfort zone within a psychrometric chart, which is a graphics that shows the thermodynamic properties of moist air, considering negligible the small changes in local barometric pressure. A example of this zone can be found in Figure 1. However, the most disseminated index for evaluating indoor thermal comfort is the PMV. In agreement with ASHRAE Standard 55-66, the following definition of thermal comfort for a person can be stated (Fanger, 1974): "that condition of mind which expresses satisfaction with the thermal environment". Thermal environment is those characteristics of the environment, which affects a person's heat loss, and, in terms of bodily sensations, it is a sensation of hot, warm, slightly warmer, neutral, slightly cooler, cool and cold. From the physiological point of view, thermal comfort occurs when there is a thermal equilibrium between the human body and the environment.

In this context, a mathematical formulae which combines environmental variables and individual parameters can be proposed. This index is based on a theoretical model combined with the results from experiments with approximately 1300 subjects, and is given by: (Fanger, 1974):

$$
\begin{aligned}
PMV &= \mathcal{F}(t_{bs}, t_{cl}, t_{rm}, h_c, f_{cl}, M, W, p_V) \\
PMV &= (0.303\,\mathrm{e}^{-0.036M} + 0.028)\{(M-W) \\
&\quad -3.05\times 10^{-3}[5733 - 6.99(M-W) - p_V]\} \\
&\quad -0.42[(M-W) - 58.15] - [1.7\times 10^{-5} M(5867 \\
&\quad -p_V)] - [0.0014M(34 - t_{bs})] \\
&\quad -\{3.69\times 10^{-8} f_{cl}[(t_{cl}+273)^4 - (t_{rm}+273)^4]\} \\
&\quad -[f_{cl} h_c (t_{cl} - t_{bs})]
\end{aligned}
\tag{1}
$$

where t_{bs} is the dry-bulb temperature (^{O}C) or just indoor air temperature, t_{cl} is the clothing surface temperature (^{O}C), t_{rm} is the mean radiant temperature (^{O}C), h_c is the convective heat transfer coefficient (W/m^2K) that is calculated as shown on equation (2). f_{cl} is the clothing area factor, which can be computed by means of a cloth index, given by I_{cl} (Fanger, 1974). M is the metabolic rate, the rate of transformation of chemical energy into heat and mechanical work by aerobic and anaerobic activities within the body (W/m^2) and W is the effective mechanical power (W/m^2).

$$h_c = 10.4\sqrt{v}, \text{ for } v < 2.6 \text{ m/s} \tag{2}$$

The vapor pressure and humidity ratio are correlated as shown on Equation 3:

$$w = 0.622\frac{p_V}{p_T - p_V} \tag{3}$$

where p_T is the local barometric pressure. The term p_V can be defined as partial vapor pressure (kPa) and can be related to the dry-bulb temperature t_{bs} and relative humidity ϕ (%) as follows:

$$p_V = \phi P_{SAT}(t_{bs}) \tag{4}$$

where the water vapor saturation pressure correlation P_{SAT} can be found in (ASHRAE, 1993). The term t_{cl} can be computed iteratively by the following equation:

$$
\begin{aligned}
t_{cl} &= 35.7 - 0.032M - 0.18 I_{cl}(3.4 f_{cl}\times \\
&\quad ((t_{cl}+273)^4 - (t_{rm+273})^4) + f_{cl} h_c (t_{cl} - t_{bs})
\end{aligned}
\tag{5}
$$

Therefore, combining equations (1) to (5), the PMV index can be written as a function of four environmental variables (temperature: t_{bs}, relative humidity: ϕ, mean radiant temperature: t_{rm} and air velocity: v) and two individual parameters (metabolic rate: M and cloth index: I_{cl}), as follows:

$$PMV = \mathcal{G}(t_{bs}, \phi, t_{rm}, v, M, I_{cl}) \tag{6}$$

Table 1 shows the relationship among PMV and thermal sensation. In 1994, this formulae was included in ISO Standard 7730 and a PMV-based criterion has been established between -0.5 and $+0.5$ as acceptable for thermal comfort in air-conditioned environments. In this Table, PPD means Predicted Percentage of Dissatisfied and is an indication of the percentage of people who could complain about the thermal quality of a given indoor environment.

Table 1. Relationship between PMV, PPD and thermal sensation.

PMV	Thermal sensation	PPD (%)
+3	Hot	100
+2	Warm	75
+1	Slightly warm	25
0	Neutral	5
-1	Slightly cool	25
-2	Cool	75
-3	Cold	100

3. MODEL PREDICTIVE CONTROL LAWS FOR THERMAL COMFORT

Model predictive controllers are defined by the following steps: first, a model is used to compute the predicted process output. Then, a cost function describes the closed-loop performance of the system and then this cost function is minimized in relation to the future control signals. Finally, the first of these control signals is applied to the process (receding horizon strategy). Several MBPC algorithms have been proposed based on these scheme and the main difference between them is the strategy used in each step described above (Clarke, 1994).

In this section, three MPC algorithms are proposed. They are characterized by two main points: *i)* The process model is SIMO, *i.e.*, the manipulated variable is the input provided to the HVAC device (a heater) and the controlled variables are temperature and relative humidity; *ii)* the cost function and minimization problem are closely related to the system performance in terms of thermal comfort.

The process model is assumed to be described by state-space equations as follows:

$$
\begin{cases}
\boldsymbol{x}(k+1) = A\,\boldsymbol{x}(k) + B\,u(k) \\
\boldsymbol{y}(k) \quad = C^T \boldsymbol{x}(k)
\end{cases}
\tag{7}
$$

where $\boldsymbol{x}(k)$ is the state vector (dimension n), $u(k)$ is the control signal sent to HVAC (i.e., the HVAC dynamics are included in the model), $\boldsymbol{y}(k) = [\ y_T(k)\ \ y_H(k)\ ]^T$ and is the output signal vector the triple (A, B, C) describes the process dynamics. By using this model, the j-step-ahead prediction equation can be derived, as it is usual in state space based MPC law. So, $\hat{\boldsymbol{y}}(k+j|k) = [\ \hat{y}_T(k+j|k)\ \ \hat{y}_U(k+j|k)\ ]^T$ is the output prediction vector at instant $k+j$, made at time k. The PMV prediction equation is made by using equation (6), when the air velocity and the individual parameters are considered constant and $t_{rm} = t_{bs}$, as follows:

$$
\begin{aligned}
\hat{y}_{PMV}(k+j/k) = \mathcal{G}(&\hat{y}_T(k+j|k), \hat{y}_H(k+j|k), \\
&\hat{y}_T(k+j|k), v, M, I_{cl})
\end{aligned}
\tag{8}
$$

The assumption $t_{rm} = t_{bs}$ is equivalent to assume that the indoor walls mean temperature are equal to the indoor air temperature.

3.1 Algorithm based on setting temperature signal boundaries and on relative humidity optimization

In this section, a control strategy is described assuming that the building occupants thermal comfort sensation is given by hard bounds on the internal temperature and a set-point for the internal relative humidity. The objective is to find the optimal value for the relative humidity signal by guaranteeing that the temperature signal lies inside limits, since this latter signal is the most relevant on the thermal sensation computation (see Section (2)). The bounds value and the set-point are defined by means of the comfort zone within a psychrometric chart.

Therefore, the control law is given by the following optimization problem:

$$\begin{aligned} \Delta u(k) = & \min_{\Delta u(k|k), \Delta u(k+1|k), \ldots, \Delta u(k+N_u-1|k)} J_k \\ & \text{s.to} \\ & \Delta u(k+j|k) = 0 \quad \forall\, j = N_u, \ldots, N_y \\ & 0 \leq u(k+j) \leq u_{max} \quad \forall\, j = 1, \ldots, N_u \\ & w_{T,min} \leq \hat{y}_T(k+j|k) \leq w_{T,max} \quad \forall\, j = N_1, \ldots, N_y \end{aligned} \tag{9}$$

where N_1 and N_y define the prediction horizon and N_u is control horizon. w_U is the set-point on the relative humidity and the interval $[w_{T,min}, w_{T,max}]$ defines the bounds for temperature. $\Delta u(k + j|k)$ is the optimal control signal variation at time $k + j$ computed at time k and u_{max} is the maximum control signal which is stipulated by the heating system. The cost function J_k is given by:

$$J_k = \sum_{j=N_1}^{N_y} (\ \hat{y}_U(k+j|k) - w_U\ )^2 \tag{10}$$

The optimal solution of (9), given by means of N_u future values for the control signal variation, is obtained by using a quadratic programming algorithm. The signal $u(k)$ applied to the process at the time instant k is obtained as: $u(k) = \Delta u(k|k) + u(k-1)$.

3.2 Algorithm based on temperature and relative humidity optimization

Now, the second control strategy is described. The building occupants thermal comfort sensation is given by defining a trade-off between the internal temperature and relative humidity errors in relation to pre-defined set-point values. The set-points for these signals are defined as the center of a comfort region within a psychrometric chart.

Therefore, the control law is given by the following optimization problem:

$$\begin{aligned} \Delta u(k) = & \min_{\Delta u(k|k), \Delta u(k+1|k), \ldots, \Delta u(k+N_u-1|k)} J_k \\ & \text{s.to} \\ & \Delta u(k+j|k) = 0 \quad \forall\, j = N_u, \ldots, N_y \\ & 0 \leq u(k+j) \leq u_{max} \quad \forall\, j = 1, \ldots, N_u \end{aligned} \tag{11}$$

In Equation 11, the cost function J_k is given by:

$$J_k = \sum_{j=N_1}^{N_y} (\ \hat{y}_T(k+j|k) - w_T\ )^2 + \rho(\ \hat{y}_U(k+j|k) - w_U\ )^2 \tag{12}$$

where ρ defines the trade-off between the temperature and relative humidity errors. It stipulates the importance given to these two variables on the thermal comfort. As before, problem (11) defines a quadratic programming problem and the control signal applied to the process is obtained as: $u(k) = \Delta u(k|k) + u(k-1)$.

3.3 Algorithm based on PMV optimization

In following, the third control strategy is described. The building occupants thermal comfort sensation is given by PMV calculation. As discussed in section 2, the closer to zero is the PMV value the better is the thermal sensation.

Therefore, the control law is given by the optimization problem presented on Equation 12, but the cost function J_k is given by:

$$J_k = \sum_{j=N_1}^{N_y} (\ \hat{y}_{PMV}(k+j|k))\ )^2 \tag{13}$$

This cost function defines a non-linear optimization programming. Similar to the previous cases, the optimal solution of (11) provides a set of optimal future variations of the control signal and the signal $u(k)$ applied to the process is computed as in the two previous cases.

4. SIMULATION EXAMPLES

In this section, the thermal comfort control system performance is analyzed with simulation examples. The problem is to heat up an indoor environment in order to keep the internal temperature and relative humidity in such a level that promotes a thermal comfort sensation. In this way, the building properties and model are described below and them the closed-loop performance of the three proposed controllers are discussed.

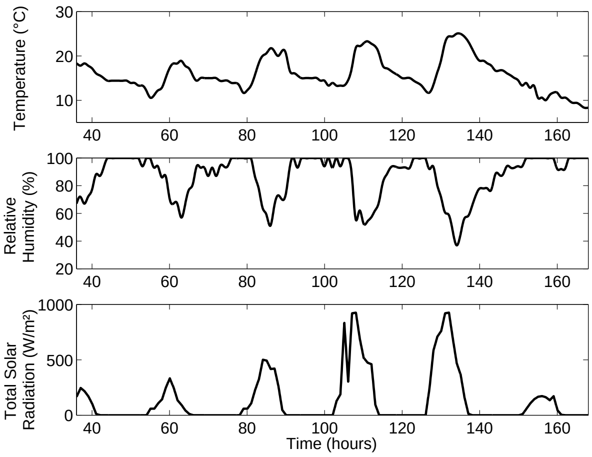

Fig. 2. External temperature, relative humidity and total solar radiation for the simulation period in Curitiba - Brazil.

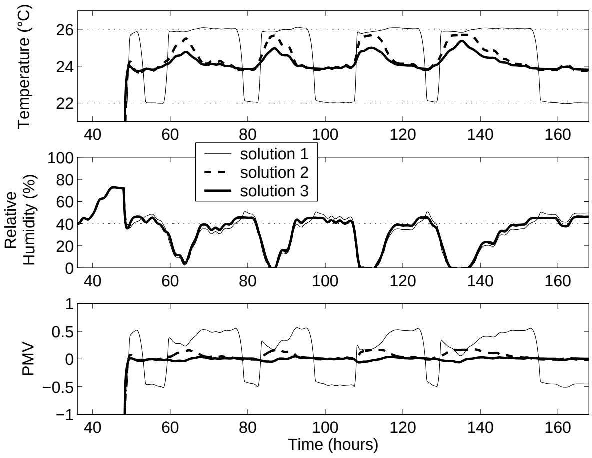

Fig. 3. Temperature, relative Humidity and PMV evolution.

The room dimensions are $5.4m \times 3.25m \times 3.00m$ for length, width and height, respectively. It is located in a building in such a way that the external surface is a $16.2m^2$ wall. Three walls divide the room with the rest of the building. The model used here is based on the one presented in (Virk and Loveday, 1994), which was obtained by means of a parametrical identification using environmental actual data. Its discrete-time transfer functions are (sampling time equal to 5 minutes):

$$\begin{gathered}(1 - 1.61z^{-1} + 0.64z^{-2} - 0.02z^{-3})y_T(z) = \\ (-0.002z^{-1} + 0.003z^{-2})\, y_U(z) \\ (+0.22z^{-1} + 0.068z^{-2} - 0.26z^{-3})\, u_T(z) \\ (+0.04z^{-1})\, T_l(z) \\ (+0.6z^{-1})\, T_0(z) + (+0.15z^{-1})\, S(z)\end{gathered} \tag{14}$$

$$\begin{gathered}(1 - 1.54z^{-1} + 0.4007z^{-2} + 0.171z^{-3})y_U(z) = \\ (-0.037z^{-1} + 0.003z^{-2})\, y_T(z) \\ (-0.71z^{-1} + 0.37z^{-2} + 0.29^{-3})\, u_T(z) \\ (-0.005z^{-1})\, T_l(z) + (-0.005z^{-1})\, T_0(z) \\ (+0.026z^{-1})\, U_0(z) + (-0.1486 \cdot 10^{-3} z^{-1})\, S(z)\end{gathered} \tag{15}$$

where the units for temperature, relative humidity and heating power are ${}^{O}C$, % and kW, respectively. The external weather parameters are $T_o(k)$, $S(k)$ and $U_o(k)$, which are the external temperature, total solar radiation (in W/m^2) and external relative humidity, respectively. $T_l(k)$ is the temperature in the room next to the one under analysis ($T_l(k) = 15^{O}C$). The TRY (Test Reference Year) weather data of Curitiba/Brazil (latitude -25.4^{O}) are used, representing the first 7 days of July. These data can be viewed in Figure 2. Now some closed-loop results are presented. In all cases, the controller is turned on at the 0-th hour of the third day (time equal to 48 hours) and the control proposals are named here as Solution 1, 2 and 3 for the algorithm presented in Sections (3.1), (3.2) and (3.3), respectively. In Solution 1, the interval $[w_{T,min}, w_{T,max}]$ is given by $24 \pm 2^{O}C$ and $w_U = 40\%$ (see Figure (1)). In Solution 2, $w_T = 24^{O}C$ and $w_U = 40\%$. $\rho = 0.005$, that is, a higher weighting factor on the temperature signal. In Solution 3, the constant parameters of the PMV formulae are: $v = 0.1\ m/s$, $I_{cl} = 0.66\ clo$ and $M = 69.78\ W/m^2$; representing an office environment. The controller parameters in the three cases are: $N_1 = N_y = 3$ and $N_u = 1$.

Figure 3 shows the internal temperature and relative humidity for the three solutions presented in Section 3. It can be noticed that, as for the Solution 1, the temperature signal varies within the pre-defined bound in order to optimize the internal humidity error in relation to the set-point value. In Solution 2 case, the weighting factor ρ is privileging the temperature errors, so this signal tends to be closer to the set-point than the relative humidity one. Figure 3 also illustrates the PMV evolution during the simulation period. It can be noticed that Solution 3 is the most efficient in leading the thermal comfort as close as possible to the ideal point, $i.e.$, $PMV = 0$. However, the three solutions are able to keep the thermal comfort inside the admissible bound, that is, $PMV \in [-0.5, 0.5]$. Although the performance of all solutions are quite good in terms of thermal comfort, such a concept is closely related with the occupants activity and/or clothing characteristics. For instance, assume the closed-loop control situation described in Figure (3) for Solution 3. If two people having different metabolic rates are inside the same environment, a seated person ($M = 69.78\ W/m^2$) and a walking-around person ($M = 119.79\ W/m^2$), the seated one will feel a neutral thermal sensation (PMV close to 0) since the controller have been tuned for such a case. However, a walking one will experiment a slightly warm sensation according to Table 1 (PMV close to 1). This means that a temperature of $24^{O}C$ may not be adequate for a walking person. To highlight the properties of the Solution 3 controller, the

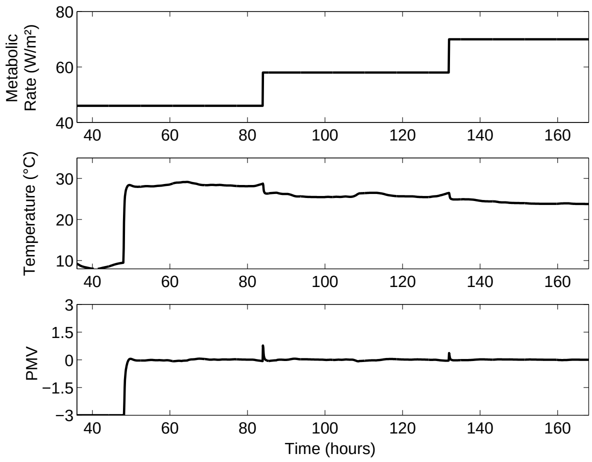

Fig. 4. Temperature evolution and PMV for different metabolic rate situations.

following case is presented. Assume that the occupants activities varies in time, the same for their metabolic rates that moves from 46 to 70 as shown in Figure 4. These is equivalent to be lie down, sit-down and stand-up. The indoor PMV behavior and temperature signal during the time, when the Solution 3 control law feeded with an adaptive adjustment of the metabolic rate (M) parameter is used, are also presented in Figure 4. It can be notice that the PMV index is always close to zero by varying the internal temperature as a function of the occupants activities profile. All these results highlight the difference between the PMV-based MPC (Solution 3) and an ordinary temperature control for promoting thermal comfort.

5. CONCLUSIONS

In this paper, indoor thermal comfort control problem in buildings equipped with HVAC (Heating Ventilation and Air Conditioning) systems have been addressed. Three MPC algorithms to optimize room air conditions focused on thermal comfort by using only-one-actuator system associated to heating equipment have been presented.

The first and second proposed control algorithms were based on defining thermal comfort as a comfort zone within a psychrometric chart. These algorithms compute the optimal control signal in such a way that both temperature and relative humidity are considered in the control law. This characteristic represents the main difference of them in relation to ordinary temperature control, since a SIMO system is defined by considering an important signal as far as thermal comfort is concern. The third proposed control algorithm was based on defining thermal comfort by using the PMV index. A control signal is computed to optimize the PMV index and the algorithm can be tuned in function of the building occupants activities and cloths. The problem of measuring some of the PMV parameters for control purposes (eg, metabolic rate) is still an one issue in the field. However, they can be estimated by means of the occupants profile and this point will be discussed in future works.

Simulation results, by using the Curitiba/BR weather data file, have shown that the three proposals are able to promote thermal comfort in a indoor environment. It have been also shown that the PMV-based model predictive control can be easily set to track optimal thermal comfort indices for different profiles of the individual parameters.

REFERENCES

ASHRAE (1993). *ASHRAE Fundamentals*. American Society of Heating, Refrigerating and Air Conditioning Engineers. Atlanta, GA.

Astrom, K. J., H. Hagglund and A. Wallenborg (1993). Automatic tuning of digital controllers with applications to HVAC plants. *Automatica* **29**(5), 1333–1343.

Clarke, D. W., Ed.) (1994). *Advances in Model Based Predictive Control*. Oxford Univ. Press.

Dumur, D., P. Boucher, K. M. Murphy and F. Déqué (1997). Comfort control in residential housing using predicitve controllers. In: *Proc of the IEEE CCA*. USA.

Fanger, P. O. (1974). *Thermal Comfort*. McGraw-Hill Inc.. New York, USA.

Gouda, M. M., S. Danaher and C. P. Underwood (2001). Thermal comfort based fuzzy logic controller. *Building Serv. Eng. Res. Technol.* **22**(4), 237–253.

Hamdi, M. and G. Lachiver (1998). A fuzzy control system based on the human sensation of thermal comfort. In: *Proc. of IEEE Conf. on Fuzzy Systems*. USA. pp. 487–492.

Kolokotsa, D., D Tsiavos, G. S. Stavrakakis, K. Lalaitzakis and E. Antonidakis (2001). Advanced fuzzy logic controllers design and evaluation for buildings occupants thermal-visual comfort and indor air quality satisfaction. *Energy and buildings* **33**, 531–543.

Oliveira, G. H. C., L. S. Coelho and N. Mendes (2003). Using fuzzy logic in heating control systems. In: *Proc. of the 6-th ASME-JSME Thermal Engineering Joint Conf.* USA.

Virk, G. S. and D. L. Loveday (1994). Model-based control for HVAC aplications. In: *Proc. of IEEE CCA*. Glasgow. pp. 1861–1866.

Yonezawa, K., F. Yamada, Y. Wada and Y. Hanada (2000). Comfort air-conditioning control for building energy-saving. In: *Proc. of the Industrial Electronics Society. 26th IEEE Annual Conf.* Vol. 3. pp. 1737–1742.