FAULT DETECTION BASED ON PROBABILISTIC ROBUSTNESS TECHNIQUES FOR BELT CONVEYOR SYSTEMS

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Abstract: In this paper, problems and application of the computation of false alarm rate (FAR) and threshold for belt conveyor systems are studied. Based on an information system which is developed to meet the requirements on monitoring and fault detection for large scale belt conveyor systems, the probability distribution of model uncertainty is assumed to be known and will be taken into account for the design of the fault detection system . The use of the probabilistic information will get a less conservative result, compared with the worst case handling of the system uncertainties. The solution and the application on the belt conveyor system will be illustrated.

Keywords: Modelling; Simulation; Fault detection; Probabilities integration; Uncertain linear systems

1. INTRODUCTION AND PROBLEM FORMULATION

The operations of belt conveyors are faced with increasing requirements on quality and productivity under the presence of many disturbances and strong parameter changes caused by the industrial surroundings, where the belt conveyors are usually in operation. For transporting high mass flows over long distances belt conveyors are widely used in mining and on large-scale building-sites. In order to reduce maintenance costs and provide a high availability of such complex devices, it is necessary to achieve the optimal operating efficiency and simultaneously ensure a high level of safety. The existing approaches to the data analysis for detecting system faults are restricted to static calculation and testing methods.

Attention of this paper is focused on the design of an expert system for monitoring and fault detection by applying the probabilistic robustness theory to the determination of thresholds and their integration in the design of the observer based fault detection system. The core of this system is based on the application of advanced model-based monitoring techniques. The rapid development in computer technology, control engineering and signal processing offers us advanced methods and technologies to solve such problems, which the model-based approaches to system simulation, monitoring and fault diagnosis are the most powerful tools (Frank and Ding, 1997),(Isermann, 1993).

The established norm-based residual evaluation allows the systematic calculation of threshold using the robust control theory. The norm-based threshold calculation covers all possible changes in the residual vector caused by the model uncertainty and unknown inputs. This solution is often achieved at the cost of a high threshold which may increase the number of undetectable faults. Different from the norm based residual evaluation methods, in which the worst case handling of model uncertainty and disturbances are adopted, the approach proposed here gives less conservative problem solutions in the probabilistic framework (Calafiore, 2002),(Calafiore, 2000). The problems related to the computation of false alarm rate (FAR) and thresholds, which are often dealing with the design of observer based fault detection systems are presented. The application of the probabilistic robustness technique for the purposes of computing thresholds and FAR are studied.

The background of this work is a R&D project initiated by the companies PC-SOFT GmbH and Vattenfall Europe AG, whose objective is to meet the requirements represented. There are demands on

- optimize the design and construction of the belt conveyor,
- on-line monitoring aiming at identifying the changes of operation parameters of the belt conveyor in operation and
- expert system for the detection of faults in the belt conveyor during the operation to adapt abrasion maintenance schemes

are continuously increasing during the recent years.

2. SYSTEM DESCRIPTION AND OBSERVER

The information system consists of a residual generator and a residual evaluator which are developed on the basis of the observer and used for the purpose of fault detection and diagnosis.

In order to model the technical-physical structure, the kinetics and the dynamics of a belt conveyor, the entire plant is generally partitioned into the following three subsystems: driving station, conveyor road and reversing station (Sader, 2004). The overall model of the belt conveyor system used is described in (Jeinsch *et al.*, 2002) and (Sader, 2004). The variables

- torque (engine moment of the driving motors $M(t)$),
- mass flow $Q(t)$,

are used as model inputs. As model outputs, simulation and estimations

- the speed $n(t)$ of the driving motor
- the belt tension $T_{sp}(t)$
- the driving force $F_{ant}(t)$
- the acceleration of the conveyor belt $a_i(t)$
- the distance $s_i(t)$ of the i-th section
- the resulting belt tension relationship $T_i(t)$ and
- the mass flow $q_i(t)$ over the entire belt conveyor.

are delivered.

To model the dynamic behavior of the whole belt conveyor, the steel cable belt is first divided into K sections with an identical length L_o, and each of them is then modelled as a spring-mass-damper system, since the stress-and extension behavior is mainly determined by the elastic characteristics of the steel cable as well as the internal material absorption (Schulz, 1995). The determination of the friction coefficient

$$f_i = c_1 \cdot k_1 \cdot v_i + c_2 \cdot k_2 \quad (1)$$

where $c1, k1, c2$ and $k2$ are constants affected from environment and technical equipment, are of special importance. To this end, a linear function of the belt velocity is assumed and the influence of the resistance to rolling, pressing in resistance and deformation resistance along a belt section i are taken into account. In (Sader, 2004) the complete dynamic model of a belt conveyor system is presented, which include the calculation of the real mass flow distribution. Mathematically, the model consists of K differential equations of second order with time-dependent coefficients and a number of algebraic equations.

Let a_i, v_i, m_i, f_i, s_i denote the acceleration, velocity of the i-th section of the conveyor belt, the mass, the friction coefficient and the distance of the mass, respectively. Then the dynamics of the i-th-section can be generally described by

$$\begin{aligned} m_i a_i = {} & k_i s_i + k_{i,v_i}(m_i) v_i + k_{i-1,s_i} s_{i-1} \\ & + k_{i+1} s_{i+1} + k_{i-1,v_i} v_{i-1} \\ & + k_{i+1,v_i} v_{i+1} + k_{i,v_i}(m_i) \end{aligned} \quad (2)$$

where $k_i, k_{i-1,s_i}, k_{i-1,v_i}$ and k_{i+1,v_i} are constants which are known, $k_{i,v_i}(m_i)$ is assumed to be a known function of m_i (Jeinsch *et al.*, 2000). Now introduce state vectors and input vector,

$$s = \begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix}, v = \begin{bmatrix} v_1 \\ \vdots \\ v_K \end{bmatrix}, u = \begin{bmatrix} M_1 \\ \vdots \\ M_p \end{bmatrix}$$

with u denoting the torque.

The major model uncertainties are

- load of the belt m_i
- kinetic resistances f_i
- environmental influence (wind, temperature,rain)
- measurement disturbances.

and the system fault states are

- operational faults of the belt,
- critical states of the drive and the reversing pulley,
- sensor faults in velocity sensors and
- faults in the driving system and the motors.

The overall system model considering the uncertainties and faults can then be expressed in terms of a state space equation

$$\begin{aligned} \dot{x}(t) &= A(m)x(t) + B(m)u(t) + E_d(m)d(t) + E_f\ f(t) \\ y(t) &= Cx(t) + F_d d(t) + F_f\ f(t) \end{aligned} \quad (3)$$

where $x \in R^n$, $u \in R^{k_u}, y \in R^m, f \in R^{k_f}$ and $d \in R^{k_d}$ denote the state, input, output and unknown input vectors respectively. We assume d and u are L_2-norm bounded

$$\|d(t)\|_2 \leq \delta_d \quad (4)$$

A, B, C, E_d and F_d are matrices with appropriate dimensions and E_f, F_f represents model uncertainties and multiplicative and additive faults in the plant. The mass and its distribution are unknown in the on-line-implementation, so we consider $m(t)$ as model uncertainty and decompose $A(m)$, $B(m)$ and $E_d(m)$ into

$$\begin{aligned} A(m) &= A + \Delta A, \quad B(m) = B + \Delta B, \\ E_d(m) &= E_d + \Delta E_d \end{aligned}$$

where A,B and E_d are constant matrices, ΔA, ΔB and ΔE represent model uncertainty due to the changes of the mass, which can be expressed by $[\Delta A \quad \Delta B \quad \Delta E_d] = E\,\Sigma(t)\,[F_1 \quad F_2 \quad F_3]$ where E, F_1, F_2 and F_3 are known matrices. Denote

$$\begin{aligned} \Omega_A &:= \left\{\Delta A \,|\, \Delta A = E\,\Sigma(t)\,F_1, \Sigma^T(t)\Sigma(t) \leq I\right\} \\ \Omega_B &:= \left\{\Delta B \,|\, \Delta B = E\,\Sigma(t)\,F_2, \Sigma^T(t)\Sigma(t) \leq I\right\} \\ \Omega_{E_d} &:= \left\{\Delta E_d \,|\, \Delta E_d = E\,\Sigma(t)\,F_3, \Sigma^T(t)\Sigma(t) \leq I\right\} \end{aligned}$$

$A + \Delta A$ is assumed to be asymptotically stable for all $\Delta A \in \Omega_A$.

For the purpose of state estimation, the observer is constructed as follows

$$\begin{aligned} \dot{\hat{x}}(t) &= (A + HC)\,\hat{x}(t) + Bu(t) - Hy(t) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned}$$

where $\hat{x} \in R^n$ and $\hat{y} \in R^n$ represent the state and output estimation vector respectively. The design parameter is the observer gain matrix H. The dynamics of the estimation error is governed by

$$\begin{aligned} \dot{e}(t) &= (A + HC)\,e(t) + \Delta Ax(t) + \Delta Bu(t) \quad (5) \\ &\quad + (E_d + \Delta E_d + HF_d)d(t) \\ z(t) &= Ce(t) + F_d d(t) \end{aligned}$$

where $e = x - \hat{x}$.

It thus becomes clear that the objective of selecting H is to make the influence of $d(t)$ on the estimation error as small as possible. To this end, we can use the well-established robust observer theory like H_∞-robust observer or observer design using μ-synthesis (Zhou, 1998), (Sader, 2004).

3. DESIGN OF OBSERVER BASED FAULT DETECTION SYSTEM

In this section, we present the design of observer based fault detection system applied at the above-described large scale belt conveyor system with model uncertainties. The FD system consists of a residual generator and a residual evaluation stage including an evaluation function and a threshold (Chen and Patton, 1999),(Gertler, 1998),(Frank and Ding, 1997).

3.1 Residual generator design

For the purpose of residual generation, observer-based fault detection system of the following form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + H\,(y(t) - \hat{y}(t)) \quad (6)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (7)$$

$$r(s) = R(s)\,(y(s) - \hat{y}(s)) \quad (8)$$

are considered, where r is the residual vector and the design parameters are the observer gain H and $R(s) \in RH_\infty$ is the so-called post-filter which is an arbitrarily selectable parametrization matrix (Frank and Ding, 1997). It can be derived that the dynamics of the residual generator (8) is governed by

$$r(s) = R(s)M_u(s)\left(G_{\bar{d}}(s)\bar{d}(s) + G_f(s)f(s)\right) \quad (9)$$

with

$$M_u(s) = I + C(sI - A - HC)^{-1}H$$

$$\begin{aligned} G_{\bar{d}}(s) &= C\,(sI - A)^{-1}\,\bar{E}_d + \bar{F}_d \\ G_f(s) &= C\,(sI - A - \Delta A)^{-1}\,E_f + F_f \end{aligned}$$

$$\bar{E}_d = [E \quad E_d]\,, \quad \bar{F}_d = [0 \quad F_d] \text{ and } \bar{d} = \begin{bmatrix} \Sigma\varphi_\Delta \\ d \end{bmatrix}$$

where $\Sigma\varphi_\Delta$ is a unknown input vector considering the model uncertainties (Sader, 2004). The solution of this problem provide the optimal parameter of the fault detection filter (Ding *et al.*, 2000).To evaluate the residual, the 2-norm of the residual signal r is used as the evaluation function and the decision logic is the mostly used on

$$\begin{aligned} \|r\|_2 > J_{th} &\Longrightarrow fault \\ \|r\|_2 \leq J_{th} &\Longrightarrow no\ fault \end{aligned} \quad (10)$$

The main objective of designing residual generators is to improve the sensitivity of the FD system to faults without loss of the robustness to disturbances. Thus the selection of the design parameters H and $R(s)$ can be formulated as an optimization problem

$$\min_{R(s),H} \frac{\left\|R(s)M_u(s)\bar{G}_{zd}(s)\right\|_\infty}{\sigma_i(R(s)M_u(s)\bar{G}_{zf}(s))} \quad (11)$$

where $\sigma_i(R(s)M_u(s)\bar{G}_{zf}(s))$ denotes some nonzero singular value of $R(s)M_u(s)\bar{G}_{zf}(s)$. The determination of adaptive threshold considering the design of the residual generator is to improve the results on fault detection and isolation.(Jeinsch, 2003)

3.2 Probabilistic approach to the threshold selection

Different from the norm-based residual evaluation methods, where J_{th} is the threshold selected as

$$J_{th} = \sup_{d,f=0} \|r\|_2 = \left\|R(s)M_u(s)\bar{G}_{zd}(s)\right\|_\infty \Delta_d \quad (12)$$

and the threshold determination is based on the worst-case handling of model uncertainty and unknown inputs, the probabilistic robustness theory is applied for the purpose of calculating thresholds and false alarm rate. This approaches will lead to a more practical design of observer based fault detection systems. Applying the norm based residual evaluation methods, the threshold J_{th} should cover all possible changes in the residual vector caused by the model uncertainty and unknown inputs. It is evident that threshold setting in the way of (12) ensures no false alarms, but causes a high threshold which may result in that the number of undetectable faults, the miss detection rate (MDR) becomes very large. This problem can also be formulated from the probabilistic viewpoint. The FAR is defined as the conditional probability that $\|r\|$ is larger than the threshold in the fault free case.

$$FAR = \Pr\{\|r\| > J_{th} \quad | f = 0\} \quad (13)$$

and the threshold be $J_{th} = \sup_{d,f=0} \|r\|$, where $f = 0$ indicate the fault-free case, then we have

$$\Pr\{\|r\| \leq J_{th} \quad | f = 0\} = 1 \Rightarrow$$

$$\begin{aligned} FAR &= \Pr\{\|r\| \geq J_{th} \quad | f = 0\} \\ &= 1 - \Pr\{\|r\| \leq J_{th} \quad | f = 0\} = 0 \end{aligned}$$

there $\Pr\{\alpha \leq \beta\}$denotes the probability of $\alpha \leq \beta$. We now set the threshold J_{th} smaller than $\sup_{d,f=0} \|r\|$, it turns out

$$\begin{aligned} FAR &= \Pr\{\|r\| > J_{th} \quad | f = 0\} \\ &= 1 - \Pr\{\|r\| \leq J_{th} \quad | f = 0\} \end{aligned}$$

If $\Pr\{\|r\| \leq J_{th} \quad | f = 0\} < 1$,then FAR will be larger than zero. Decreasing the threshold leads to increasing the number of detectable faults. For this reason, it is in practice often the case that a compromise between the FAR and the MDR is made. The idea of this study is to make use of the knowledge of random matrix Σ (i.e. its probability distribution) to establish a relationship between the FAR and the threshold and to achieve a suitable determination of the threshold for the information system of the belt conveyor.

3.3 Detailed problem solution for belt conveyor

For the application on belt conveyor systems the following problem of estimation of the FAR if $J_{th} \leq \sup_{d,f=0} \|r\|$ is studied. Furthermore we consider uniform and gaussian distribution, which are fitted from real measurement data , for study the relationship between the FAR and the threshold to improve the FD system on belt conveyors using practical knowledge of random matrix Σ.

In this note, the L_2-norm based residual evaluation defined by

$$J := \|r\|_2 = \left(\int_0^\infty r^T(t)\, r(t)\, dt\right)^{1/2} \quad (14)$$

is considered and the H_∞ system norm is defined as follows: Given LTI system $\zeta = \Psi\varpi, \|\Psi\|_\infty = \sup\{\|\zeta\|_2 : \|\varpi\|_2 \leq 1\}$(Scherer *et al.*, 1997),(Boyd *et al.*, 1994).

Due to the limitation of this note we will show first results of this idea taking into consideration the model uncertainty m and $d = 0$. To solve the formulated problem it follows from (Ding *et al.*, 2003) and (13) that

$$\begin{aligned} FAR &= 1 - \Pr\{J \leq J_{th} \mid f = 0\} \leq 1 - \vartheta \\ \vartheta &= \Pr\left\{\sup_u \|\varphi_u\|_2 \leq J_{th}/\|R_1\|_\infty\right\} \end{aligned} \quad (15)$$

with

$$\sup_u \|\varphi_u\|_2 = \|\Psi_u\|_\infty \|u\|_2 \quad (16)$$

$$\begin{aligned} \Psi_u &: \dot{x}_u = (A + \Delta A)x_u + (B + \Delta B)u, \\ \varphi_u &= \Sigma(F_1 x_u + F_2 u) \end{aligned}$$

where φ_u is the output of system Ψ_u and R_1 is the transferfunction from φ_u to the residual $r(s)$(Ding *et al.*, 2003). The estimating of FAR is now reduced to find an estimation for ϑ for an adjusted threshold J_{th}. To solve this, the computation of the probability that the inequality related to the random matrix Σ holds (15) is required. We consider according to (15) the inequality

$$\sup_u \|\varphi_u\|_2 \leq J_{th} / \|R_1\|_\infty \tag{17}$$

where the right side of (17) is independent of the model uncertainty related to Σ. The solution of (17) is realized by computing the H_∞-norm of a system, using LMI technics which are known from the robust control theory (Scherer *et al.*, 1997), defining $\|R_1\|_\infty = \min \beta_1 := \beta_{1\,\min}$ and $\|\Psi_u\|_\infty = \min \gamma_1 := \gamma_{1\,\min}(\Sigma)$.

Our problem can be solved by the following steps:

- generation of N matrix samples $\Sigma^{j=1\dots N}$ using the chosen randomized algorithm (uniform and gaussian), where N is the number of samples satisfy

$$N \geq \frac{\log \frac{2}{\delta}}{2\epsilon^2} \tag{18}$$

  with δ describes the confidence and ϵ describes the accuracy of the estimate

$$\Pr\{|\widehat{\rho} - \rho| \leq \epsilon\} \geq 1 - \delta, \tag{19}$$
$$\rho = \Pr\{\sup_u \|\varphi_u\|_2 \leq (J_{th} / \|R_1\|_\infty\}$$

  The empirical estimation of the probability of inequality (17) is given by $\widehat{\rho} = \frac{n_\varphi}{N}$ where n_φ is the number of samples for which $\gamma_{1\,\min}(\Sigma^j)\,\|u\|_2 \leq (J_{th}/\beta_{1\,\min})$ holds.
- Use the indicator functions for given J_{th} for $j = 1, ..., N$

$$I_2(\Sigma^j) = \begin{matrix} 1, \text{ if } \gamma_{1\,\min}(\Sigma^j)\,\|u\|_2 \leq (J_{th}/\beta_{1\,\min}) \\ 0, \text{ otherwise} \end{matrix}, \tag{20}$$

- compute the empirical probability

$$\widehat{\rho}_N = \frac{1}{N}\sum_{j=1}^{N} I_2(\Sigma^j)$$

The result is an estimation for the $FAR_e = 1 - \widehat{\rho}_N$, under selected δ and ϵ. This solution is suitable for the implementation and analysis of the given fault detection system of the belt conveyor.

4. APPLICATIONS TO A REAL BELT CONVEYOR SYSTEM

In this section an example is presented where the solution of a probabilistic approach to estimate the FAR of an observer based fault detection system is applied on a real belt conveyor system

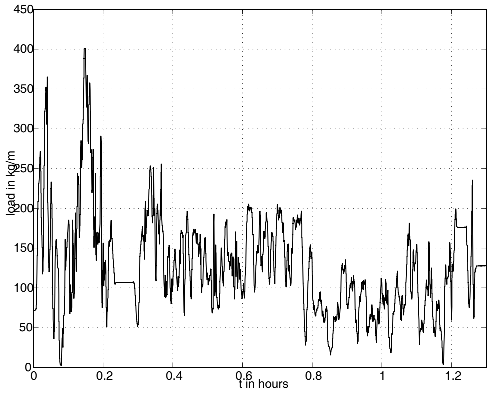

Fig. 1. Measured date of a real massflow

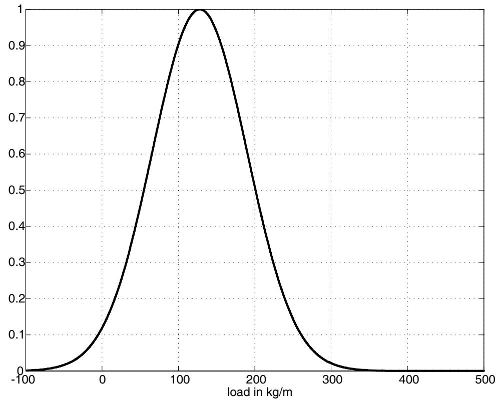

Fig. 2. Gaussian distribution of the massflow

(Sader, 2004). We consider two cases of different probability distribution f_Σ of Σ.

- case 1, assumption that Σ is uniform distributed,
- case 2, use knowledge about the distribution of the real massflow, where a gaussian distribution is assumed and μ and σ are analyzed by real measured data (Fig. 1).The computed parameter of the gaussian distribution are $\mu = 128$ and $\sigma = 62$ (Fig. 2).

The sample size $N \geq 10000$ follows from (18) for $\delta = 0.02, \epsilon = 0.01$. The table shows the FAR for case 1 and 2. Applying the gaussian distribution approach of the load, the FAR_e for a given J_{th} is smaller then in case 1. For the practical application that means a lower threshold can be chosen on an acceptable level of FAR. Also the MDR will decrease and a higher number of faultdetections are feasible. That means on the one hand a loss of accuracy of the FD system, but on the other hand the information about small failures and faulty states of our system increases. An other important application is to compare a system with a similar one. Focused on the fault detection and the derivation of states of maintenance, the presented results will support the maintenance schemes and the engineering of such complex systems.

Treshold J	case 1 FAR_e *in* %	case 2 FAR_e *in* %
0.04	81.6	81.2
0.06	71.2	65.2
0.08	60.8	46.2
0.1	49.6	26.1
0.14	24.3	2.1
0.18	19.9	0.4
0.2	18.1	0.4
0.3	11.0	0
0.5	1.0	0
0.9	0	0
1.0	0	0

5. CONCLUSION

In this paper, the problems and the application of estimation of false alarm rate in relation to the determination of thresholds are presented. The results demonstrate that the problem solution in the probabilistic framework and the computation of FAR offers a more practical application of the fault detection system. Especially if the statistical knowledge of the model uncertainty is available, the FAR can adjusted to an acceptable level while the threshold is very low. The number of detectable faults of the information system based on the probabilistic robustness theory is much higher than in the worst case handling of the system uncertainty. The on-line implementation and the required on-line computations for the observer are at an acceptable level so that they do not cause any trouble for a successful industrial application.

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